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REINFORCED CONCRETE CONSTITUTIVE
RELATIONS

S. Adham, et al

Agbabian Associates

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REINFORCED CONCRETE CONSTITUTIVE RELATIONS

S. Adham, A. Bhaumik, J. Isenberg

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El Segundo, CA 90245

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ABSTRACT (cont'd)

obtained under uniaxial, biaxial and triaxial states of stress. Additional laboratory experiments were performed under this contract to investigate the bond-slip relation in tension and in compression. The model stress/strain relation in each principal stress direction is affected by the other two principal stresses. The form of the stress/strain relation and the manner in which it is affected by the lateral principal stresses is different in tension and compression. Until cracking occurs, properties of the model depend on the entire stiffness of steel and concrete. After cracking, a composite modulus is used which reflects the combined stiffness of steel and concrete and takes into account the extent to which bond between steel and concrete is broken. This model is incorporated in a dynamic inelastic finite-element code and is used to analyze the response of reinforced concrete members such as beams, deep beams, beam columns, and ring beams, subjected to static and dynamic loading. The results of analysis are in general agreement with available experimental observations. Recommendations for improvement in the material model are made as a result of the correlation studies.

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LIST OF SYMBOLS

A, B	Transformation matrices (in Eq. 91 ff.)
\bar{A}	Coefficient of dowel action
A_1, A_2, \dots, A_6	Coefficients used in incremental stress-strain relations (Ref. 51)
A_s	Cross sectional area of steel
$A_{s_I}, A_{s_{II}}$	Areas of steel reinforcement in concrete (Ref. 1)
A_i^c	Effective concrete ratio in the i-direction
A_i^s	Effective steel ratio in the i-direction
C	Matrix for stress/strain relations
C_{ijkl}	Elastic stress/strain relationships
D	Intact bonded length of concrete after cracking (Ref. 1)
D	Poisson's ratio multiplier
E	Tangent modulus in uniaxial loading
E_1, E_2	Tangent moduli in direction 1, 2
E'_{1b}	Initial tangent modulus in biaxial compression
E_c	Initial tangent modulus for concrete
E_i, E_j	Tangent moduli in direction i, j
E_s	Young's modulus of steel
E_s	Secant modulus of concrete at ultimate load (Ref. 53)
\bar{E}_i	Effective composite modulus
E_{oc}	Initial tangent modulus for concrete in unconfined compression

LIST OF SYMBOLS (continued)

E_{os}	Elastic Young's modulus of steel bars (Ref. 1)
E_{pc}	Tangent modulus for concrete in the inelastic range
E_{ps}	Plastic Young's modulus of steel bars (Ref. 1)
E_{sc}	Secant modulus based on ultimate stress and corresponding strain
E_i^c	Effective concrete modulus
F_1, F_2	Yield surface functions (ref. 51)
G	Dowel shear modulus
G_{12}	Composite shear modulus (Ref. 1)
G_s	Shear modulus of steel
G_{ij}	Biaxial shear modulus of concrete
H	Axial force
I_2^p	Second invariant of the plastic-strain deviator (Ref. 51)
J_1	First invariant of stress (Ref. 51)
J_2^i	Second invariant of stress deviator (Ref. 51)
K_1, K_2	Hardening parameters (Ref. 51)
K_h	Axial stiffness
K_v	Transverse stiffness
L	Length of specimen or element
M	Moment (Ref. 58)

LIST OF SYMBOLS (continued)

N	Axial tension force of bar (Ref. 65)
N_y	Axial tensile force inducing yield in pure tension (Ref. 65)
P	Applied load
P_x, P_y	Load applied in x and y directions, respectively (Ref. 4, 20, 21)
T	Tension force (Ref. 58)
T_f	Failure load of dowel shear
V	Transverse force
V_s	Transverse shear (Ref. 58)
W	Unit weight of concrete (ACI Code 318-71)
W_{ave}	Crack width (Ref. 33)
X, Y, Z	Global coordinate for entire structure
X, Y, Z	Submatrices of C matrix (Eq. 71 - 76)
Z_c	Shear contribution of concrete
Z_s	Shear contribution of steel
a^*	Effective length of longitudinal cracked zone (Ref. 58)
d	Unbonded (cracked) length of concrete (Ref. 1)
$d\epsilon_{kl}$	Total strain increment
$d\sigma_i$	Stress increment in the i -direction
$d\sigma_{ij}$	Incremental stress
f	Maximum stress at which idealized stress-strain curves change slope

LIST OF SYMBOLS (continued)

f'_c	Unconfined compressive strength of concrete
\bar{f}_c	Experimental parameter for the unconfined compressive strength of concrete
f_o	Concrete stress (Ref. 84)
$f_o(\max)$	Max concrete stress (Ref. 84)
f_s	Yield strength of steel (Ref. 1)
f_t	Unconfined tensile strength of concrete
f'_t	Biaxial tensile strength of concrete
f_y	Yield stress of steel in a uniaxial tension test
$g_1(J_1)$	Function of J_1 (Ref. 51)
h_1, h_2	Functions of the stress and plastic strain invariants (Ref. 51)
n	Dowel coefficient of local compression of concrete (Ref. 65)
p_{xs}, p_{ys}, p_{zs}	Steel ratios in steel directions
r	Constant for plastic capped model (Ref. 51)
t	Thickness of concrete cover (Ref. 33)
x, y, z	Local coordinates for element
x_s, y_s, z_s	Steel directions
$\gamma_1 \text{ -- } \gamma_{25}$	Constants used for yield surfaces (Ref. 51)
Δ	Displacement (Ref. 58)
α	Dowel factor
$\alpha_x, \beta_x, \gamma_x$	Angles between x-axis and X-, Y-, and Z axes, respectively

LIST OF SYMBOLS (continued)

$\bar{\alpha}$	Dowel factor multiplier
β	An experimental parameter corresponding to angle of internal friction
β_i	Ratio of principal stress in the orthogonal direction to the principal stress in the direction considered
γ	Dowel constant (Ref. 65)
γ_{shear}	Dowel shear strain
δ	Angle of stirrups in degrees (Ref. 65)
ϵ_1	Concrete strain in the 1-direction
ϵ_h	Relative displacement parallel to steel reinforcement
ϵ_i	Strain in direction i
ϵ_p	Peak strain
ϵ_s	Steel strain
ϵ_v	Relative displacement perpendicular to steel
ϵ_{ij}^p	Elastic strain increments
ϵ_{vc}^p	Volumetric plastic strain
η	Slope of cracking envelope
θ	Cracking angle (extent of bond deterioration (Ref. 1))
λ	Ratio of concrete contributing to overall stiffness
λ'	Biaxial compression factor (Ref. 53)
$\bar{\lambda}$	Bond-slip parameter in compression
ν	Poisson's ratio
ν_{ij}	Poisson's ratio of concrete in biaxial directions

LIST OF SYMBOLS (concluded)

ν_{st}	Poisson's ratio for steel (Ref. 1)
ρ	Dowel action coefficient (Ref. 65)
σ_i	Concrete stress in the i-direction
σ_c	Cube strength of concrete (Ref. 65)
σ_h	Axial stress
σ_i	Effective average stress in i-direction
σ_o	Unconfined compressive strength of concrete (Ref. 53)
σ_p	Peak stress
σ_s	Stress in steel reinforcing bar
σ_v	Transverse stress
σ_y	Yield stress of steel (Ref. 65)
σ_i^c	Stress in concrete in i-direction
τ_f	Failure stress of dowel shear
ϕ	Bar size (Ref. 65)
ϕ	Inclination of steel reinforcement to global coordinates (Ref. 1)

SECTION I

INTRODUCTION

The purpose of this study was to include a number of recent advances in theoretical and experimental behavior of reinforced and plain concrete in a computer program for analyzing plain or reinforced concrete structures. The properties of reinforced concrete are represented by variable moduli that combine the stiffness of steel and concrete and can change due to (1) cracks, (2) progressive failure of bond, (3) orthotropy due to different tangent moduli in different directions, (4) general orthotropy due to inelasticity of concrete in compression and cracks in tension, (5) confining effects, and (6) inelasticity of steel.

Although a model of this type was originally developed by Isenberg and Adham in 1969 (Ref. 1), the present study aims at specific types of improvements as follows:

- Improved constitutive properties of plain concrete, especially in multiaxial compression
- Improved representation of dowel action
- Improved representation of bond slip, including review of previous work and some new experiments
- Inclusion of rebonding capability
- Development of a computer program that incorporates the constitutive properties and that can be inserted with few or no changes into a wide variety of continuum finite element and finite difference programs

The present study was carried in four primary steps as follows:

STEP 1. The purpose of this step was to refine the formulation of the material model proposed by Isenberg and Adham (Ref. 1) on the basis of experimental and analytical data that became available after this model was proposed

in 1969. The work included: (1) literature review, (2) evaluation of data that are pertinent to the improvement of the Isenberg-Adham model, (3) modification of the model based on the above pertinent data.

STEP 2. The purpose of this step was to develop a set of subroutines for the plain and reinforced concrete models developed under Step 1. The subroutines are designed to be used with finite element or finite difference codes and can handle plane stress, plane strain, axisymmetric, and three-dimensional states of stress or strain.

In the intended application a solution is obtained by the step-by-step method of integration. The subroutines calculate stress and strain at the end of the current step, based on the current strain increment. However, the capability of calculating strains from a known state of stress was also included.

STEP 3. The purpose of this step was to obtain new experimental data on bond slip and to compare the results with the assumptions contained in the mathematical model. Cylindrical specimens of concrete were each cast with one reinforcing bar cast concentrically. Strain gages were placed on the inner surface of the bar and the outer surface of the concrete. One experiment was performed on bond slip in tension, while another experiment was performed on bond slip in compression. Each experiment used two instrumented specimens and one noninstrumented specimen on which only total elongation/contraction were measured. Auxiliary tests were made on noninstrumented specimens to determine the physical properties of concrete.

STEP 4. The purpose of this step was to incorporate the computer package developed under Step 2 into an existing dynamic, inelastic, two-dimensional continuum finite element code (FEDIA), which has been developed by Agbabian Associates. The new version of FEDIA, which includes the new material package, is called FEDRC. The FEDRC code was used to analyze several reinforced concrete structures in order to demonstrate the capabilities of the material model developed under this study. Five static cases and four dynamic cases were analyzed, for a total of nine demonstration cases. Cases 1 and 2 represented static behavior of deep beams, while static cases 3, 4, and 5 were aimed at

representing the extreme condition of beam columns under static loads. The first dynamic case was a deep beam subjected to impact loading, while the second case demonstrated dynamic beam column response. The third dynamic case was aimed at demonstrating the capability of the code to model the behavior of reinforced concrete silos. The fourth dynamic case represented the behavior of reinforced concrete beams under impact loading. The results of these demonstration cases were compared with available experimental data.

The main application of the present work is to the analysis of the static or dynamic response of reinforced concrete structures. Although the examples reported are two-dimensional, the computer program is capable of analyzing three-dimensional structures.

Section II of this report is a review of current approaches to analyzing composite models of reinforced concrete.

Section III is a discussion of the recent data on parameters to construct a composite reinforced concrete model.

Section IV describes the method of analysis used in this project. The fundamental assumptions and the formulation of the model are given.

Section V includes a description of the experimental studies.

Section VI contains a detailed description of the cases studied and discussion of the results.

Sections VII and VIII provide a summary and conclusions, derived from this study, and offer recommendations.

The computer package MATPAC (Material Package), used in this study, is described in Appendix I. The procedure for setting up input for the analytical model is given in Appendix II.

Units of measurement are given throughout in both English and System International units.

SECTION II

REVIEW OF CURRENT APPROACHES USED TO FORMULATE COMPOSITE MODELS

1. BACKGROUND AND SCOPE OF REVIEW

A rigorous analysis of a reinforced concrete structure requires the idealization of the structure as a composite model. The formulation of such a model is complicated by the following factors.

- The reinforced concrete member is composed of two materials: steel and concrete.
- The stress/strain relationship for reinforced concrete is nonlinear.
- The data on confining effects and failure theory for concrete under biaxial or three-dimensional states are limited.
- The influence of creep and shrinkage may need to be accounted for.
- The inception and propagation of cracks, the associated bond-slip relationships, and change in topology* are difficult to model.
- The effect of stirrups, dowel action, and aggregate interlock at cracks on shear transfer mechanism is not clearly understood.
- The orthotropy due to reinforcement, cracks, material nonhomogeneity, and anisotropy may change the effective properties during a solution updating, thus requiring successive evaluation and transformation of composite properties.

Significant progress has recently been made in developing analytical methods to examine reinforced concrete structures. The finite element method has provided the basis for most of the new analytical methods.

There are presently two distinct types of constitutive equations that are used in finite element or finite difference models of reinforced concrete. The discrete cracking element approach, advanced by Ngo and Scordelis (Ref. 2) and

*Topography of the surface.

by Nilson (Ref. 3), uses a special linkage element to represent the bond between steel and concrete. A different approach, proposed by Isenberg and Adham (Ref. 1), develops a composite modulus from individual properties of concrete and steel, and the bond between them.

As a result of the differences in techniques used to account for composite behavior, the two approaches have developed along different lines. The discrete cracking element approach allows a much simpler model of concrete properties to be formulated because the cracks occur between finite elements rather than within them. The equivalent continuum approach requires the crack directions to be computed and stored on an element-by-element basis. The properties of the continuum elements become orthotropic as cracking and inelasticity progress.

Both the scope of application and formulation of the discrete cracking element approach have been expanded and refined by several authors (Ref. 4) to include beams and shear walls subjected to static loading. The equivalent continuum approach has been applied to cylindrical concrete silos embedded in rock through which the static equivalent of dynamic ground shock loading is applied (Ref. 5). The finite element method has provided the basis for most of the analytical solutions that use these two approaches.

2. BASIC FINITE ELEMENT MODELS USED FOR THE ANALYSIS OF REINFORCED CONCRETE STRUCTURES

The finite element method has provided an extremely powerful tool for the analysis of complex structures and continuous media (Refs. 6-11). This method has been used to solve an extensive range of elastostatic, as well as elastodynamic, problems (Refs. 11-14). The method proved successful also in the solution of elastic-plastic problems (Refs. 15-17).

A comprehensive survey of the literature on application of the finite element method to the analysis of reinforced concrete structures was made by Scordelis (Ref. 4). The following material presents a summary of finite element models based on the linkage element and continuum approaches. In addition, brief summaries are included of some pertinent applications of the finite element method which are not necessarily based on the two approaches under discussion.

a. Original Linkage--Element Model

The first finite element model for the analysis of a reinforced concrete beam was described by Ngo and Scordelis (Ref. 2). Simple beams were represented by two-dimensional triangular finite elements. A special linkage element was developed to represent bond-slip phenomena. This element, which is illustrated in figure 1, links continuum elements representing steel reinforcing to those representing concrete at discrete points. The linkage element has two degrees of freedom corresponding to relative displacement between steel and concrete along the reinforcement (ϵ_h) and perpendicular to the reinforcement (ϵ_v). The bond stress-strain relationship is given by

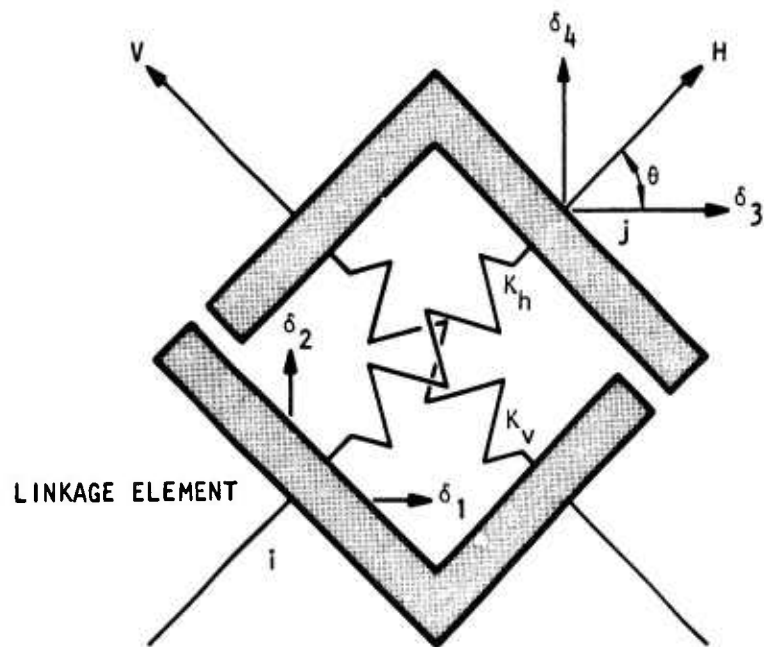
$$\begin{Bmatrix} \sigma_h \\ \sigma_v \end{Bmatrix} = \begin{bmatrix} K_h & 0 \\ 0 & K_v \end{bmatrix} \begin{Bmatrix} \epsilon_h \\ \epsilon_v \end{Bmatrix} \quad (1)$$

The axial and transverse stiffness, K_h and K_v , are prescribed in the natural coordinate system, which is parallel and perpendicular to the axis of the bar. The strains and displacements are related by the displacement transformation illustrated in figure 1. In order to add the stiffness of the linkage element to the global stiffness matrix, transformation to global coordinates is performed.

With the aid of the linkage element, it was possible to construct an analytical model for the study of reinforced concrete beams. A typical result from this analysis method is shown in figures 2 and 3. The result agrees with intuition in that stress in the steel bar is maximum at a crack, the bond force is zero at the crack and reaches maximum of opposite sign on either side of a crack.

b. Refined Linkage Element Model

Ngo, Scordelis, and Franklin (Ref. 18) extended the work of reference 2. Shear in beams with diagonal tension cracks was studied as illustrated by the example given in figures 4 and 5. The model was refined by representing the concrete and main longitudinal reinforcement by quadrilateral elements

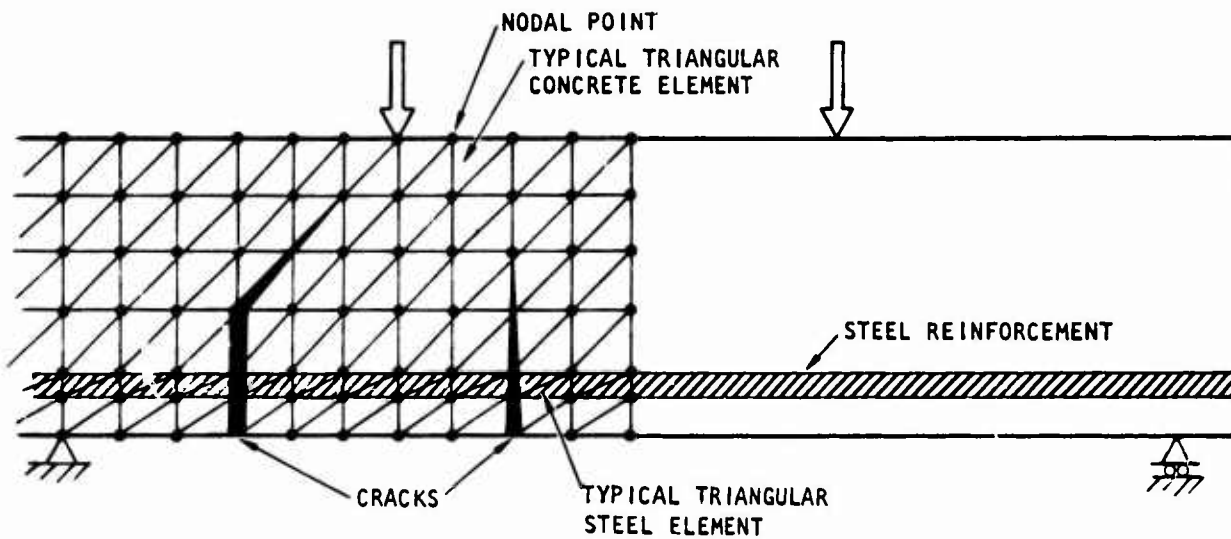


$$\begin{Bmatrix} e_h \\ e_v \end{Bmatrix} = \begin{bmatrix} -C & -S & C & S \\ S & -C & -S & C \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix}$$

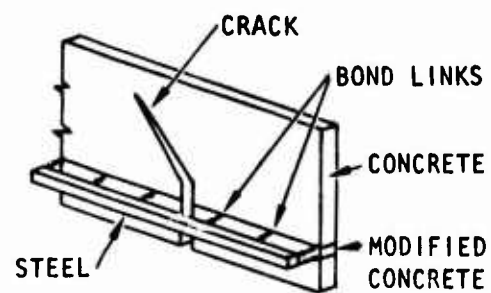
$$C = \cos \theta \quad S = \sin \theta$$

$\delta_{1,2,3,4}$ = ABSOLUTE DISPLACEMENTS, EXPRESSED IN LOCAL COORDINATES, OF NODAL POINTS OF CONTINUUM ELEMENTS TO WHICH LINKAGE IS ATTACHED

Figure 1. Linkage Element (Ref. 2)



(a) Finite element mesh



(b) Analytical model

Figure 2. Finite Element Mesh and Analytical Model (Ref. 2)

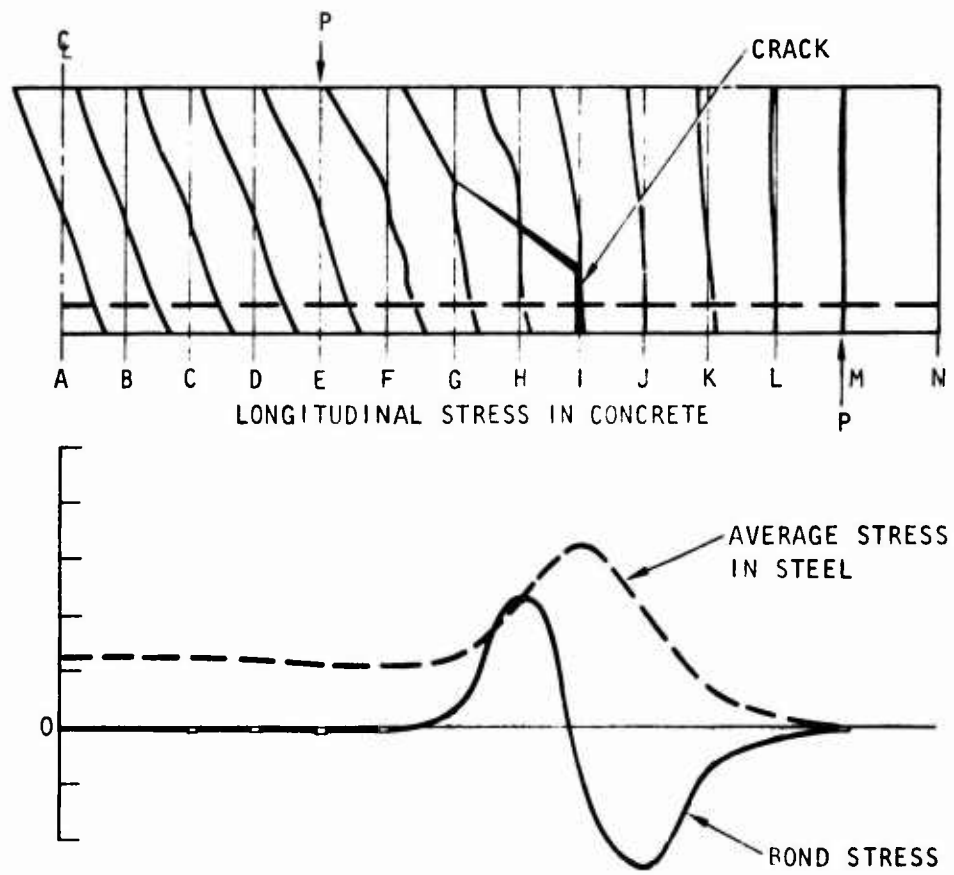
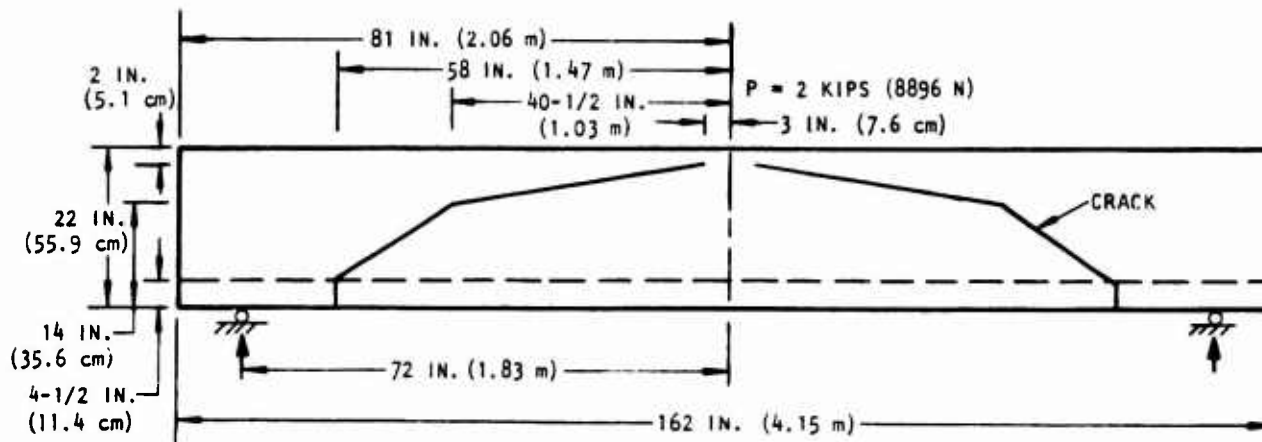
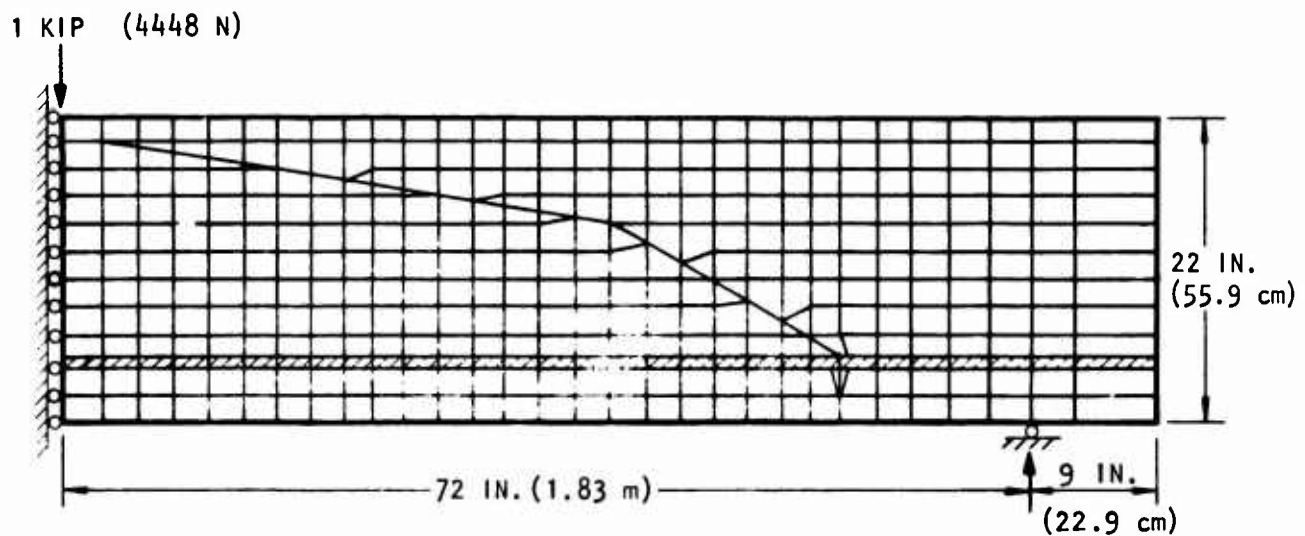


Figure 3. Variation in Bond and Steel Stresses in Beam (Ref. 2)

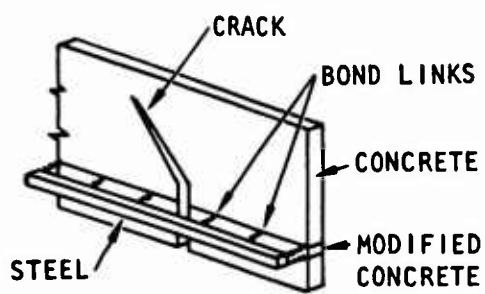


(a) Symmetrical idealized single diagonal crack
(beam width = 9 in. [22.86 cm])

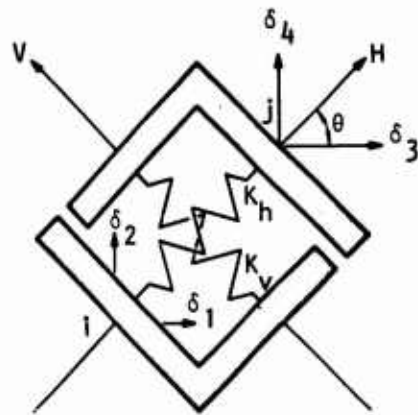


(b) Finite element mesh layout

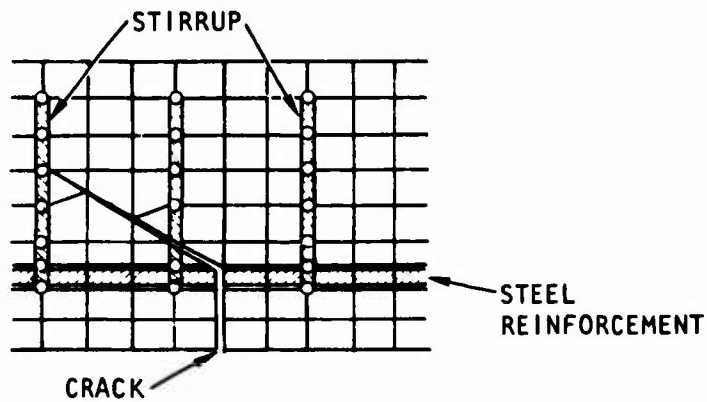
Figure 4. Linear Analysis of a Beam with a Predefined Crack (Ref. 18)



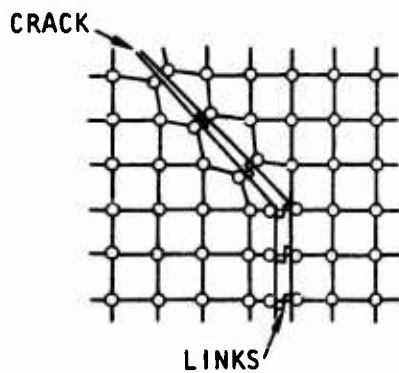
(a) Analytical model



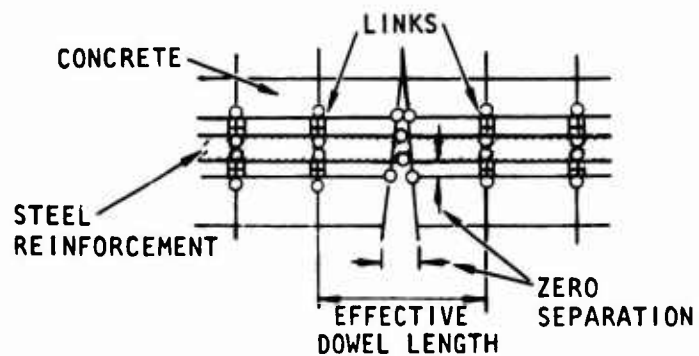
(b) Linkage element



(c) Reinforcement representation



(d) Crack and aggregate interlock



(e) Effective dowel length

Figure 5. Analytical Models and Linkage Elements (Ref. 18)

consisting of two constrained linear-strain triangles (Ref. 14), and the influence of stirrups, dowel shear, aggregate interlock and horizontal splitting along reinforcement near the support were considered. One-dimensional bar elements were used for the vertical stirrups.

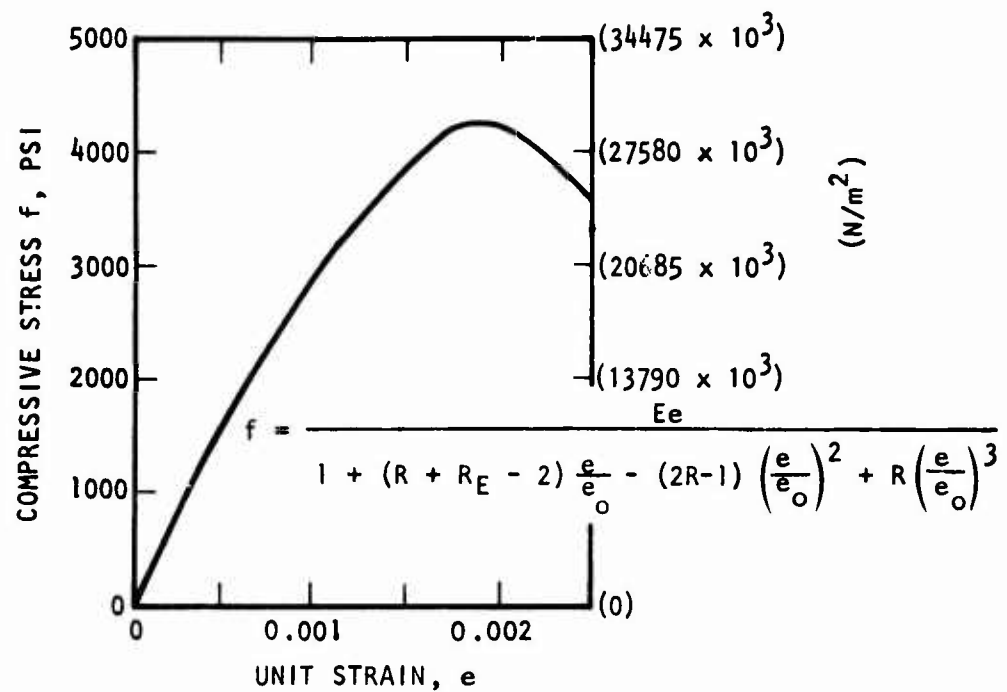
c. Nonlinear Model

Nilson (Refs. 3 and 19) refined the work of reference 2 by introducing nonlinear material properties and a nonlinear bond-slip relationship into the model. He accounted for these nonlinearities by applying the load in small increments (Figs. 6 and 7). Improved quadrilateral plane stress finite elements were used. Cracking was accounted for by stopping the solution when an element indicated a tensile failure. The new cracked structure was redefined, the revised information was then input into the computer, and the structure was reloaded incrementally. The method was applied to concentric and eccentric reinforced tensile members that were subjected to loads applied longitudinally through the reinforcing bars, and the results were checked against experimental data. Nilson found bond stress patterns that are similar to those found by Ngo and Scordelis (Ref. 2).

The main shortcoming of Nilson's work, besides approximate treatment of constitutive properties of plain concrete, is the difficulty of handling tensile cracking. While it may be possible to terminate a static calculation, remesh, and begin loading again from the beginning, it is extremely difficult to do this in a dynamic calculation.

d. Refined Nonlinear Models

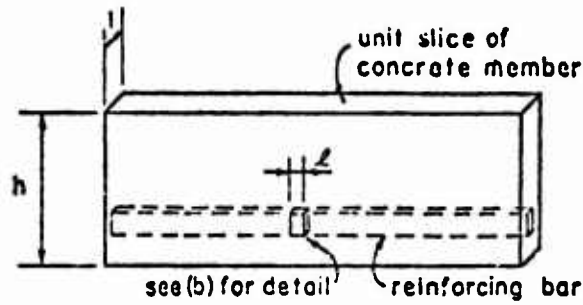
Franklin (Ref. 20) performed a nonlinear analysis of the beam, as shown in figure 8, in which cracking within the finite elements and redistribution of stresses was automatically accounted for, so that the response from initial loading to failure was obtained in one continuous computer analysis. Incremental loading, with iterations within each increment, were used to account for cracking and nonlinear material properties. Unlike the preceding studies, the crack is not predefined, and progressive cracking is assumed to occur over an entire element normal to a principal stress direction rather than along a



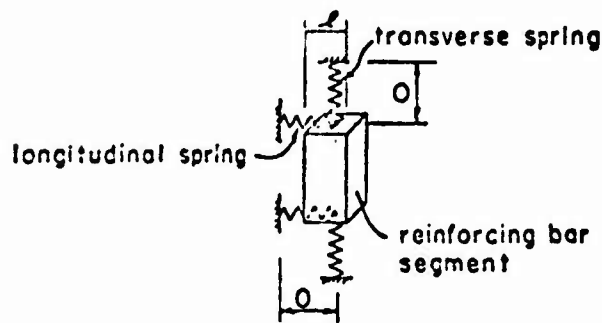
TERMS

- f = STRESS AT STRAIN e
- f_f = FAILURE STRESS AT FAILURE STRAIN e_f
- e_o = STRAIN CORRESPONDING TO MAXIMUM STRESS f_o
- E = THE INITIAL TANGENT MODULUS
- E_s = THE SECANT MODULUS = $\frac{f_o}{e_o}$
- $R_E = \frac{E}{E_s}$
- $R_f = \frac{f_o}{f_f}$
- $R_e = \frac{e_f}{e_o}$
- $R = \frac{R_E (R_f - 1)}{(R_e - 1)^2} - \frac{1}{R_e}$

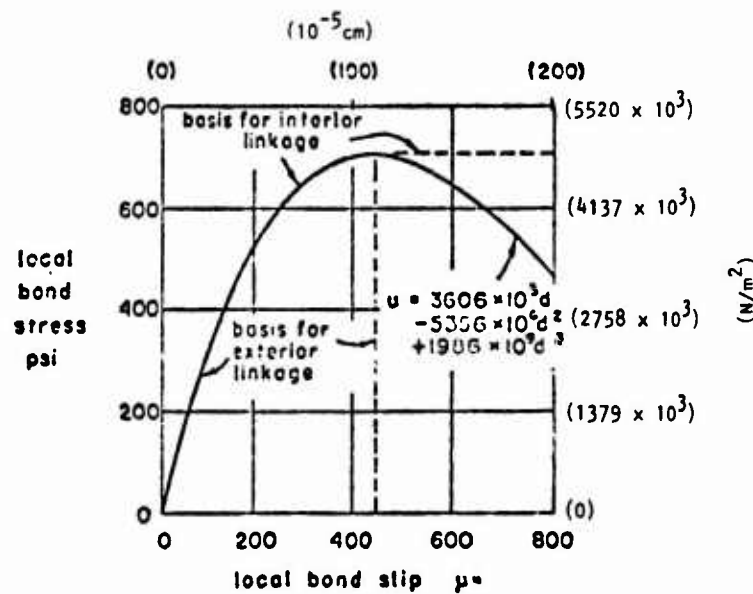
Figure 6. Concrete Uniaxial Stress/Strain Relations in Compression (Saenz Equation, Ref. 19)



(a) Model of reinforced concrete member

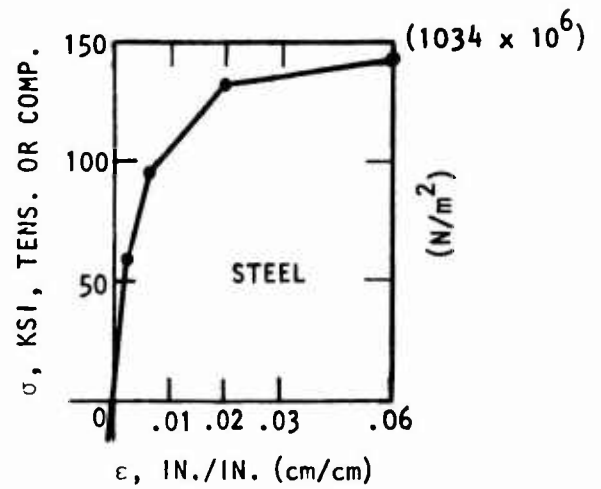
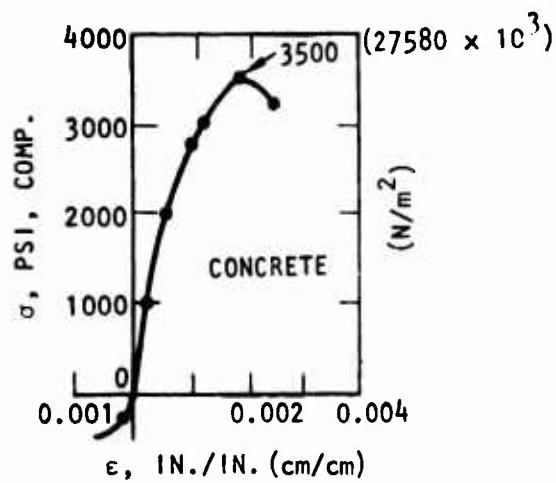


(b) Detail of bond linkage

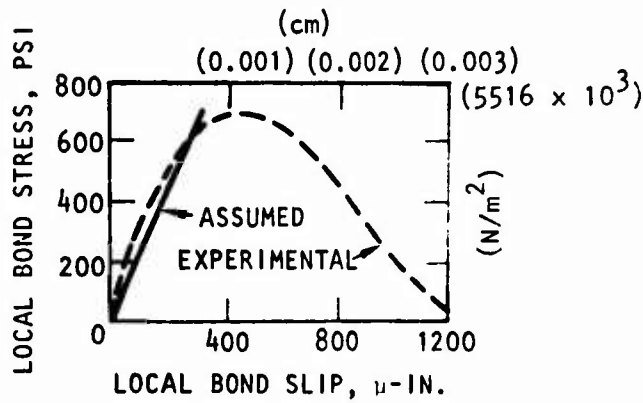


(c) Bond-slip curves used in analysis

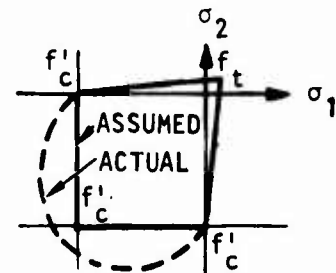
Figure 7. Bond-Slip Model (Ref. 19)



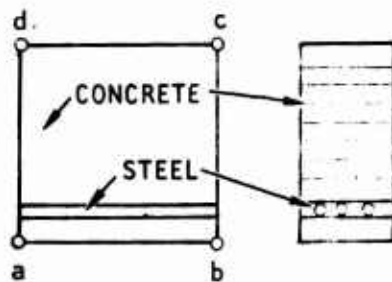
(a) Material uniaxial stress/strain curves in tension



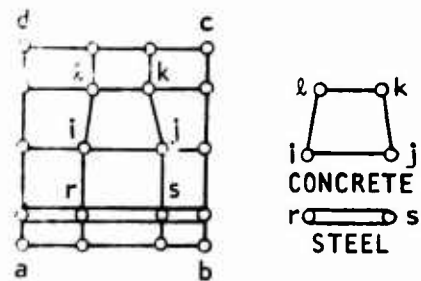
(b) Bond stress vs. slip curve



(c) Concrete failure criteria



(d) One-dimensional layered frame element



(e) Two-dimensional quadrilateral elements and rod elements

Figure 8. Nonlinear Analysis of a Beam with Progressive Cracking (Refs. 4 and 20)

single line. This permits the use of the same structural nodal point topology throughout the solution, rather than the necessity for splitting the nodes after cracking and establishing a new topology, as would be required by Nilson's nonlinear approach.

The segmental material uniaxial stress-strain curves used for the concrete, steel, and bond stress/slip are shown in figures 8(a) and (b). Two different modeling schemes used in the analysis are shown in figures 8(d) and (e).

In the first scheme, figure 8(d), the beam is divided longitudinally into a series of "frame elements," each having a total depth equal to that of the beam. The frame element is subdivided into ten concrete layers and may have reinforcement at up to four layers over the depth. Utilizing an assumption of plane sections remaining plane at sections a-d and b-c, the stiffness of the frame element subjected to axial force, shear, and moment at each section can be evaluated for material properties that may vary vertically from layer to layer, but are constant over the length of the element. For this purpose, each layer is assumed to be in a state of uniaxial stress. The nonlinear analysis proceeds in the usual manner with the modulus of elasticity of each concrete or steel layer, taken from the stress-strain curves of figure 8(a), being dependent on the strain existing at the start of the load increment. When a concrete layer cracks in tension, its modulus is set to zero and the stress is redistributed to the remaining structure. Obviously, for this idealization only vertical cracks can occur in the beam. Also, no account is taken of bond slip.

For plane stress systems in which diagonal cracks occur and the two-dimensional stress state dominates the behavior, the second scheme (shown in Fig. 8(e)) is more realistic. Here the beam segment a-b-c-d is subdivided into series of concrete quadrilateral plane stress elements i-j-k-l, made up of two constrained linear strain triangles and one-dimensional steel bar elements r-s. Bond may be simulated by adding linkage elements between double nodes introduced at points such as r. In the present example, Franklin (Ref. 20) assumed isotropic behavior during each increment of loading prior to cracking.

He used a single tangent modulus E taken from the uniaxial stress-strain curve (Fig. 8(a)) as the smaller of the tangent moduli E_1 and E_2 corresponding to the two current principal strains. The modulus E is used to define his isotropic two-dimensional constitutive relation. On the other hand, Nilson (Refs. 3 and 19) treated the material as orthotropic and used tangent moduli E_1 and E_2 . Franklin assumed a modified biaxial failure envelope for the concrete which approximates the actual envelope (Fig. 8(c')). When tensile cracking failure occurs, the element is cracked normal to the principal stress direction, and this stress is redistributed to the remaining structure. Subsequently, the element is assumed to be anisotropic and to have zero modulus of elasticity normal to the crack.

Experimental and analytical load versus midspan deflection curves are shown in figure 9. It can be seen that the analytical cracking load is lower in all cases than the experimental load, indicating that the actual tensile strength of the concrete was higher than assumed, or that at low loads the "tension stiffening" effect of the concrete between cracks in an element, which was ignored, cannot be neglected. All analytical curves indicate a stiffer model after cracking than found experimentally. Only the analytical model with a low bond link failure limit produced a failure load less than the experimental value, all others being much higher. Scordelis (Ref. 4) concluded that for beams that fail primarily because of diagonal tension cracking, the bond, dowel shear, and aggregate interlock must be modeled more accurately before a reliable finite element prediction can be made of the failure load.

e. Wall Panel Study

In this example, the response of a wall panel specimen was studied experimentally and analytically by Cervenka (Ref. 21) and also analytically by Franklin (Ref. 20). Dimensions, loading, reinforcing scheme, and method of loading are shown in figure 10. Because of symmetry, each half of the deep beam test specimen can be considered similar to a wall panel subjected to a single transverse load.

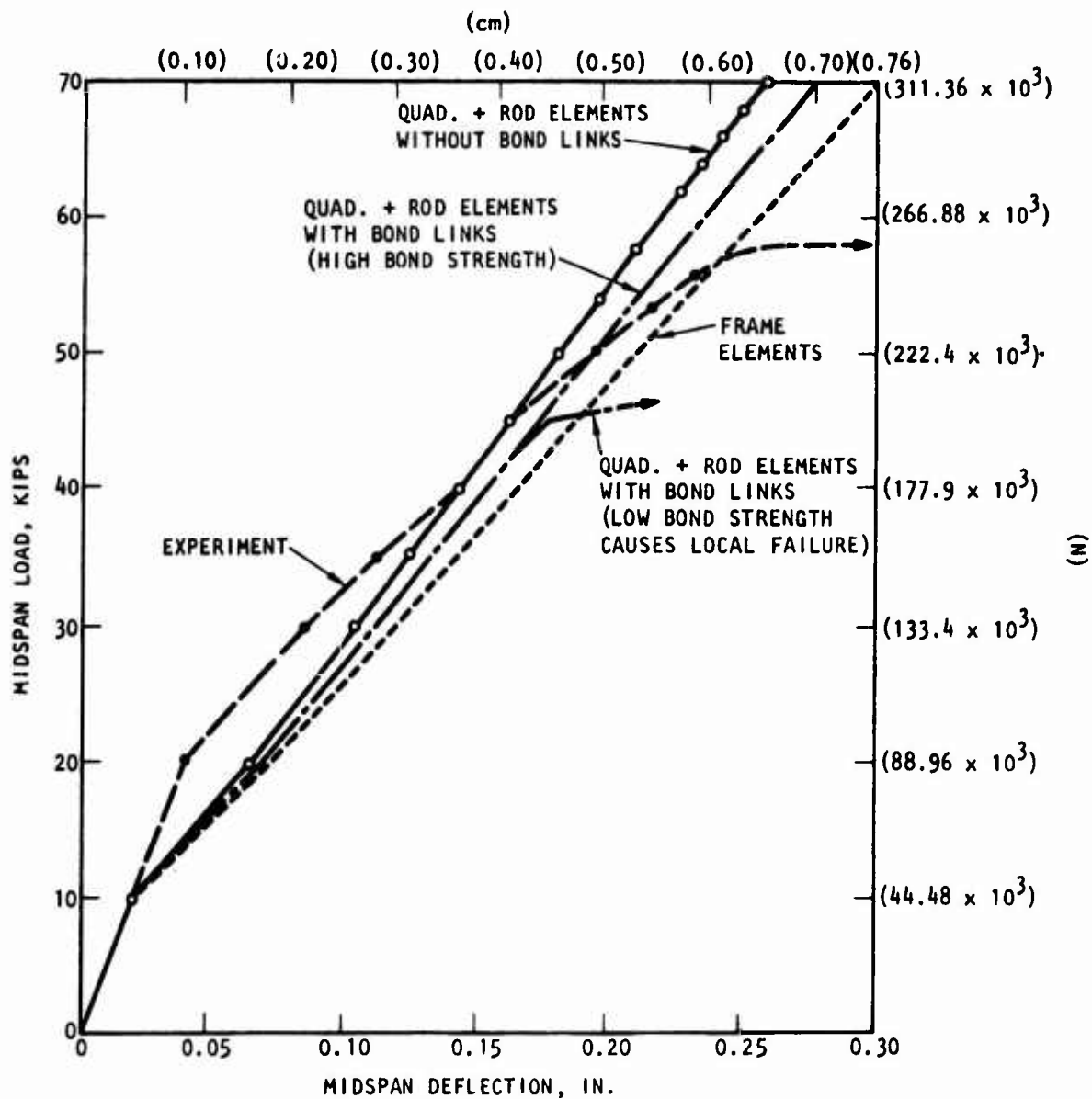
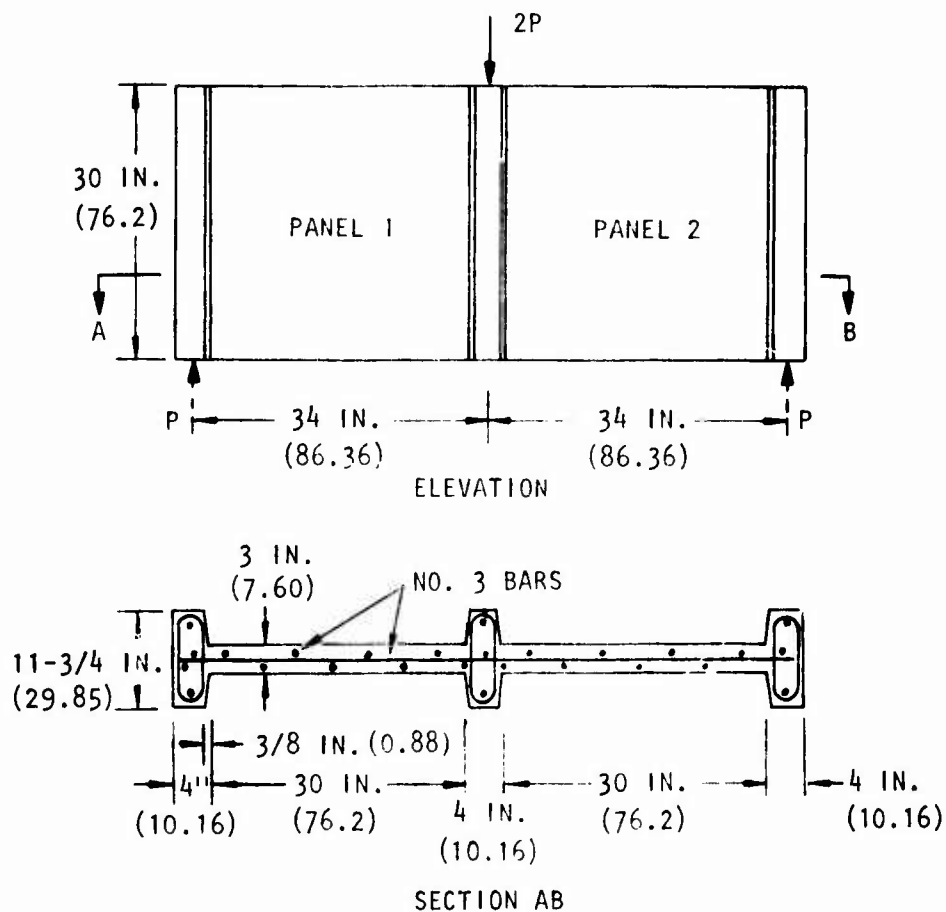
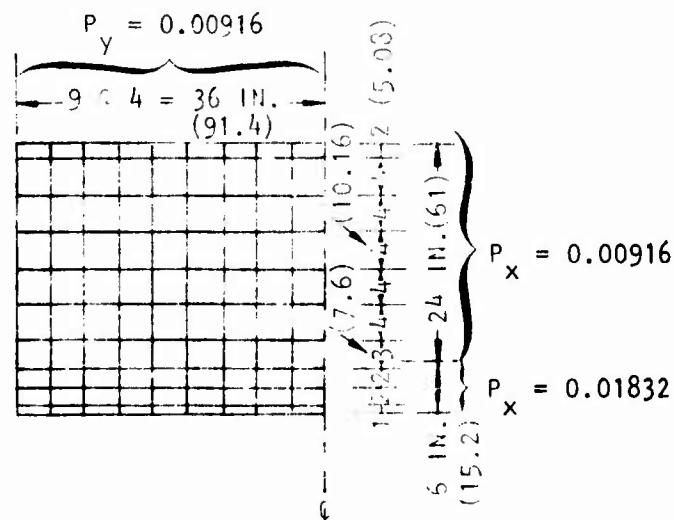


Figure 9. Experimental and Analytical Load versus Deflection Curves (Refs. 4 and 20)



(a) Dimensions and loading of test specimen

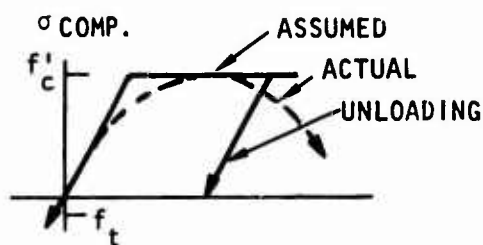


(b) No. 3 reinforcement for test specimen

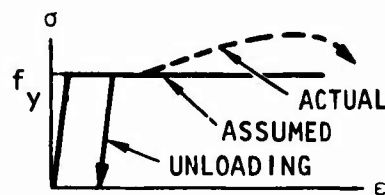
Figure 10. Wall Panel Study (Refs. 4, 20, 21)

In Cervenka's analysis, constant-strain triangular finite elements were used to model the concrete and steel reinforcement. The steel was assumed to be uniformly distributed over the element in two orthogonal directions, and a composite material constitutive relation was developed in order to determine the element stiffness matrix. Both the concrete and steel were assumed to have elastic-perfectly plastic stress/strain curves (Figs. 11(a), 11(b)). For the concrete, $f'_c = 3880$ psi (26753×10^3 N/m²), $f_t = 529$ psi (3647×10^3 N/m²), $E_c = 2900$ ksi (19996×10^6 N/m²); and for the steel, $f_y = 51,200$ psi (353024×10^3 N/m²), $E_s = 27,300$ ksi (188233×10^6 N/m²). The assumed biaxial stress failure criteria shown in figure 11(c) was adopted for the concrete. An incremental nonlinear analysis was used which accounted for tensile cracking and subsequent stress redistribution. Plasticity of the uncracked concrete, which is in a biaxial state of stress, was assumed to obey the von Mises yield condition and associated flow rule, and was used in the finite element analysis to account for plastic deformations under biaxial compressive yielding. No account was taken, either in this analysis or the one that follows, of bond slip, aggregate interlock, or tension stiffening of concrete in the cracked zone. Cervenka compared his analytical results with Peter's experimental studies on reinforced concrete wall panels and spandrel beams (Ref. 22). He concluded that reasonable correlation with experimental results can be obtained in problems by monotonically increasing the loading. However, in the problems with cyclic loading, nonlinear effects of bond slip and crack surface deterioration should be properly accounted for, in order to achieve good correlation with experimental measurements.

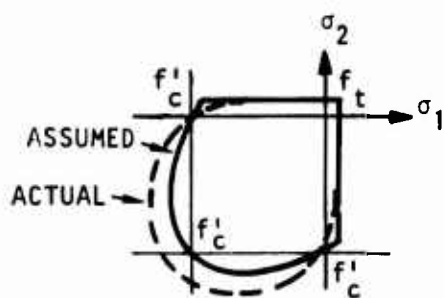
In the analysis using Franklin's program (Ref. 20), quadrilateral elements made up of two constrained linear strain triangles (Ref. 14) were used for the concrete and one-dimensional bar elements were used for the steel reinforcement. The same uniaxial stress/strain curves (Figs. 11(a) and 11(b)) were used for the concrete and steel, while a slightly different failure criterion (Fig. 8(c)) was assumed for the concrete under biaxial stress. Tensile cracking was accounted for and the concrete was assumed to have failed when it reached f'_c in compression with no subsequent plasticity.



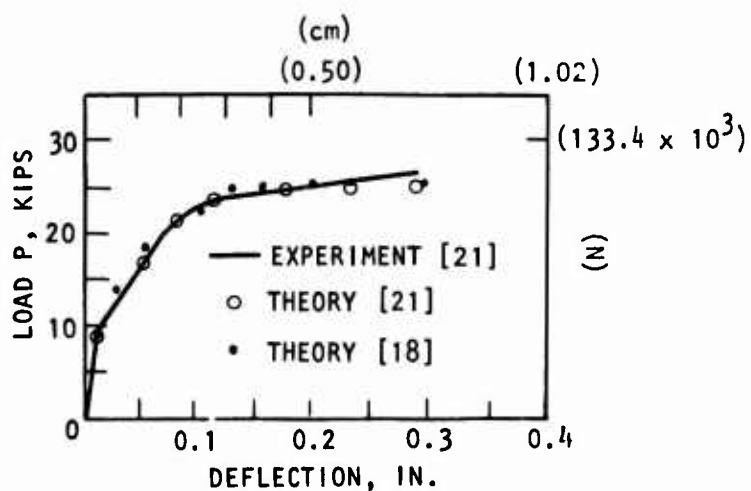
(a) Uniaxial stress/strain curve for concrete



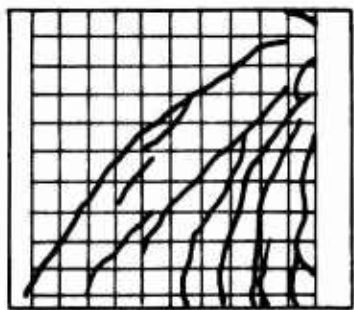
(b) Uniaxial stress/strain curve for steel



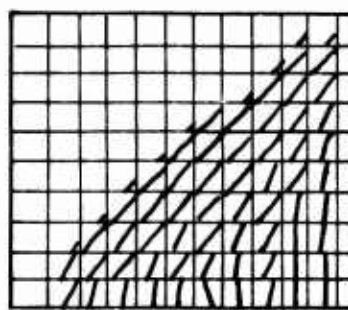
(c) Concrete failure criteria



(d) Experimental and analytical load vs. deflection curves



$P = 25.5 \text{ KIPS } (113.4 \times 10^3 \text{ N})$
(e) Experimental crack pattern



$P = 25.5 \text{ KIPS } (113.4 \times 10^3 \text{ N})$
(f) Analytical crack pattern

Figure 11. Material Idealization and Selected Results (Ref. 21)

The two analyses both seemed to predict experimental load-deflection curve and crack pattern quite well (see Figs. 11(d), 11(e), and 11(f), respectively). From these results it appears that for wall panels in which the reinforcement is distributed rather uniformly over the entire wall panel, the effects of bond slip, dowel shear, aggregate interlock, and tension stiffening of the concrete in a cracked zone are of much less importance than for the case of a beam in which the major reinforcement is concentrated along a single line.

3. CONTINUUM METHOD

General constitutive equations that consider the following aspects of reinforced concrete behavior were developed by Isenberg and Adham (Ref. 1).

- Inelastic stress/strain properties of plain concrete including tensile cracking and compressive crushing
- Inelastic stress/strain properties of steel
- Progressive deterioration of bond
- Anisotropy (different effective moduli in different directions) due to cracking and crushing

These equations have been adapted to a plane strain, static finite element code, which was applied to determine quasi-static aspects of hardened structure response in the SAMSO RockTest II program.* Comparison between predictions and measurements, reported in reference 5, was encouraging.

The continuum approach requires composite moduli to be defined. Due to the tendency for concrete to develop large-scale, discrete cracks under tensile stress, these models must be capable of representing anisotropy. Thus, tensile stress/strain properties perpendicular to a crack should be different from those parallel to the crack. A central problem is to define the principal directions of anisotropy--that is, directions in the material for which fundamental stress/strain properties can be derived and subsequently transformed to

*The RockTest II program sponsored by the U.S. Air Force included design and validation testing under a HEST/DIHEST environment of facility subsystems for an advanced fixed-base, large payload missile system.

other directions. A fundamental assumption of the Isenberg and Adham approach is that the principal directions of anisotropy coincide with the principal axes of stress until tensile cracking occurs. Thereafter, principal directions of anisotropy are assumed to be parallel and perpendicular to the direction of first cracking. Within this framework of orthotropy, tangent moduli for the composite material are defined in the principal directions of orthotropy. These moduli depend on the tangent stiffnesses of the concrete and steel and on the state of bond between them. A variable modulus model, described below, was developed to represent data available in 1968 (Ref. 1). A variable modulus model of the reinforcing steel was also defined. These two contributions are combined, using area-averaging as modified by the bond-slip relation, into composite moduli of a section. The development of this model includes:

a. Compressive-Stress/Strain Relations for Plain Concrete
(Tangent Modulus)

The relation between increments of stress and strain for increasing compressive stress was considered. Experimental data from which the tangent modulus could be obtained were available at that time for the following states of stress:

- Uniaxial Compressive Stress $(\sigma_1 < 0; \sigma_2 = \sigma_3 = 0)^*$
(Barnard, Ref. 23 and Kabaila, Ref. 24)
- Triaxial Compressive Stress $(\sigma_1 < \sigma_2 = \sigma_3 < 0)$
(Balmer, Ref. 25)
- Biaxial Tensile and Compressive Stress $(\sigma_1 < 0; \sigma_2 > 0; \sigma_3 = 0)$
(Isenberg, Ref. 26, 27)

*Throughout this report the stress will be compressive if preceded by a negative sign.

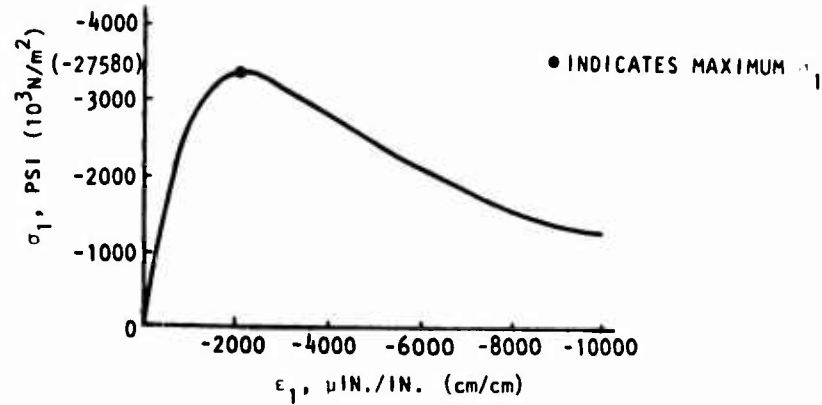
The relevant data from these experiments are illustrated in figure 12. The stress/strain (σ_1/ϵ_1) relation for unconfined compression, figure 12(a), is nonlinear before maximum stress is reached and, when testing is performed in a stiff machine, the curve has a falling branch.

The triaxial compression experiments, figure 12(b), were performed in the usual way, whereby an initial all-around confining pressure is applied to solid cylindrical specimens. This is followed by applying additional stress (additional σ_1) in the direction of the long axis of the specimen. The data of interest are the σ_1/ϵ_1 curves developed during application of the additional stress. Balmer's experiments apparently were not performed in a stiff testing machine. Hence there is no reason to expect a falling branch in the σ_1/ϵ_1 curves that he measured. The existence of a falling branch under triaxial compressive stress has been neither proved nor disproved.

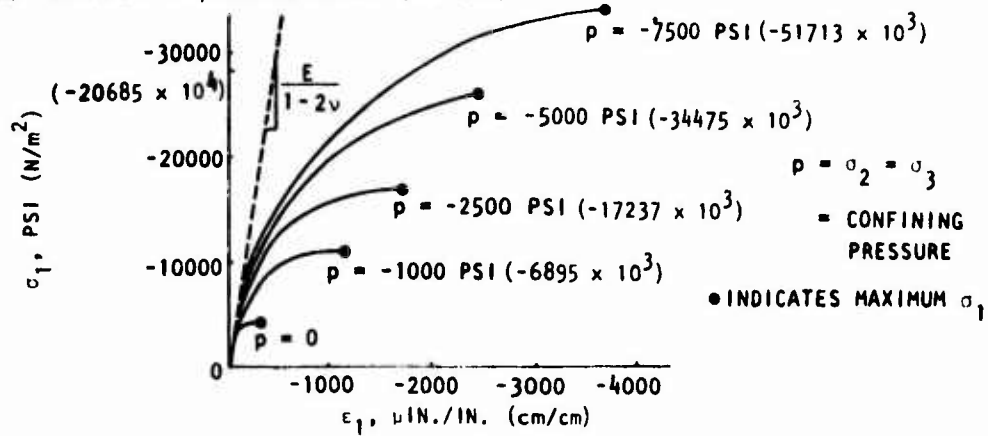
The biaxial tension and compression experiments were performed by subjecting tubular specimens to combined torsion and compression. The technique is reported in reference 27. The data indicate that the shape of the σ_1/ϵ_1 curves are roughly similar to those measured in unconfined compression and triaxial compression. However, the magnitude of stress at a particular strain and the maximum value of σ_1 depend heavily on the state of stress. Although the σ_1/ϵ_1 curves in figure 12(c) contain a small Poisson's-ratio effect because of the fact that ϵ_2 is changing, the effect is small and

$$\epsilon_1 \approx \epsilon_1 - \nu \epsilon_2 \quad (2)$$

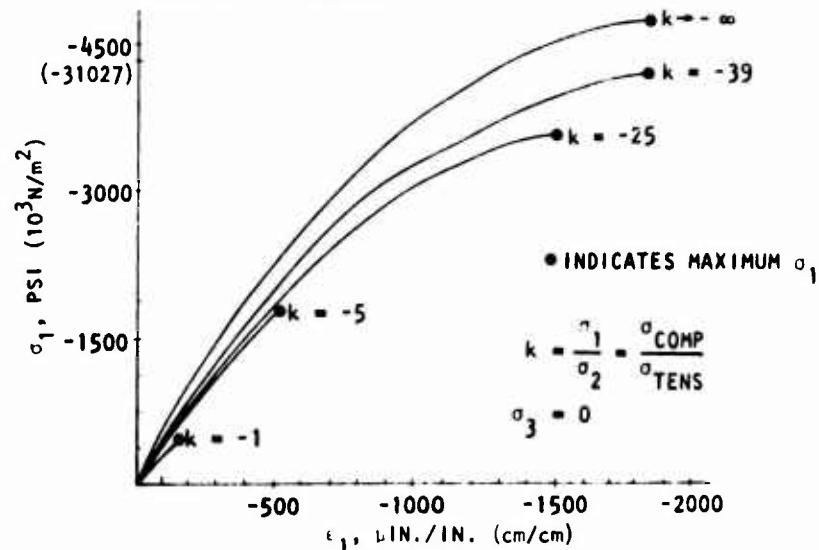
These data suggest that, if an ideal series of tests covering a wide range of stress states could be performed on a group of identical specimens, the σ_1/ϵ_1 curves shown in figure 13(b) might be obtained. These curves represent loading conditions where the lateral stresses σ_2 and σ_3 are held constant. Hence, the slope of the σ_1/ϵ_1 curve at any point is the tangent modulus



(a) Uniaxial compressive stress (Barnard)

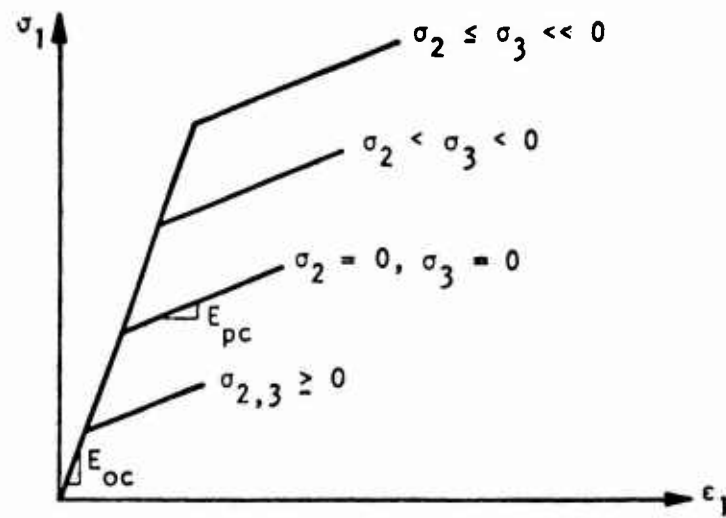


(b) Triaxial compressive stress (Balmer)

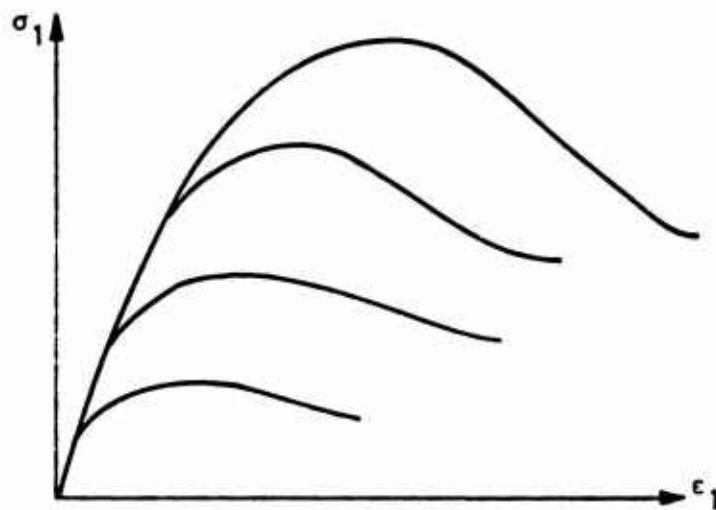


(c) Biaxial tensile and compressive stress (Isenberg)

Figure 12. Experimental Compressive Stress/Strain Curves for Three States of Stress



(a) Bilinear model



(b) Model based on modified Kabaila equation

Figure 13. Idealized Stress/Strain Curves for Plain Concrete

$$\left(\frac{\partial \sigma_1}{\partial \epsilon_1} \right)_{\sigma_2, \sigma_3 = \text{const}} = E_1 \quad (3)$$

which is prescribed as a function of the current values of σ_1 , σ_2 and σ_3 .

The bilinear model shown in figure 13(a) is attractive because it is simple and represents the broad characteristics of the experimental curves up to maximum stress. The state of stress at which the model σ_1/ϵ_1 curves change slope is given by

$$f = \bar{f}_c + \beta \left(\frac{\sigma_2 + \sigma_3}{2} \right) \quad (4)$$

The tangent modulus E_1 is defined as follows:

$$\begin{aligned} \text{if } \sigma_1 > f & , \quad E_1 = E_{oc} \\ \sigma_1 < f & , \quad E_1 = E_{pc} \end{aligned} \quad (5)$$

where E_{oc} and E_{pc} are the tangent moduli for concrete in the elastic and inelastic regimes, respectively, and

β = An experimental parameter corresponding to angle of internal friction

\bar{f}_c = An experimental parameter related to the unconfined compressive cylinder strength of concrete, with the value of \bar{f}_c considered to lie between 0.85 and 1.0 times the unconfined compressive strength f'_c .

Values of β and \bar{f}_c may be obtained for a specific type of concrete by plotting maximum σ_1 versus $(\sigma_2 + \sigma_3)/2$ and finding the slope and intercept. This has been done for the maximum combined stress data shown in figure 12 and some additional data obtained in biaxial compression, reference 28 ($\sigma_1 = \sigma_2 < 0$; $\sigma_3 = 0$). The result is shown in figure 14, where an attempt has

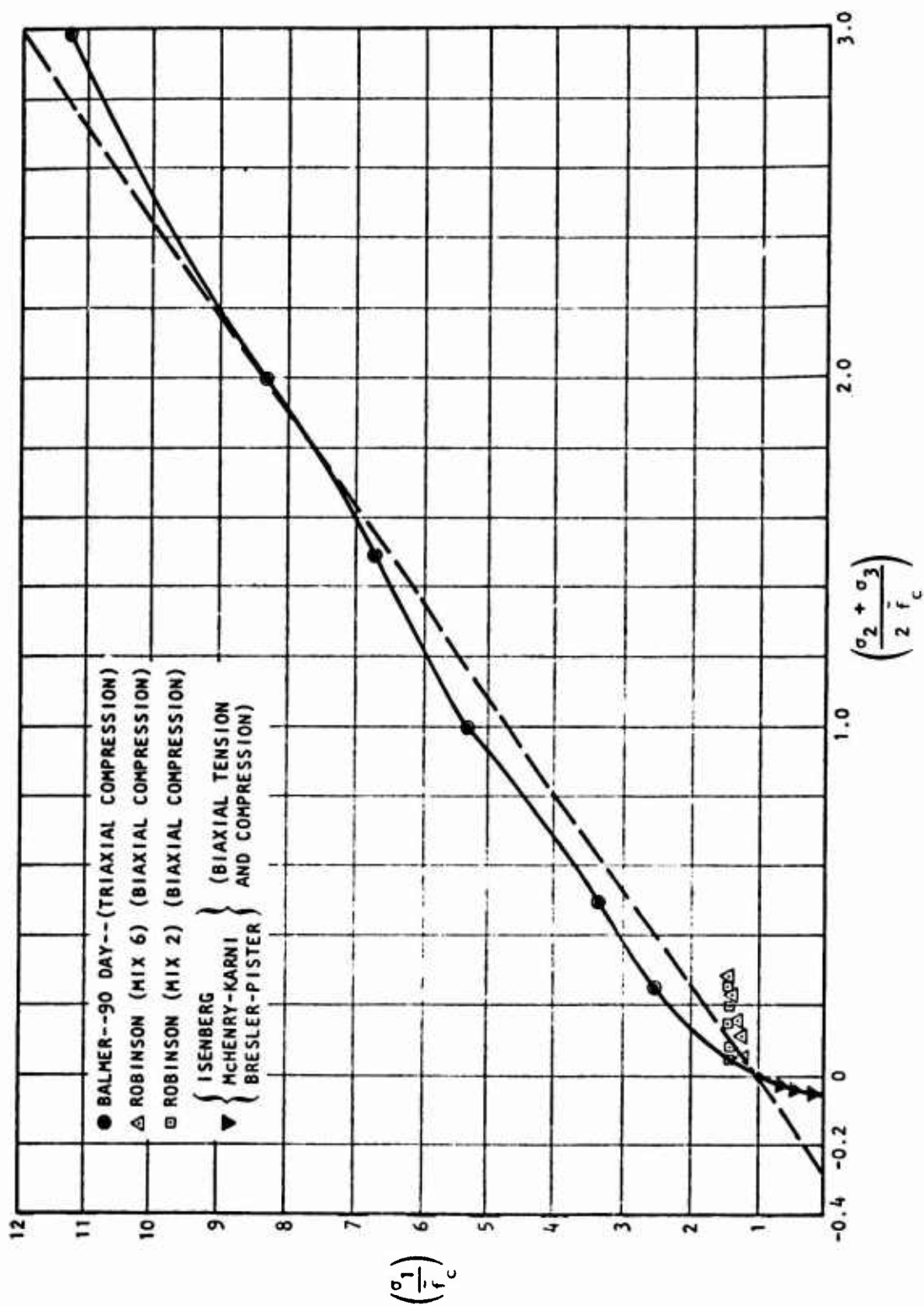


Figure 14. Maximum Compressive Stress (σ_1) as a Function of the Other Compressive Stresses (σ_2, σ_3)

been made to take into account the different properties of test specimens used by the various investigators. In each individual test series, the companion values of σ_1 and $(\sigma_2 + \sigma_3)/2$, are divided by \bar{f}_c , where \bar{f}_c is here defined as 1.0 times unconfined compressive strength in that test series. The results of tests in biaxial tensile and compressive stress are so closely grouped in a small area of the graph that they are represented by only three symbols.

In determining the parameter β from experimental data, more weight is given to the results of triaxial compression tests than to biaxial compression tests. This is done when the model is being applied to plane strain situations, where the most frequent states of stress are closer to triaxial than to biaxial compression. For plane stress analyses, β should be evaluated on the basis of biaxial compression tests.

b. Tensile Stress/Strain Relations for Plain Concrete (Tangent Modulus)

The relation between increments of stress and strain for increasing tensile stress is considered next. The states of stress for which data on the tangent modulus are available are:

- Uniaxial tensile stress, reference 29
- Biaxial tensile and compressive stress, reference 27

These data indicate that the tensile stress/strain relations are approximately linear up to maximum combined stress, although they are not exactly linear. It was decided to represent the tangent modulus in tension by a constant

$$\left(\frac{\partial \sigma_1}{\partial \epsilon_1} \right)_{\sigma_1, \tau} = E_{oc} \text{ (a constant)} \quad (6)$$

where E_{oc} is equal to the initial tangent modulus in unconfined compression.

In addition to tensile stress/strain relations, it is also necessary to have a criterion of maximum tensile stress or cracking stress. The sense and magnitude of the lateral stresses σ_2, σ_3 affect the maximum tensile stress, as experiments under biaxial tensile and compressive tests show (Refs. 27, 30, and 31). However, at the time of conducting the study reported in reference 1, there were no data on maximum tensile stress as a function of two lateral compressive stresses ($\sigma_2 < 0, \sigma_3 < 0$) or as a function of one lateral tensile stress ($\sigma_2 > 0$) and one lateral compressive stress ($\sigma_3 < 0$). In the absence of such experimental data, a criterion based on data from the biaxial tensile and compressive stress state was assumed. The mathematical statement of the tensile strength criterion is as follows:

$$\begin{aligned} \text{For} \quad & \sigma_2 < 0 \text{ and } \sigma_2 < \sigma_3 \\ & \sigma_1 \leq f'_t, \\ \text{where} \quad & f'_t = f_t - \eta \sigma_2 \end{aligned} \quad (7)$$

and

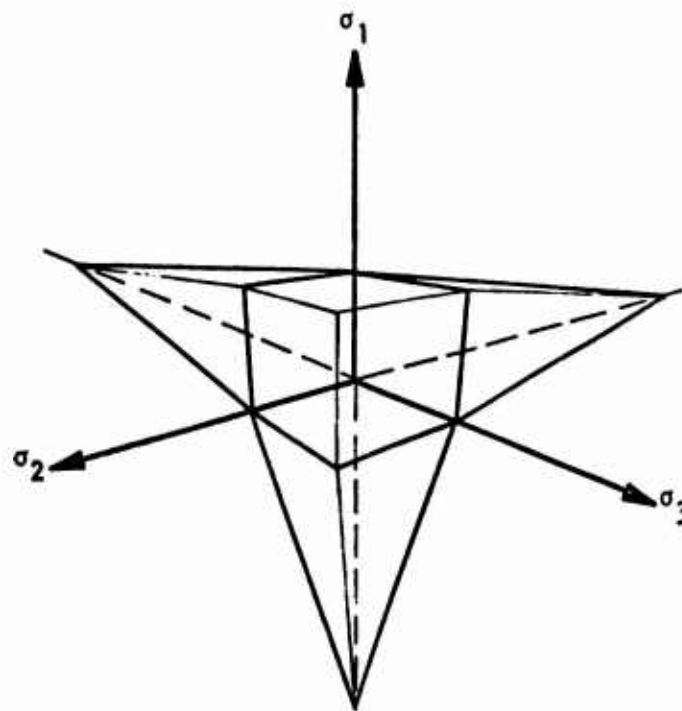
$$\begin{aligned} f_t &= \text{Unconfined tensile strength of concrete } (> 0) \\ \eta &= \text{Slope of cracking envelope} \end{aligned}$$

If $\sigma_1 = f'_t$, maximum tensile stress has been reached and cracking is said to have occurred.

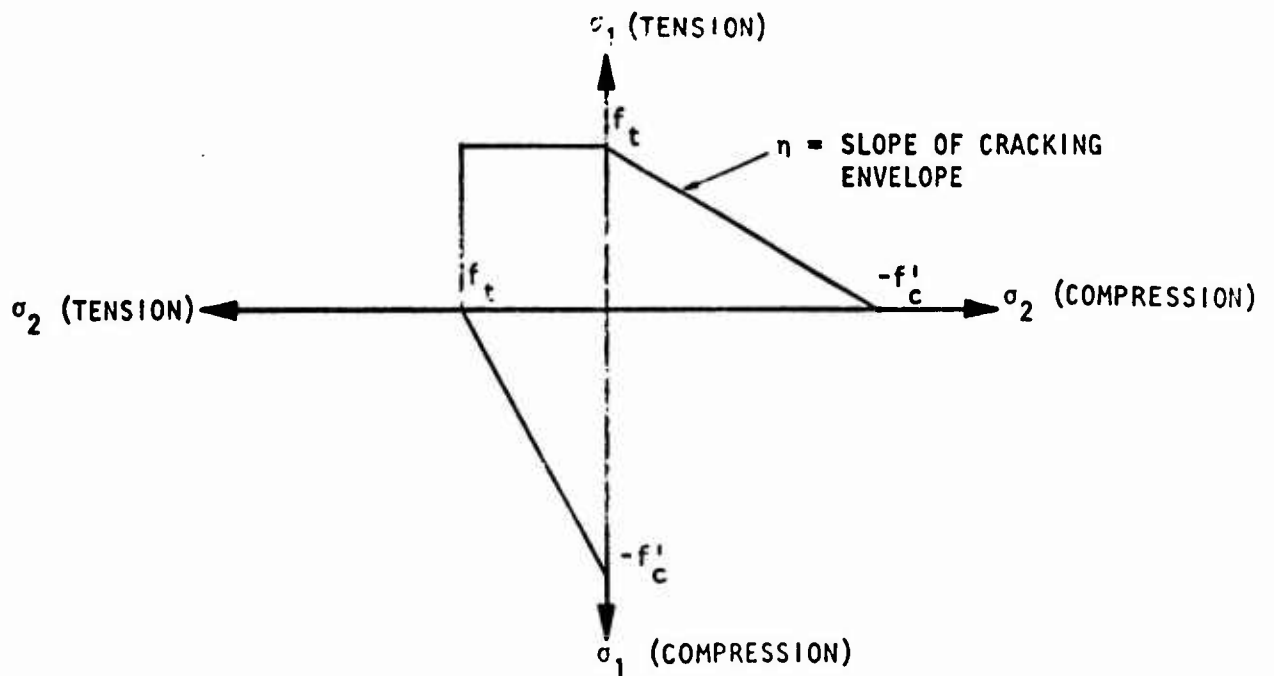
For both $\sigma_2, \sigma_3 > 0$

$$f'_t = f_t \quad (8)$$

The graphical representation of this criterion is illustrated in figure 15. The projection of this criterion in σ_1/σ_2 plane is shown for convenience in the same figure and correlates well with the actual criterion shown in figure 8(c).



(a) Three-dimensional cracking criteria



(b) Projection of tensile cracking criterion in the $\sigma_1 - \sigma_2$ plane

Figure 15. Tensile Cracking Criteria

c. Poisson's Ratio

Experimental data on Poisson's ratio are available mainly from tests in triaxial compressive stress. The data in reference 32 clearly show that under this state of stress, Poisson's ratio is approximately a constant up to 50 to 90 percent of maximum combined stress and that it increases rapidly as maximum stress is approached.

No attempt has been made to represent this type of variation in Poisson's ratio. Instead, it has been assumed to be initially constant, equal in all directions and equal to the value measured in unconfined compression near zero stress. This assumption is justifiable because the model is intended to apply in situations of plane strain or nearly uniaxial strain. These situations differ sufficiently from triaxial compressive stress, where lateral expansion is not prevented.

The initial value of Poisson's ratio for plain concrete is used for the composite material. When inelasticity occurs, one or more tangent moduli decrease. Consequently, in order to preserve the symmetry of the finite element stiffness matrix, one or more Poisson's ratios of the composite material are also decreased. This procedure is justified mathematically but does not necessarily represent physical processes. The steps required to maintain a symmetric stiffness matrix are described below.

d. Properties of Steel

The required properties of the steel are the tangent modulus, the yield stress, and Poisson's ratio. These properties are assumed to be the same in tension and compression. It is also assumed that yielding of a steel bar is governed only by the stress in the direction of the bar. The influence on yielding of stresses perpendicular to the bar is neglected.

No specific experimental data were used in the Isenberg/Adham model (Ref. 1) to define the properties of the steel. Engineering handbook values of elastic Young's modulus, Poisson's ratio, and yield strength were used. The experimental tensile stress/strain curve of the steel bars were idealized

as shown in figure 16. The broken line is the stress/strain curve used in the model. The parameters which characterize the model are:

E_{0s} = Elastic Young's modulus

E_{ps} = Plastic Young's modulus

f_s = Yield strength of steel

In addition, Poisson's ratio for steel, ν_{st} should be specified.

e. Composite Behavior

The composite model combines the variations in steel and concrete properties discussed above and accounts for:

- The combined properties of steel and concrete
- The change in principal directions of stresses and the orientation of cracks
- Progressive bond failure between steel and concrete
- Material orthotropy
- Mechanical orthotropy
- Orthotropy due to cracking

The present model considers the finite element system as a continuum in which properties are uniform within an element. To demonstrate the method, the linear strain triangle is used in the present analysis.

f. Idealization of a Reinforced Concrete Structure

A typical finite element idealization of a reinforced concrete structure and the surrounding medium is shown in figure 17. The external loads act in the x-y plane, which is also the plane of primary reinforcement (Fig. 18). For a reinforced concrete element the reinforcement areas A_{sI} and A_{sII} in an arbitrary orthogonal orientation are given. The angle ϕ is the inclination of the reinforcement relative to the global x-axis as shown in figure 18(b).

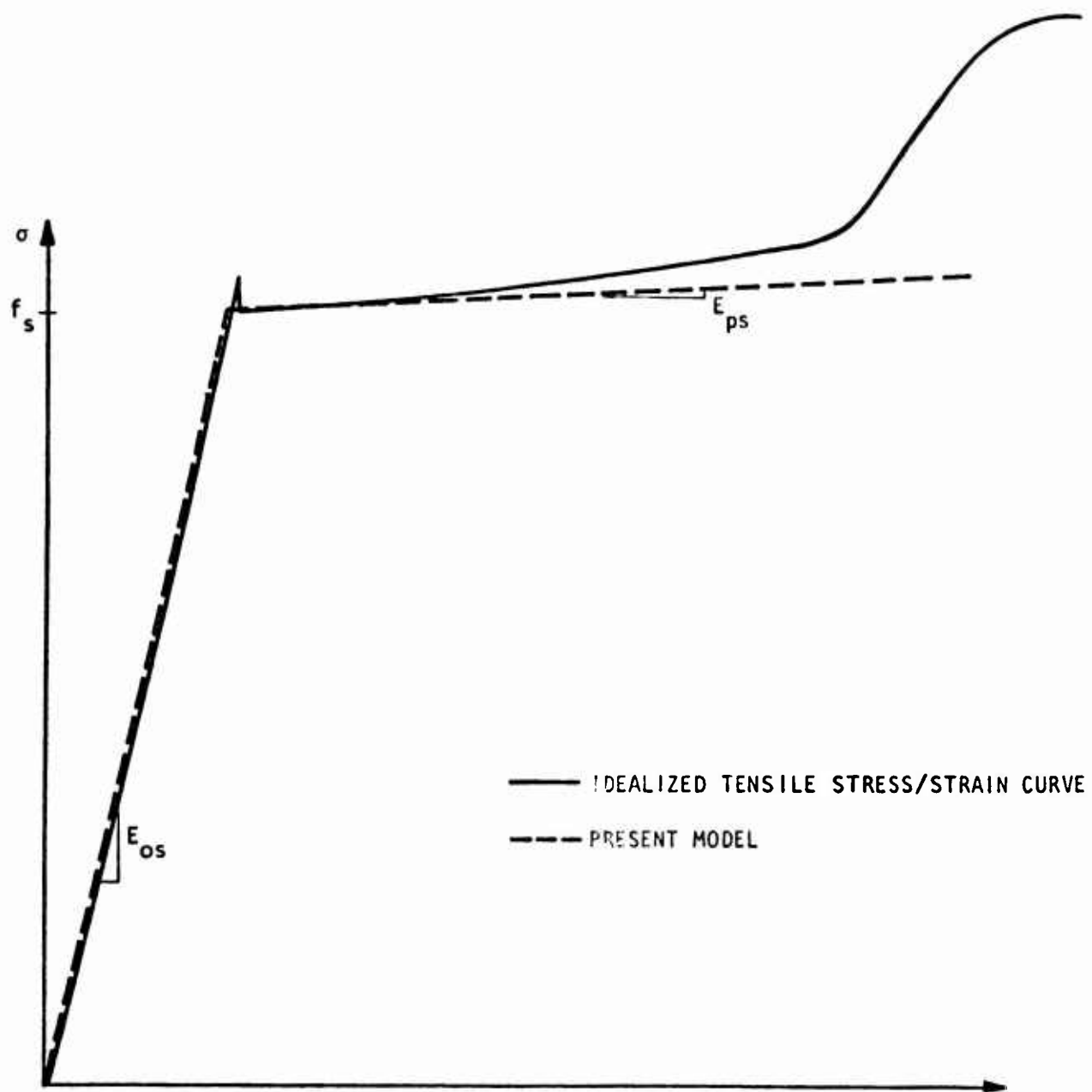


Figure 16. Tensile Stress/Strain Curves for Reinforcing Steel

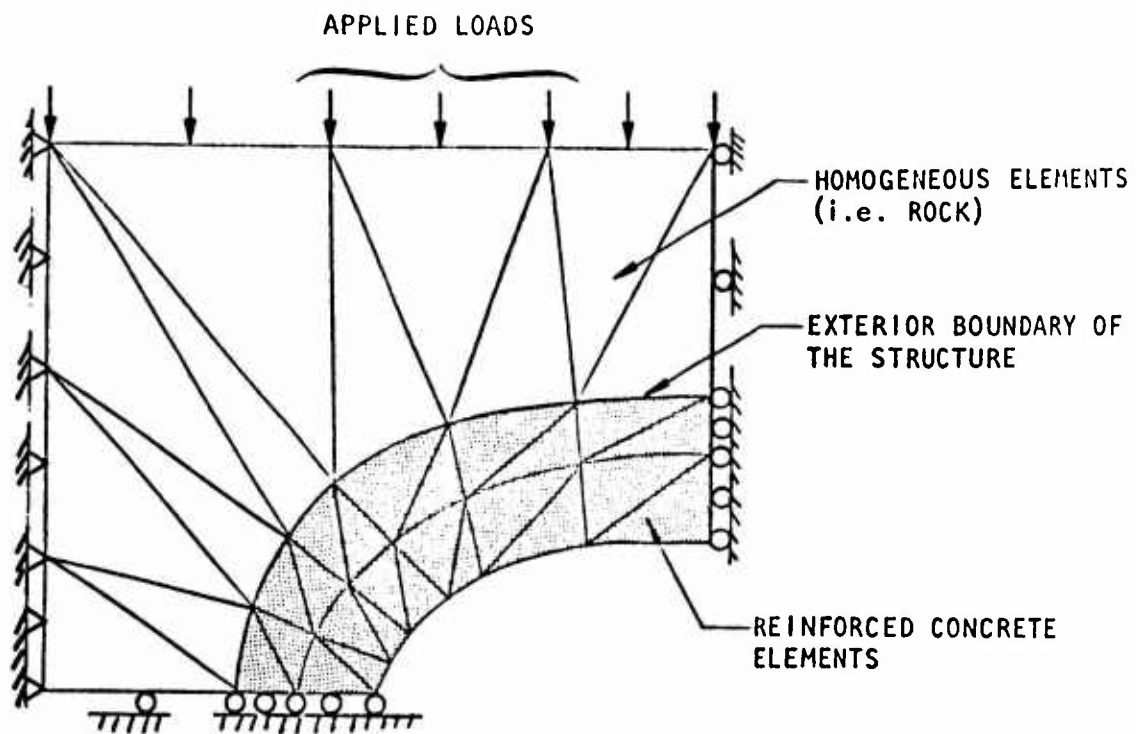


Figure 17. Simplified Finite-Element Representation of a Reinforced Concrete Structure Embedded in a Homogeneous Medium (i.e., Rock)

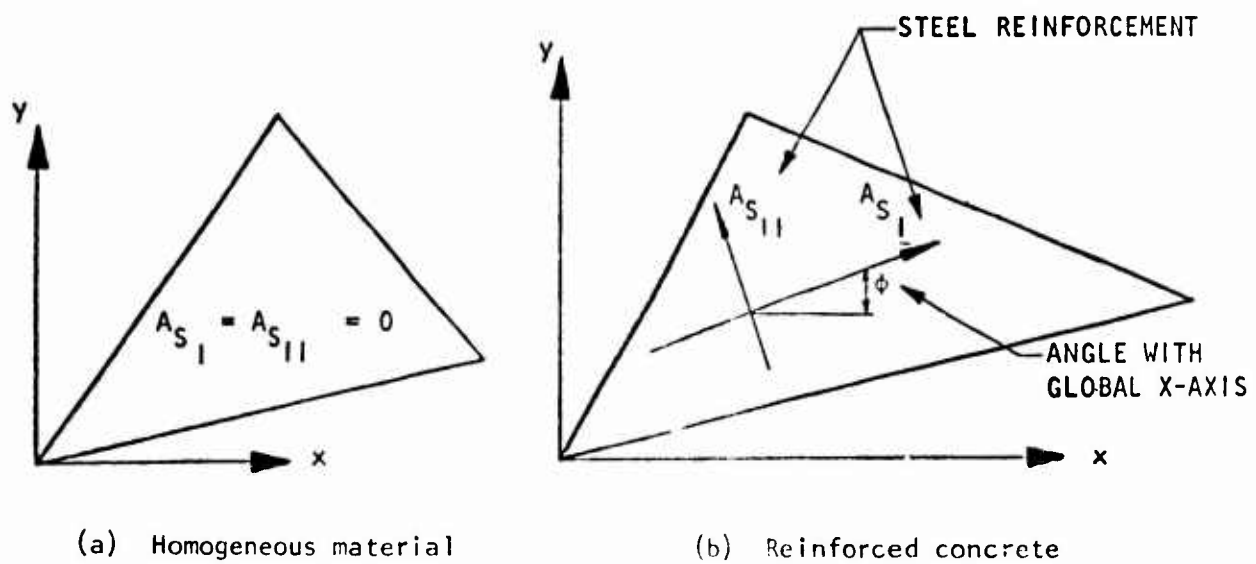
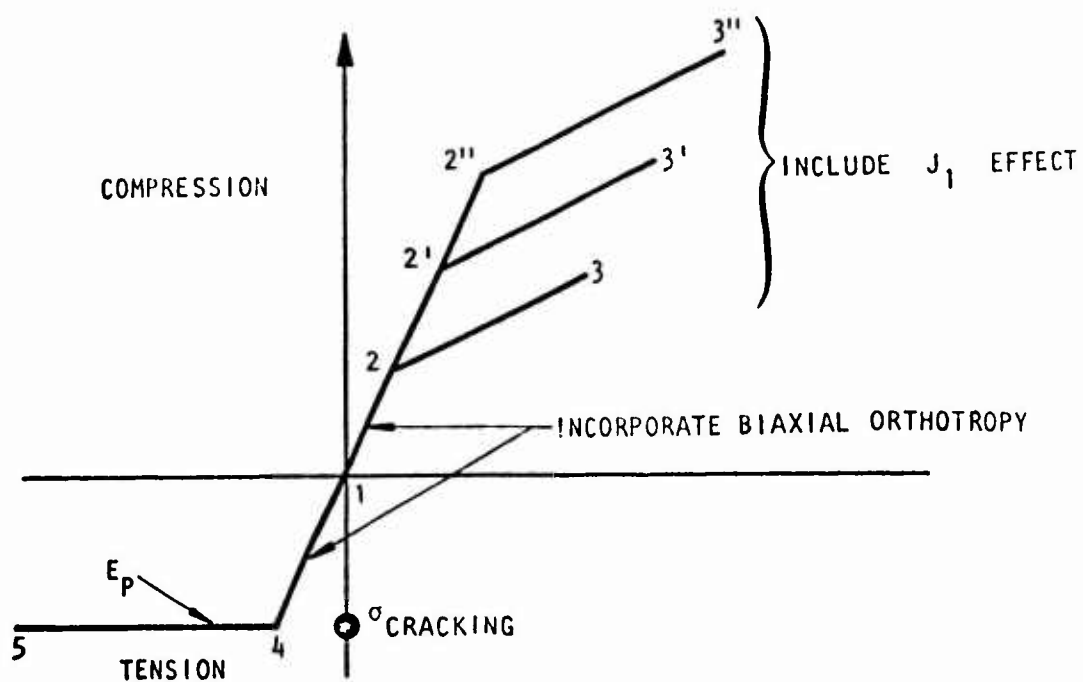


Figure 18. Finite Elements

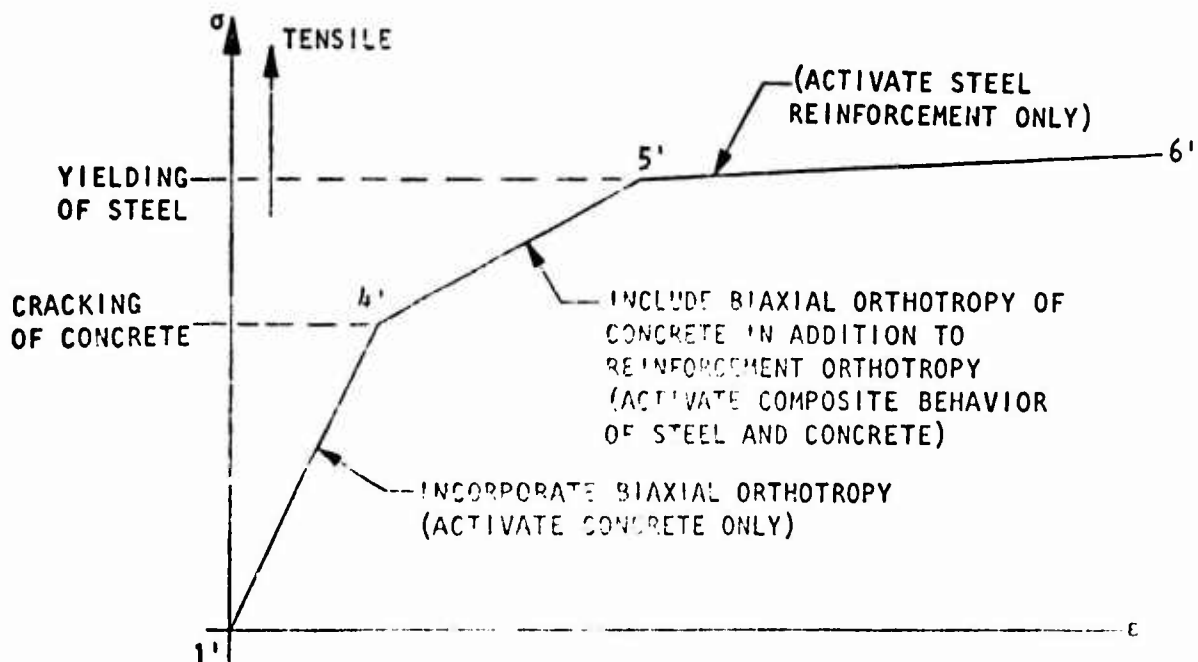
Both A_{s1} , A_{s11} , and ϕ may vary from element to element. While the method is addressed to reinforced concrete elements, it can also be applied to homogeneous elements of plain concrete or rock, etc., by eliminating from the analysis the terms corresponding to the reinforcement as shown in figure 18(a). The objective of the reference 1 investigation is illustrated by figure 19. The compressive paths (1-2-3), (1-2'-3'), etc., of figure 19(a) are applicable for defining the tangent moduli for compressive states of stress for both composite and homogeneous elements, as discussed above. The tensile path (1-4-5) in figure 19(a) applies to homogeneous elements only. The composite behavior of a reinforced concrete element in tension is illustrated in figure 19(b). In this figure, the portion (1'-4') shows the composite behavior before cracking. After primary cracks occur, a reinforced concrete element behaves differently. The bond between steel and concrete allows the concrete to continue adding a portion of its stiffness to that of the reinforcing steel, resulting in a composite tensile modulus as illustrated by the segment (4'-5') in figure 19(b). The portion (5'-6') of figure 19(b) represents the condition when bond is completely broken and the composite modulus is determined by the plastic modulus of steel. At this stage, strains may become excessive.

Upon application of load, three components of stress develop in the element. The average element stresses σ_1 , σ_2 , and σ_3 in the principal directions of orthotropy are used to check and update the properties of the entire element. At the end of each load step, a check is performed in each of the three orthotropic directions, and the properties of the elements are changed according to the stress magnitudes, the loading/unloading history, and the orthotropic properties along the three directions. The three-way check can be summarized as follows.

If σ_1 is compressive, the modulus E_1 is obtained as shown above. However, if σ_1 is tensile and does not exceed cracking stress f'_t (see equations 7 and 8), an initial tensile modulus equal to the initial unconfined compressive modulus E_{oc} is used. In this case, the principal axes of orthotropy coincide with principal axes of stress. This behavior is represented by segments (1'-4') of figure 19(b), and the principal axes of orthotropy



(a) Idealized stress/strain relations for concrete (homogeneous material)



(b) Composite concrete tensile stress/strain relation

Figure 19. Composite Stress/Strain Relations

still coincide with the principal axes of stress. If, however, σ_1 exceeds the cracking stress f'_t , the principal directions of stresses are recorded as the initial cracking directions. The effective areas of steel A_{s_1} and A_{s_2} are found by resolving A_s and $A_{s_{II}}$ to the cracked direction and the cracking angle θ is recorded, as illustrated in figure 20. The cracking directions are thus recorded and used as a permanent reference for calculating and updating the orthotropic properties of the element. After cracking, the principal axes of stress may differ from the cracking axes. The present analysis assumes that these deviations are small and that fixing the cracking directions approximates the actual behavior of the structure under consideration. The check on cracking is repeated again for the σ_2 and σ_3 stresses, following similar steps.

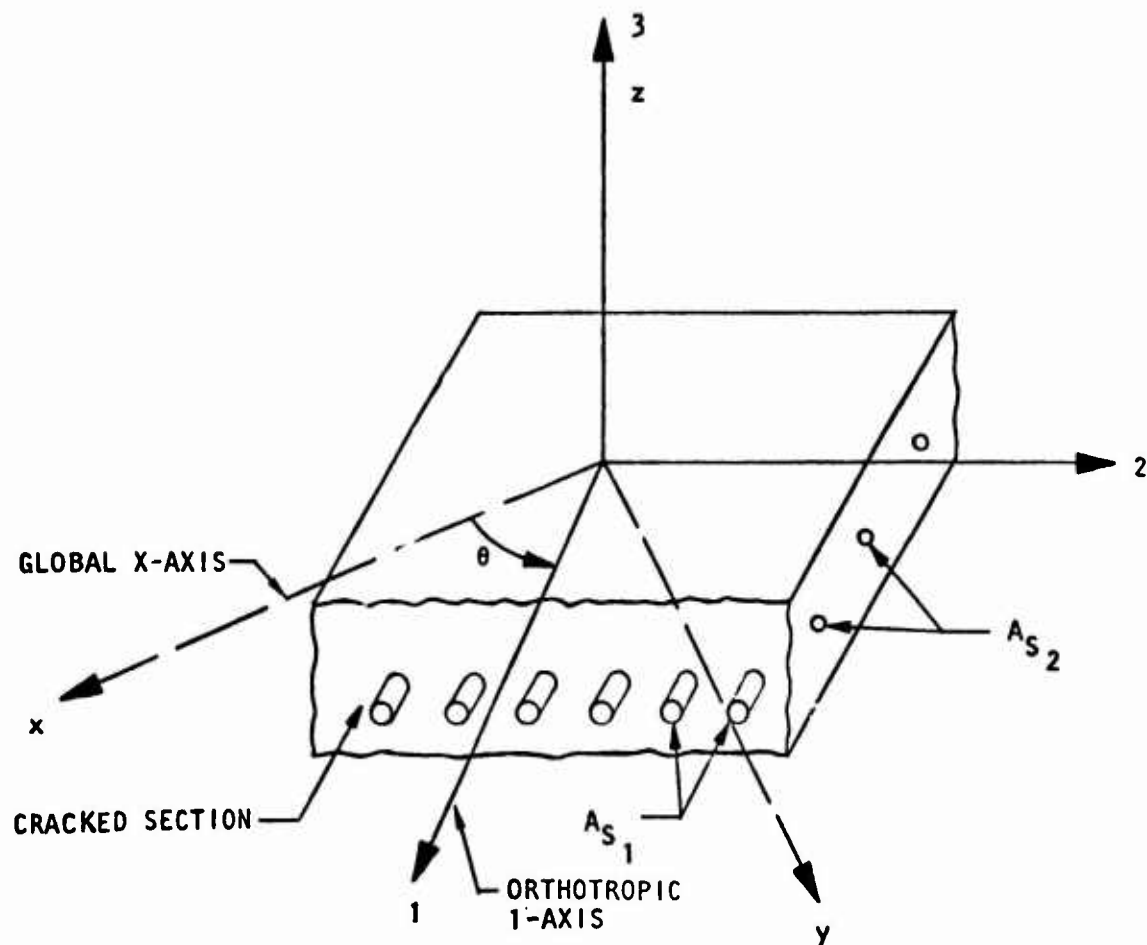
For increased stresses, a postcracking hypothesis is adopted. The present analysis used available experimental data (Ref. 33) on the extension of cracks in reinforced concrete members subjected to tension loads. It is based on the premise that, after initial cracking, the concrete contributes to the overall stiffness of the section by an amount proportional to the bonded length D , and that a variable λ can be used to estimate the reduced stiffness of the section at various stages of loading. The variable λ is defined as

$$\lambda = \frac{D}{d+D} \quad (9)$$

where D is the bonded length of concrete, and d is the unbonded cracked length of concrete.

The variable λ is used to compute the composite stiffness E_1 of the section, which is given by:

$$E_1 = \frac{1}{\left[\frac{1-\lambda_1}{E_s A'_{s_1}} + \frac{\lambda_1}{E_s A'_{s_1} + E_{c_1}} \right]} \quad (10)$$



$x, y, z \equiv$ GLOBAL COORDINATE SYSTEM

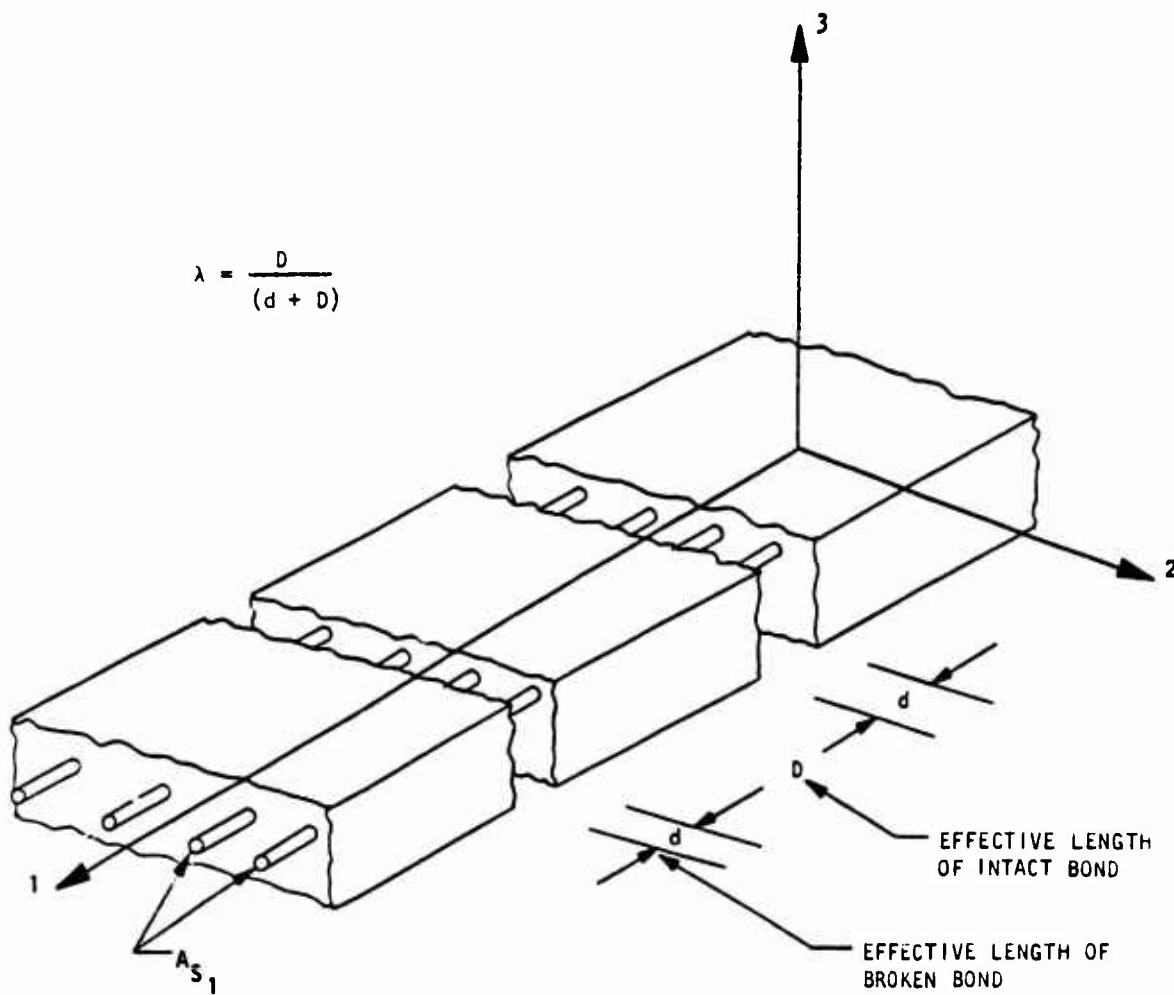
$1 \equiv$ COORDINATE PERPENDICULAR TO THE CRACKING PLANE (WEAK DIRECTION)

$\theta \equiv$ ANGLE BETWEEN 1 AND x AXES. (POSITIVE COUNTERCLOCKWISE)

$A_{S1}, A_{S2} \equiv$ REINFORCING STEEL IN THE DIRECTION AND PERPENDICULAR TO THE CRACKS

(a) Orientation of orthotropic axes 1, 2, 3 as defined by the initial cracking

Figure 20. Initiation of Cracks and Bond Slip in the Principal Direction



(b) Deterioration of bond produces decrease in λ

Figure 20. Initiation of Cracks and Bond Slip in the Principal Direction (concluded)

where

- E_s = Modulus of steel (E_{os} or E_{ps})
- A'_{s1} = Percentage of steel in the 1-direction
- E_{c1} = Concrete initial modulus in tension (E_{oc})
- λ_1 = Cracking variable as computed for direction 1

The above formula was used successively to compute the composite moduli E_1 , E_2 and E_3 in the three orthotropic directions.

Before cracking, the value of λ is one and the concrete and steel contribute all their respective stiffnesses. However, after cracks occur, the first term in the denominator of equation 10 increases with the decrease of λ , while the second term decreases with decrease of λ . When bond becomes entirely broken, $\lambda = 0$, the second term disappears and the composite stiffness becomes the stiffness of the steel bars. Thus continuous change of λ accounts for the continuous loss of bond due to increased loads and provides a simple way for updating the composite properties of the element.

The variable λ describes the extent of bond deterioration. Bond begins to deteriorate at the onset of cracking of concrete and ends when the concrete ceases to add any stiffness to the assemblage. This range varies among materials and should be part of the input to the problem. The onset of yielding of reinforcing steel is the upper bound for the cracking range.

The composite shear modulus G_{12} was computed in a similar manner.

$$G_{12} = \frac{1}{2} \left\{ \left[\lambda_1 \frac{E_{c1}}{2(1 + \nu_{21})} + \frac{E_s}{2(1 + \nu_s)} A'_{s1} \right] + \left[\lambda_2 \frac{E_{c2}}{2(1 + \nu_{12})} + \frac{E_s}{2(1 + \nu_s)} A'_{s2} \right] \right\} \quad (11)$$

For a homogeneous element (plain concrete, rock, etc.), the second and fourth terms in the above equation vanish.

The form of equation 11 is somewhat analogous to the equation for shear modulus of an isotropic material, $G = \frac{E}{2(1 + \nu)}$. Equation 11 has two basic parts. First, a shear modulus is calculated as if the element isotropically possessed the effective E and ν in the 1-1 direction. A second calculation is made using the effective E and ν in the 2-2 direction. The average of the two shear moduli is assumed to be the effective shear modulus for the element. In several practical calculations of structural response, G_{12} based on equation 11 varied from 0.75 to 1.0 times the shear modulus for plain concrete near zero stress.

4. SUMMARY OF BASIC MODELS

A summary of the models that are discussed above is given in table 1.

From this summary it was concluded that the advantage of the discrete cracking element approach is its simplicity for computer programming and its ability to represent detailed bond-slip behavior at the exact position of the reinforcing. Disadvantages of this approach are the following:

- The parameters of the cracking element do not have a clear physical meaning and are therefore difficult to evaluate.
- The independent degrees of freedom of the cracking elements are the absolute displacements. It would therefore be possible for continuum elements on either side of the cracking element to move past each other if there were a reversal in sign of the load.
- In order for the cracking direction to be completely general, it is necessary to supply a cracking element for every nodal point in the assemblage.

Table 1. Summary of Basic Models

Model	Linkage Element (a)	Refined Linkage Element (b)	Nonlinear Model (c)	Refined Nonlinear Model (d)		Wall Panel Studies (e)		Continuum (f)
	Ngo & Scordelis	Ngo, Scordelis, & Franklin	Nilson	Franklin's 1st Scheme	Franklin's 2nd Scheme	Cervenka	Franklin	Isenberg & Adham
Finite Element	CST	CLS	CST	Frame	Q + Rod	Lattice	Q + Rod	LST
Linkage Element	Yes	Yes	Yes	No	Yes	No	No	No
Bond Slip	Linear--crude	Linear--crude	Nonlinear	No	Linear	No	No	Linear
Concrete Stiffness after Cracks	Crude	Crude	Yes	0	0 Normal to cracks	0	0	Yes
State of Stress	Uniaxial	Uniaxial	Biaxial	Uniaxial	Uniaxial	Biaxial	Biaxial	Plain strain
Basis of Stress-Strain Relation	Linear	Linear	Nonlinear Uniaxial	Nonlinear	Nonlinear	Nonlinear Uniaxial	Nonlinear Uniaxial	Bilinear Triaxial
Cracks	Predefined	Predefined	Calculated and remeshed	Vertical	Diagonal	Calculated	Calculated	Calculated
Dowel Shear	No	Crude	No	No	No	No	No	Linear
Aggregate Interlock	No	Crude	No	No	No	No	No	No
Unloading	No	No	No	No	No	Yes	No	Yes
Experimental Correlation	Fair (beam)	Fair (beam)	Fair (axial)	Poor (beam)	Poor (beam)	Good (wall panel)	Good (wall panel)	Good for adequate reinforcing

The advantage of the equivalent continuum approach is that it offers complete generality with respect to directions of cracking and orthotropy and with respect to the constitutive properties of plain concrete. A disadvantage is complexity of computer programming that is needed due to the orthotropic formulation of composite stress/strain relations.

5. OTHER MODELS OF REINFORCED CONCRETE

A brief summary of other studies of reinforced concrete behavior are included here for completeness. Bresler and Bertero (Ref. 34) studied the effects of repeated loads on reinforced concrete members. Selna (Refs. 35 and 36) developed a method of analyzing one-dimensional beams and frames as layered systems, assuming linear elastic behavior up to tensile cracking or perfectly plastic yielding. Time-dependent shrinkage and creep effects were included. Sandhu, Wilson, and Raphael (Ref. 37) performed two-dimensional stress analyses of plain concrete dams, including the effect of creep. Similar studies have been undertaken by several other investigators. Valiappan and Doolan (Ref. 38) made two-dimensional stress studies that included the effects of tensile cracking and elastic-plastic behavior in compression using an initial stress approach. The postcracking stiffness of cracked finite elements was set equal to zero.

In a recent report Yuzugullu and Schnobrich (Ref. 39) used the finite element method to study the inelastic behavior of shear wall-frame systems. They assumed that cracked concrete does not carry any tensile forces perpendicular to the cracks, but maintains some amount of shear stiffness because of the irregular surface of the crack. Uniaxial, perfectly plastic behavior was assumed for both the cracked concrete and reinforcing steel. A linkage element similar to that developed by Ngo and Scordelis (Ref. 2) was used.

Taylor et al. (Ref. 40) presented a finite element analysis that used an incremental, iterative solution to predict the nonlinear behavior of reinforced and prestressed concrete structures subject to cracking. The initiation and propagation of cracks were accounted for by redefining (within

each iteration) the finite element grid lines so that they more nearly coincided with the predicted crack paths. Linear strain elements were used to model the reinforcement. A linkage element (Ref. 2) was used to model the bond/slip mechanism. The solution is path-independent, and no attempt was made to discuss cyclic stress-strain states.

Studies of reinforced concrete slabs by the finite element method have been presented by Jofriet and McNiece (Ref. 41) and by Bell and Elms (Ref. 42), which incorporate progressive cracking in triangular or quadrilateral plate bending elements. Changes in the bending stiffness of elements due to cracking normal to the principal moment direction are accounted for by using a reduced flexural rigidity in forming the element stiffnesses. Comparisons between computed and experimental results are presented.

Wahl and Kasiba (Ref. 43) applied the finite element method to the analysis of prestressed concrete nuclear reactor structures, which were treated as axisymmetric solids. Rashid (Ref. 44) discussed an analysis method (more refined than that of Wahl and Kasiba) that accounts for cracking, temperature effects, creep, and load history.

A study was undertaken by Endebrock and Traina (Ref. 45) to determine the behavior of plain concrete under combined stresses and to formulate computer-oriented constitutive relations for concrete. One nominal concrete strength was tested under various loading combinations. The loading combinations included the uniaxial, biaxial, and triaxial states of stress with various combinations of compressive and tensile stresses. The test specimens were three-inch cubes. The test information was obtained as stress/strain records for the three principal directions of the cubical test specimens.

During the testing program it was noted that somewhat different strength values were obtained depending upon the orientation of the cube with respect to the applied loads and the direction of casting. The uniaxial compressive strength was not greatly affected by the orientation of the cubes; however, the biaxial and triaxial test results were noticeably affected by the cube

orientation. The failure mode of the cubes was identical to that described by several investigators. It was noted that the slightest confinement of a test cube in an intended uniaxial test would noticeably affect the maximum strength.

In triaxial compressive tests, it was noted that the maximum stress was greatly affected by the magnitude of the minor stress and somewhat affected by the intermediate stress. The strains were generally affected in the same manner.

The test results from this investigation were compared to available test results reported by other investigators. The scatter in results reported is rather large; however, many of the investigators used different testing procedures, equipment, and different shaped test specimens. The results of this investigation were bounded by the results reported by other investigators.

A model to predict concrete behavior was developed. The mathematical development of the model was theoretical; however, empirical results were incorporated into the model such that the test results were simulated. The model was used to predict the constitutive relations for concrete subjected to combined loads. The loads can be tensile or compressive. The model solution was incorporated in a computer program.

The experimental data generated by this study represent a substantial addition to the information available on multiaxial loading. However, the model developed uses one concrete strength only and does not account for the effect of loading and unloading under combined stresses.

SECTION III

RECENT DEVELOPMENT IN METHODS OF OBTAINING PARAMETERS OF A COMPLETE MODEL

One of the greatest difficulties in any attempt to analyze a reinforced concrete member is to select a material model general enough to allow good correlation with data from various types of loading tests and to evaluate the parameters appearing in such a model. Recently reported progress, summarized below, indicates that suitable descriptions of the behavior of plain concrete and of the influence of reinforcing are available. These elements will be combined in Section IV to obtain a model of reinforced concrete.

1. CONSTITUTIVE PROPERTIES OF PLAIN CONCRETE

Considerable work has been conducted on uniaxial behavior of plain concrete. However, most of this work does not account for the combined stress states encountered in refined analysis by the finite element method. The paper by Popovics (Ref. 46) on stress/strain relationships for concrete under uniaxial loading provides an excellent review of the literature. Information on the response of concrete under combined stresses is much more meager (Refs. 1, 4, 45, and 47 through 51), particularly with respect to general stress/strain relationships. Significant experimental work was reported by Kupfer, Hilsdorf and Rusch (Ref. 50). Buyukozturk, Nilson, and Slate (Ref. 52) have used the finite element method to develop an analytical model of plain concrete composed of aggregate elements and paste elements to study its behavior under two-dimensional combined stress states. Liu, Nilson, and Slate (Ref. 53) proposed an analytical form of stress/strain relationship of concrete in biaxial compression. In addition, orthotropic constitutive relationships were stated in a matrix form adapted to finite element models. They also compared their biaxial relation to those developed by Saenz (Ref. 54) and by Desayi and Krishnan (Ref. 55).

The following constitutive relations developed by Liu, Nilson, and Slate for concrete in biaxial compression are adopted.

$$\sigma_1 = \frac{\epsilon_1 E}{(1 - \nu\beta_1) \left[1 + \left(\frac{1}{1 - \nu\beta_1} \frac{E}{E_{sc}} - 2 \right) \left(\frac{\epsilon_1}{\epsilon_p} \right) + \left(\frac{\epsilon_1}{\epsilon_p} \right)^2 \right]} \quad (12)$$

where secant modulus at ultimate load $E_{sc} = \sigma_p / \epsilon_p$, and σ_p , ϵ_p are the peak stresses and strains in biaxial compression respectively, and

σ_1 = Stress in the 1-direction

ϵ_1 = Strain in the 1-direction

$$\mu_1 = \frac{\sigma_2}{\sigma_1}$$

E = Initial tangent modulus in uniaxial loading for unconfined compression

ν = Poisson's ratio in uniaxial loading for unconfined compression

The value of E can be computed from the equation of Pauw (Ref. 56) as recommended in the American Concrete Institute Code (318-71):

$$E = 33 W^{3/2} \sqrt{f'_c} \text{ psi} \quad (13)$$

$$(E = 42.7 W^{3/2} \sqrt{f'_c} \text{ N/m}^2)$$

in which W is the unit weight of the hardened concrete, in pounds per cubic foot (kg/m^3); and f'_c = ultimate cylinder strength of concrete in uniaxial compression, in pounds per square inch (Newton/m^2).

Pauw (Ref. 56) examined Poisson's ratio ν in a series of tests and found values near 0.20. Liu, Nilson, and Slate (Ref. 53) found that the ultimate strength σ_p could be computed from:

$$\left. \begin{array}{ll} \beta < 0.2 & \frac{\sigma_p}{\sigma_o} = 1 + \frac{\beta}{1.2 - \beta} \\ 1.0 \geq \beta \geq 0.2 & \frac{\sigma_p}{\sigma_o} = 1.2 \\ 5.0 \geq \beta \geq 1.0 & \frac{\sigma_p}{\sigma_o} = \frac{1.2}{\beta} \\ \beta > 5.0 & \frac{\sigma_p}{\sigma_o} = \frac{1}{\beta} \left(1 + \frac{1}{1.2\beta - 1} \right) \end{array} \right\} \quad (14)$$

and

$$\beta = \beta_1 = \sigma_2/\sigma_1 \text{ if the 1-direction is considered,}$$

and ϵ_p is computed from

$$\left. \begin{aligned} \beta &\leq 1 & \epsilon_p &= 0.0025 \\ \beta &> 1 & \epsilon_p &= (-500 + 0.55 \sigma_p) \times 10^{-6}, \sigma_p \text{ in psi} \\ & & &= (-500 + 7.98 \times 10^{-5} \sigma_p) \times 10^{-6}, \sigma_p \text{ in N/m}^2 \end{aligned} \right\} \quad (15)$$

where

$$\sigma_o = \text{Unconfined compressive strength}$$

In addition to Young's moduli, which must be computed for each of the three orthotropic directions, three Poisson's ratios must be computed: $\nu_{12} = \nu_{21}$, $\nu_{23} = \nu_{32}$, and $\nu_{31} = \nu_{13}$.

When cracking occurs, a simplification can be made. For example, for cracking in the plane perpendicular to the 1-direction, the coupling effects are removed and

$$\nu_{21} = \nu_{31} = 0 \quad (16a)$$

For simplicity it is assumed that

$$\nu_{23} = \nu_{32} = 0 \quad (16b)$$

Three concrete shear moduli are also required. Assuming that the 1-2 plane is uncracked,

$$G_{12} = \frac{E'_{1b} E'_{2b}}{E'_{1b} + E'_{2b} + 2E'_{2b} \nu} \quad (17)$$

in which

$$E'_{1b} = \frac{E \left[1 - \left(\frac{\epsilon_1}{\epsilon_p} \right)^2 \right]}{\left[1 + \left(\frac{1}{1 - \nu \beta_1} \frac{E}{E_{sc}} - 2 \right) \left(\frac{\epsilon_1}{\epsilon_p} \right) + \left(\frac{\epsilon_1}{\epsilon_p} \right)^2 \right]^2} \quad (18)$$

where

$$\nu_{12} = \nu_{21} = \nu_{31} = \nu_{13} = \nu_{23} = \nu_{32} = \nu \quad (19)$$

for biaxial compression and is equal to E for biaxial tension and tension-compression cases (for concrete in tension, it can be assumed without significant loss of accuracy that linear behavior is obtained up to failure; the elastic slope will be taken equal to the initial tangent modulus in compression)

and

$$E'_{2b} = \frac{E \left[1 - \left(\frac{\epsilon_2}{\epsilon_p} \right)^2 \right]}{\left[1 + \left(\frac{1}{1 - \nu\beta_2} \frac{E}{E_{sc}} - 2 \right) \left(\frac{\epsilon_2}{\epsilon_p} \right) + \left(\frac{\epsilon_2}{\epsilon_p} \right)^2 \right]^2} \quad (20)$$

for biaxial compression, and is equal to E for biaxial tension, and to E_2 for tension-compression case, where

$$E_2 = E \text{ in the 2-direction}$$

$$\beta_1 = \sigma_2/\sigma_1 \text{ for biaxial compression}$$

$$\beta_2 = \sigma_1/\sigma_2 \text{ for tension/compression}$$

Assuming that there is a crack in the plane perpendicular to the 3-direction,

$$G_{13} = G_{23} = 0 \quad (21)$$

It follows that

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \lambda' \frac{E'_{1b}}{E'_{2b}} & \lambda' \nu & 0 \\ \lambda' \nu & \lambda' & 0 \\ 0 & 0 & \frac{E'_{1b} E'_{2b}}{E'_{1b} + E'_{2b} + 2E'_{2b} \nu} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_{12} \end{bmatrix} \dots\dots\dots (22)$$

where

$$\lambda' = \frac{E_1' b}{\frac{E_1' b}{E_2' b} - \nu^2} \quad (23)$$

Liu, Nilson, and Slate (Ref. 53) recommended equation 22 for the constitutive relation of concrete in biaxial loading and indicated that it produces results that compare well with experimental data as illustrated in figures 21 and 22. However, they also indicated that no experimental data are presently available to check the shear term in the proposed relationship.

2. DOWEL ACTION OF REINFORCEMENT CROSSING CRACKS IN CONCRETE

If a shear force acts on the cracked specimen, the cracked surfaces may slip, the force being counteracted only by the crosswise bars. During slipping, the bars develop shears that cause flexural stresses. This resistance of the bars to slipping is called "dowel action."

Dowel action has been studied by several investigators. Jones (Ref. 57), Parmelee (Ref. 58), Krefeld and Thurston (Ref. 59), and Lorensten (Ref. 60) have studied dowel action in reinforced concrete beams.

Parmelee (Ref. 58) proposed that the transverse force-displacement relationship of reinforcement at an inclined crack can be approximated by considering a semi-infinite elastic rod subjected to tension, T , and transverse shear, V_s , and restrained at the end of the gap by embedment in an elastic medium (Fig. 23). The displacement, Δ , of such a rod depends to some extent on the effective width, a^* , of the longitudinal cracked zone (Fig. 24).

A brief summary of the previous work on dowel action has been presented by Bresler and MacGregor (Ref. 61).

The studies reported above have neglected the action of the stirrups and the effect of horizontal cracking of the concrete cover. No consideration has been given to the dowel action in the postcracking stage.

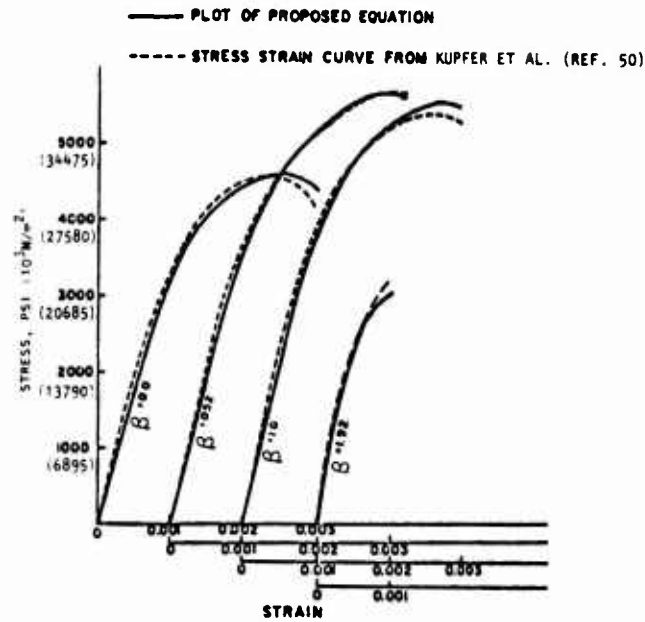


Figure 21. Comparison of Plot of Proposed Equation with Those Obtained from Biaxial Compression Tests (Ref. 50)

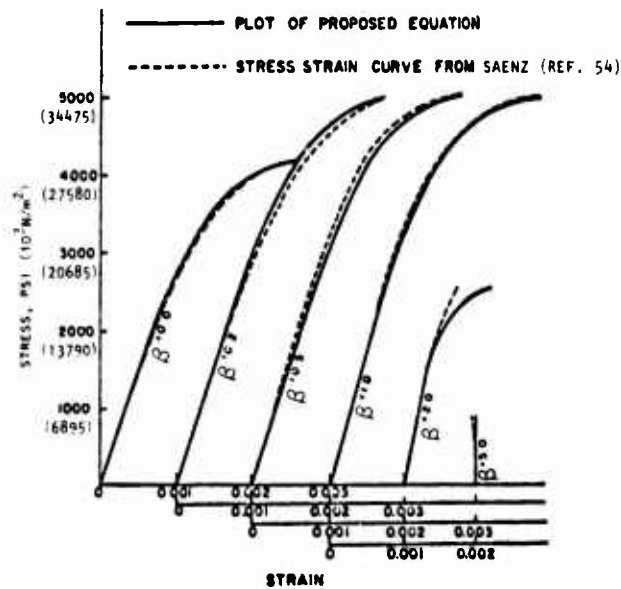


Figure 22. Comparison of Plot of Proposed Equation with Those Obtained from Biaxial Compression Tests (Ref. 54)

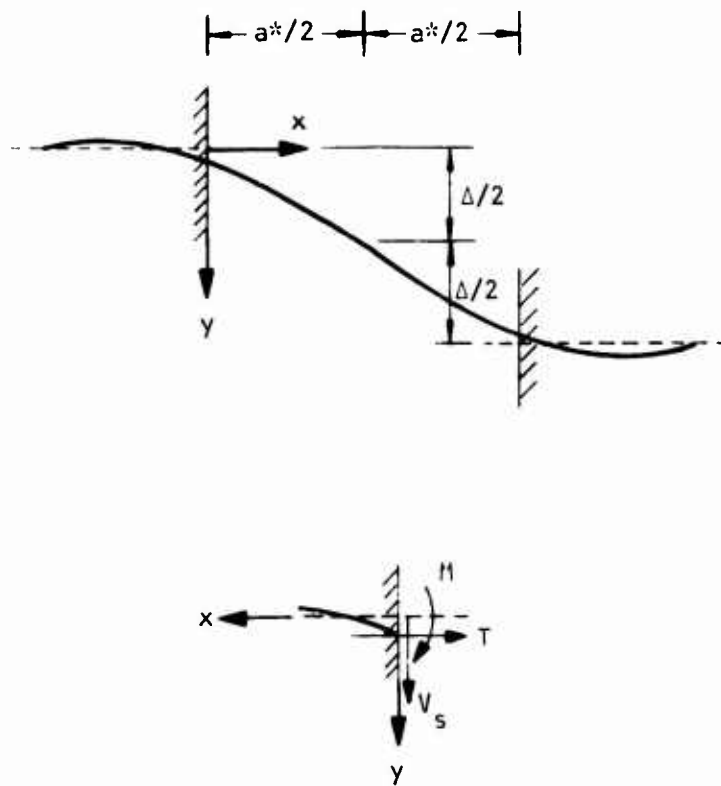


Figure 23. Dowel Action-Displacement

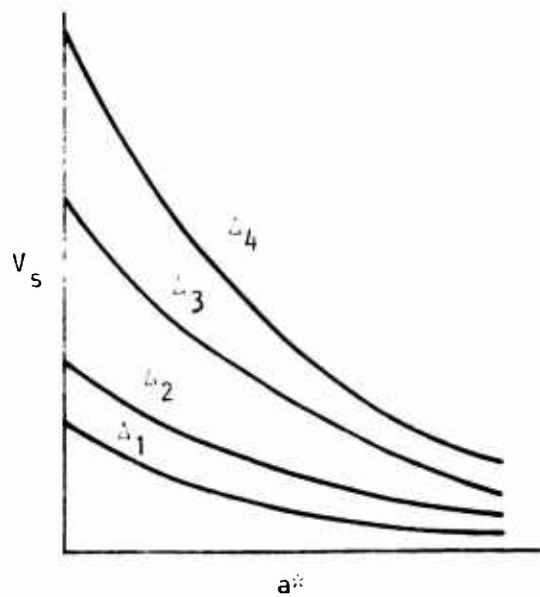


Figure 24. Dowel Force-Displacement Relationships
(after Parmelee--Ref. 58)

Dowel action in precracked and uncracked specimens was studied by Hofbeck, et al. (Ref. 62), Johnston and Zia (Ref. 63), and Peter (Ref. 22). It was also discussed in the ACI-ASCE Joint Committee 426 report (Ref. 64).

In a recent paper Dulacska (Ref. 65) conducted a test program to examine the phenomenon of dowel action in order to establish theoretical load-deformation relationships. The Dulacska tests were made using specimens designed as shown in figure 25. To simulate cracks, two layers of 0.0078-in.-thick (0.019812 cm) sheet brass, which were connected in the middle by a skewed steel stirrup, were embedded in the test specimens. During testing, relative slip along the simulated crack and opening of the crack perpendicular to the direction of the load were recorded.

Experimental results for the failure load in the bar were found to correlate well with the following relationship:

$$T_f = .\rho\phi^2\gamma\sigma_y n \sin \delta \left[\sqrt{\left(1 + \frac{\sigma_c}{3\rho\gamma^2\sigma_y n \sin^2 \delta}\right)} - 1 \right] \quad (24)$$

where

T_f = Failure load of dowel shear

ϕ = Bar size

δ = Angle of stirrups in degrees

σ_y = Yield stress of steel

σ_c = Cube strength of concrete

n = Coefficient of local compression of concrete

γ = Constant

$\rho = 1 - d^2/d_y^2$

d = Axial tension force of bar

d_y = Axial tensile force inducing yield in pure tension

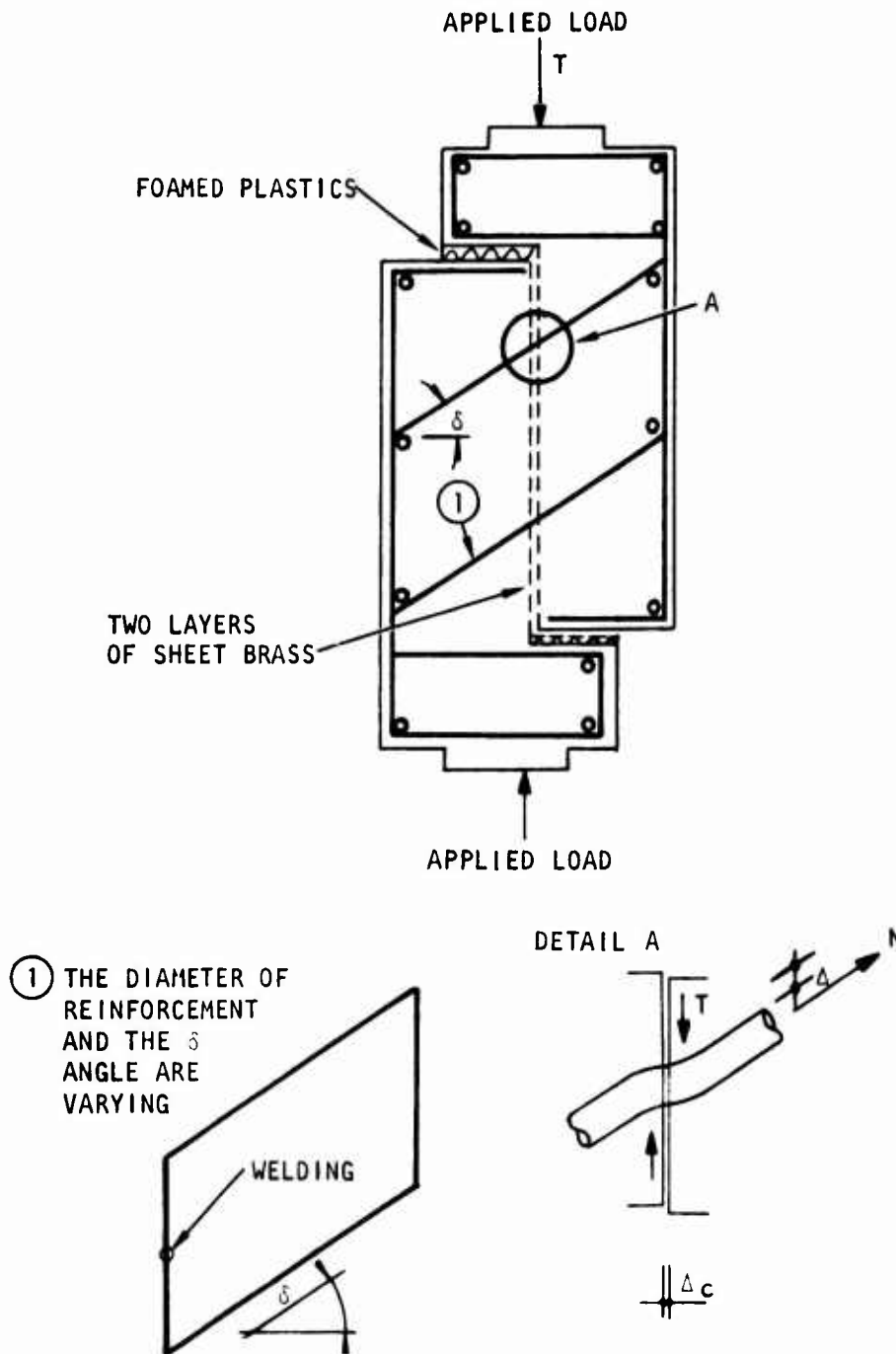


Figure 25. Details of Test Specimen Construction (Ref. 65)

From test results, it was found that slip in inches (cm) can be predicted by the approximate expression:

$$\Delta = \frac{\bar{A}T}{\phi} \sqrt{\frac{1}{\sigma_c} \tan \left(\frac{T}{T_f} \frac{\pi}{2} \right)} \quad (25)$$

where

\bar{A} = Constant

T = Dowel shear load

T_f = Failure load of dowel shear calculated from equation 24

ϕ = Bar size

σ_c = Cube strength of concrete

when T , ϕ , and σ_c are in consistent units.

Equation 25 was plotted as a function of T and Δ for two values of $\delta = 10^\circ$ and 40° respectively, as shown in figure 26.

The results indicate that an increase in the angle δ results in a decrease of the dowel shear force T and increase in the bar's normal force N . Also, an increase in the concrete strength σ_c results in an increase of dowel shear capacity T_f .

3. BOND SLIP

Bond slip enters constitutive equations for reinforced concrete through the effective moduli of the composite material. It may be defined quantitatively as the percentage of its own stiffness that concrete contributes to the stiffness of a composite element.

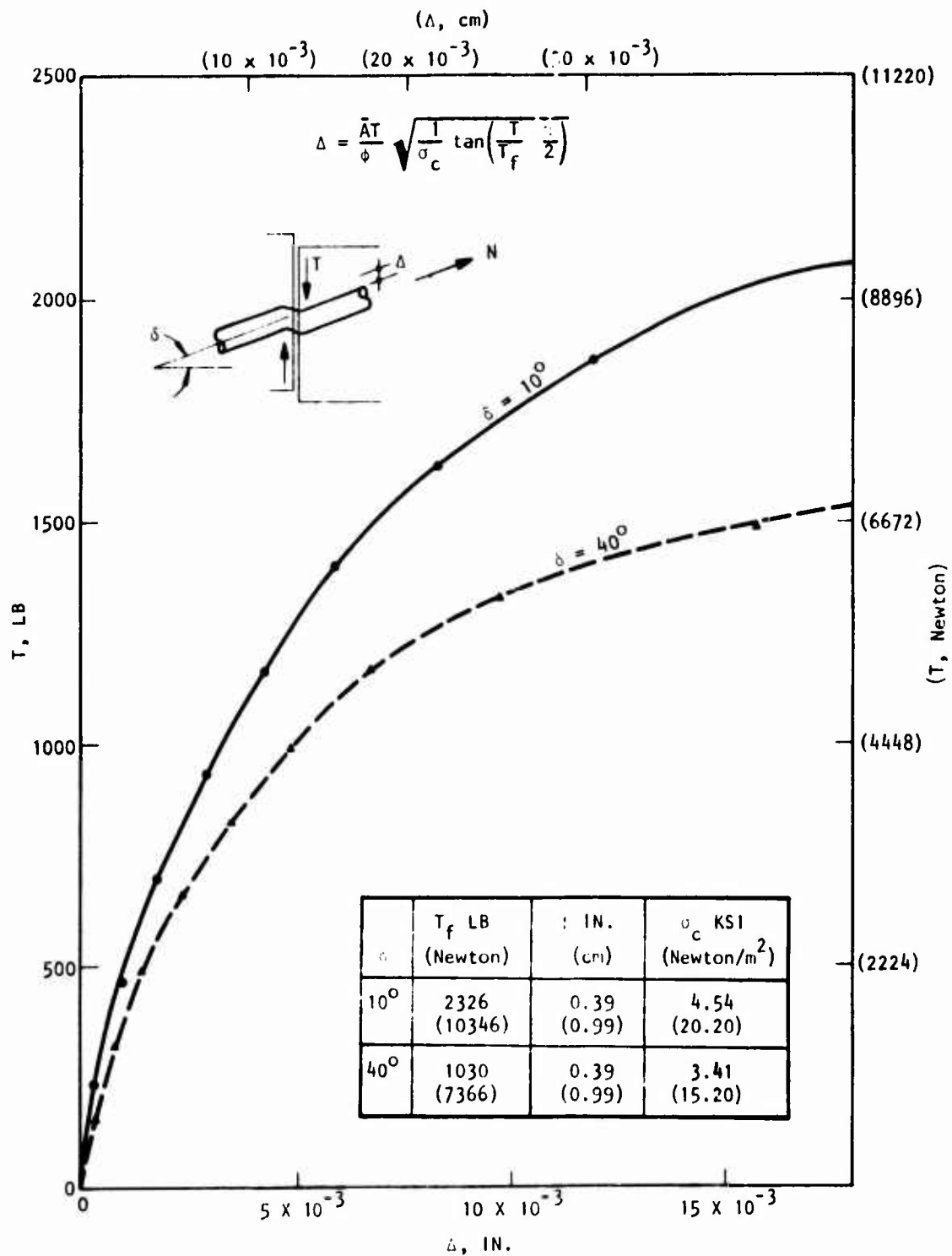


Figure 26. Dowel Shear Force-Slip Relationship (Ref. 65)

The new work bearing on this subject is that of Ismail and Jirsa (Ref. 66). An expression for λ , the ratio of concrete contributing to overall stiffness of the section was defined as follows:

$$\lambda = 1 - \frac{\int_0^L \epsilon_s dx}{\frac{TL}{A_s E_s}} \quad (26)$$

where

ϵ_s = Steel strain

T = Tensile force

A_s = Cross-sectional area of steel

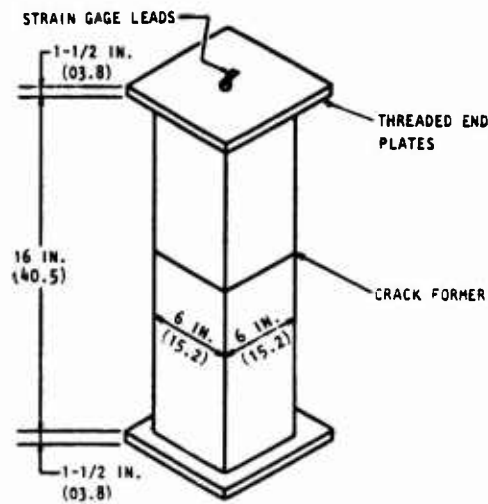
E_s = Young's modulus for steel

L = Length of specimen

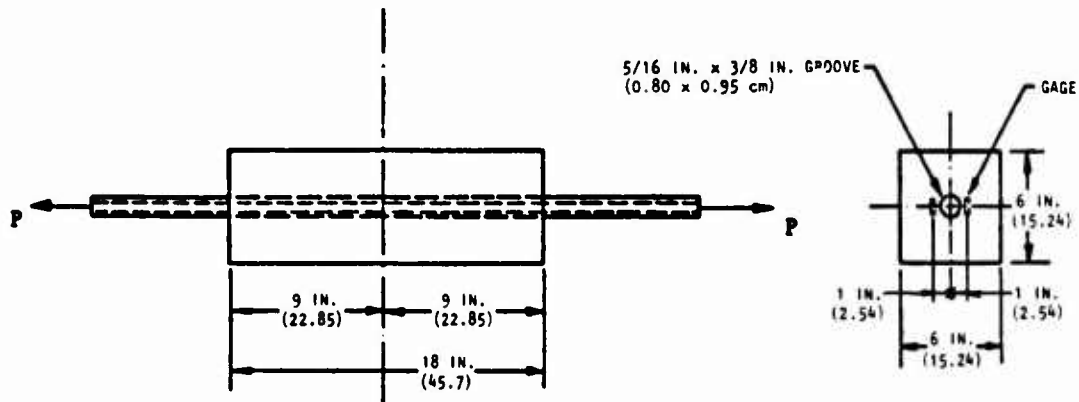
The length of test specimen was 16 in. (40.6 cm), selected to limit the formation of secondary cracks between the ends and the primary crack at the midlength. The location of the primary crack was determined by inserting greased sheet metal strips (crack formers) in the form prior to casting, as illustrated in figure 27(a). The concrete strength f'_c was 4.75 ksi ($32751 \times 10^3 \text{ N/m}^2$) for two specimens and 3.25 ksi ($22409 \times 10^3 \text{ N/m}^2$) for the other two. Data from this work are shown in figure 27 as a relationship between λ and σ_s/σ_y , where σ_s is the stress in the steel reinforcing bar and σ_y is yield stress of this bar. The data indicate that λ can be presented as a function of

- Concrete strength
- Steel stress (fraction of yield strength)
- Cycle of loading/unloading

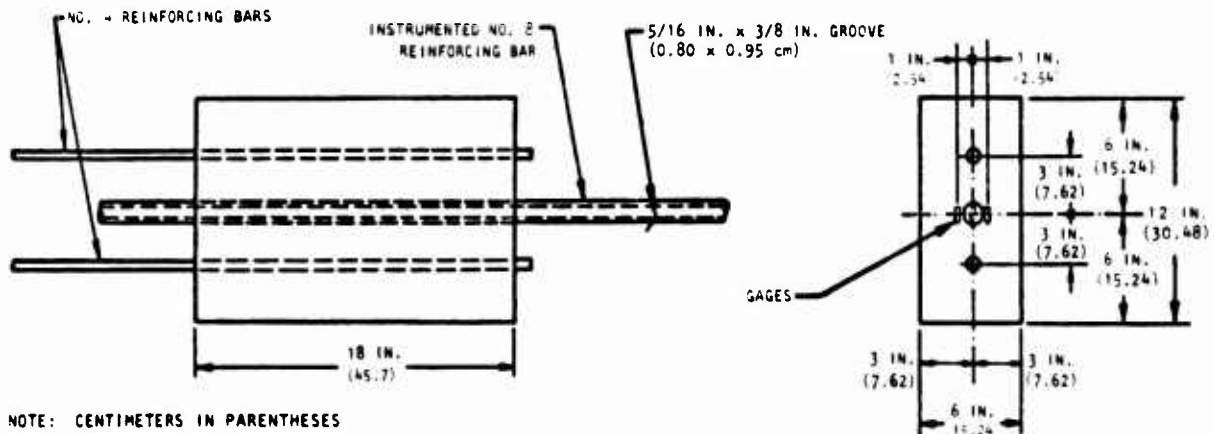
The experimental work by Tanner (Ref. 67) represents a valuable addition to the data on bond slip. The concentric pullout specimen, figure 27(b), was used to simulate conditions obtained in the constant moment region of a



(a) Dimensions of test specimen used by Ismail and Jirsa (Ref. 66)



(b) Specimen used in test series II by Tanner (Ref. 67)



NOTE: CENTIMETERS IN PARENTHESES

(c) Specimen used in test series III by Tanner (Ref. 67)

Figure 27. Test Specimens of Bond Slip (Refs. 66, 67)

reinforced concrete beam between tensile cracks. The lapped-splice specimen, figure 27(c), was designed to simulate spliced reinforcement in a constant moment region of a beam between tensile cracks.

Data from both tests are plotted for different concrete strength in figure 28. The results of Tanner's concentric pullout tests (11-B) are bounded by the results of Ismail and Jirsa (Ref. 66). However, the results of Tanner's lapped-splice specimens were considerably higher than those obtained from concentric pullout tests performed both by Tanner and by Ismail and Jirsa.

The linear bond-slip relation used by Isenberg and Adham (Ref. 1) is also indicated in figure 28.

The effect of loading and unloading on λ was studied by Ismail and Jirsa (Ref. 66) and indicates that λ is reduced by cyclic loading whenever the peak moments in the preceding cycle are greater than in any previous cycle.

4. CRACK WIDTH AND CRACK SPACING

Extensive investigations have been carried out concerning crack widths and crack spacing in reinforced concrete members (Refs. 33 and 68-84) and several theoretical and empirical formulas have been developed (Refs. 68-69, 75-76, and 84-86).

One of the simplest and most useful studies found in the references cited above is the one conducted by Broms (Ref. 33). Data on crack widths were obtained from flexural and tensile tests of members reinforced with steel bars, figures 29 through 33.

A tension member is shown in figure 29(a). The crack spacing at the level of the reinforcement was observed to decrease rapidly with increasing applied load. After the axial stress in the reinforcement had reached a certain critical value, the spacing of visible cracks remained approximately constant. It has been shown by Broms (Ref. 33) that high calculated axial tensile stress will be present within an area located inside a circle that is inscribed between

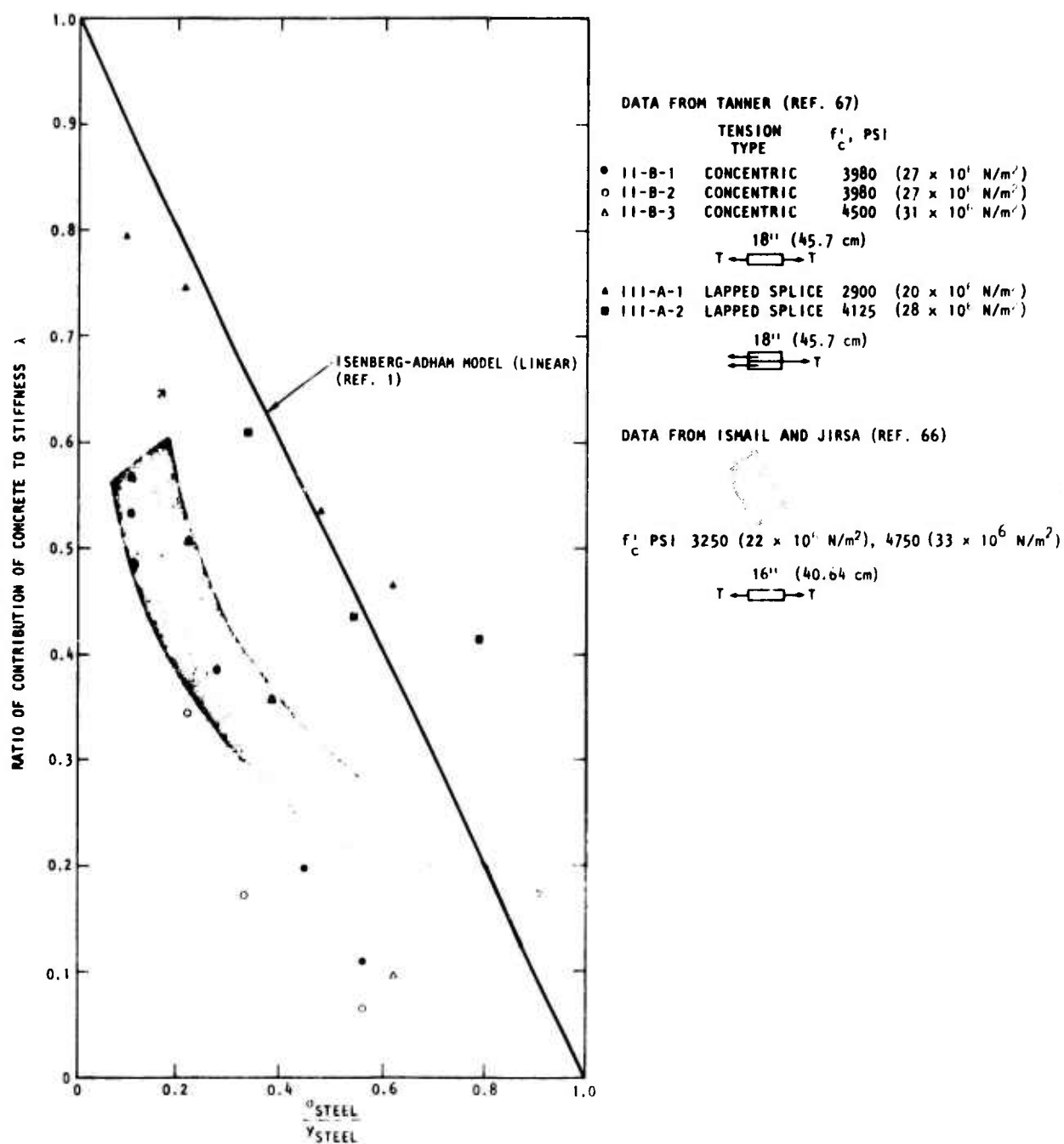


Figure 28. Experimental Bond-Slip Results (Refs. 66, 67) Compared with Analytical Results (Ref. 1)

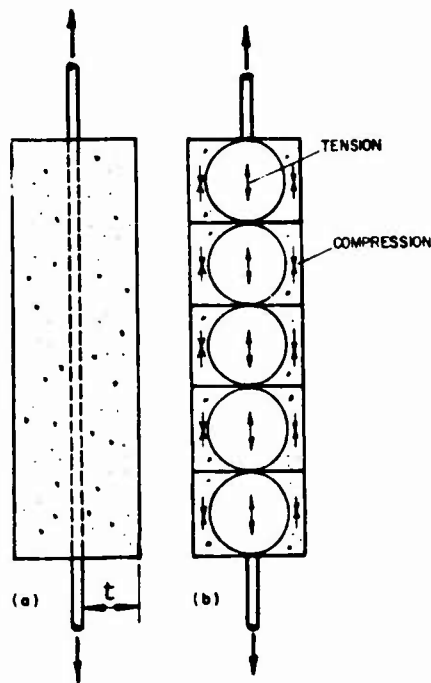


Figure 29. Tension Members (Ref. 33)

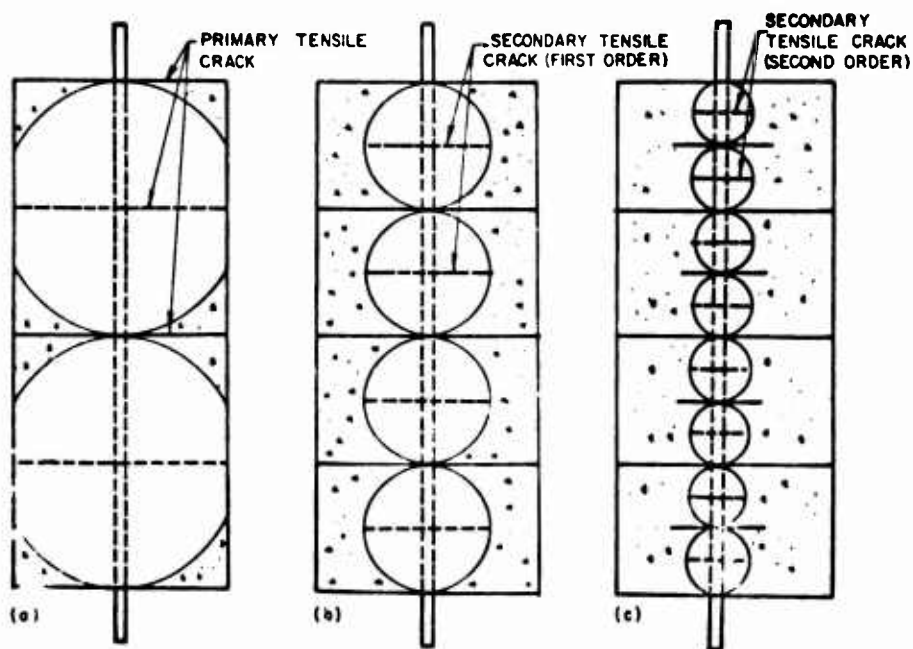


Figure 30. Mechanism of Tension Cracking (member reinforced with one bar) (Ref. 33)

two adjacent preexisting cracks. Outside this stress circle, compressive stresses or very small tensile stresses will be present, as illustrated in figure 29(b).

According to Broms (Ref. 33),

When the maximum tensile stress within any one stress circle exceeds the tensile crack strength of the concrete, a new tensile crack develops. This tensile crack will spread laterally until the average tensile stress at the root of the crack decreases to a value smaller than the tensile strength of the concrete. This appears to occur when the crack approaches the periphery of the corresponding inscribed circle. For the case when the member is reinforced with a single reinforcing bar, the length of this new tensile crack will be governed by the diameter of the circle inscribed between two adjacent preexisting cracks and thus, by the crack spacing. If the diameter of the inscribed circle is equal to or larger than the total width of the member shown in figure 30(a), then the normal crack will traverse the total section of the member. Such a crack is defined as a primary crack. If on the other hand, the diameter of the inscribed circle is less than the total width as shown in figure 30(b), then the new crack (which forms halfway between two existing primary cracks) will extend over only part of the total member width. This crack will be defined as a secondary crack. Therefore as cracking proceeds, the length of subsequent cracks will decrease in proportion to the crack spacing as shown in figure 30(c).

The length of the new cracks that develop in a member reinforced with several bars will depend on the spacing of the individual bars and on the primary crack spacing. In the case when the primary crack spacing is larger than the spacing of the reinforcement (Figs. 31(a) and 31(b)), the individual stress circles corresponding to each reinforcing bar overlap. As a result, the tensile cracks that develop at each individual bar join into a single crack that extends over part or all of the width of the member. The new tensile crack will extend to the unloaded vertical sides of the member (and will become a primary crack) if the stress circle corresponding to the bar located closest to the side of the member reaches the side of the member (Fig. 31(a)). This condition occurs when primary crack spacing is larger than twice the thickness of the side cover. When the primary crack spacing is less than twice the concrete cover (Fig. 31(b)), then the new tensile crack will not reach the surface of the member and will become a secondary crack.

The preceding analysis suggests that the absolute minimum visible crack spacing will be equal to the distance from the surface to the center of the bar located closest to the surface of the member. This distance is defined as the distance t . Thus, it is suggested that the theoretical minimum crack spacing will be equal to the thickness of the concrete cover (measured from the center of the reinforcing bar located closest to the considered face).

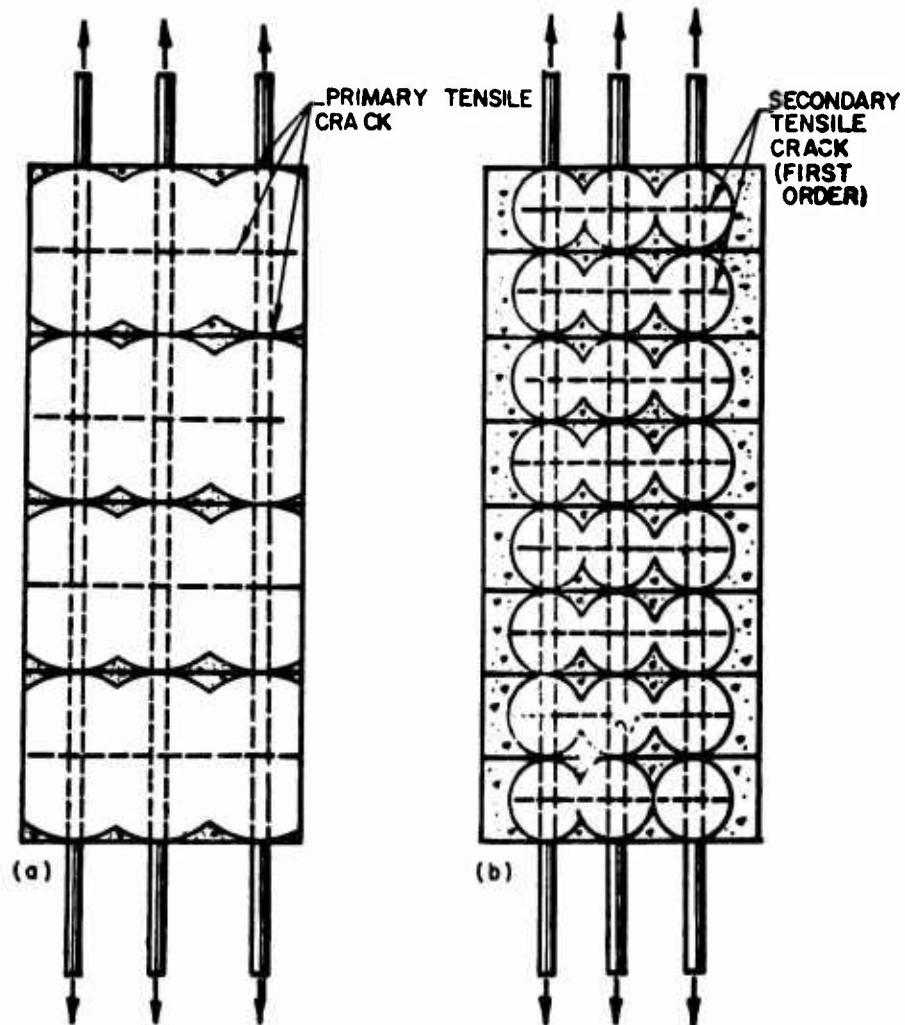


Figure 31. Mechanism of Tension Cracking (member reinforced with several bars) (Ref. 33)

The formation of flexural cracks affects the longitudinal stress distribution in flexural members. The stress distribution within a cracked flexural member can be calculated approximately from theory of elasticity. It can be shown that high tensile stresses will be present within the circular area located between two adjacent cracks, as illustrated in figure 32. Outside this circular area, compression stresses or small tensile stresses will be present. The stress circles governing the stress distribution for the tension zone have been redrawn in figure 33(a). When a set of cracks occurs approximately halfway between existing primary cracks, the new cracks will spread laterally until they reach the periphery of the corresponding stress circles where the intensity of the average axial tensile stress is low. If the primary crack spacing is less than twice the distance from the level of the reinforcement to the neutral axis, then the length of the new cracks will be less than that of the original primary flexural cracks (which extend to the neutral axis) and the new flexural cracks will become secondary flexural cracks.

The secondary flexural cracks (shown in figure 33(b)) will cause a stress redistribution in the immediate vicinity of the reinforcement. A new set of cracks, secondary flexural cracks of the second order, develops when the maximum tensile stress in the concrete exceeds the tensile strength of the concrete. The lengths of these new tensile cracks will be governed by the diameter of the corresponding stress circles (the distance between two adjacent tensile cracks). If the crack spacing for adjacent cracks is less than twice the thickness of the bottom cover of the flexural members, then the corresponding stress circles and the new set of tensile cracks will not reach the bottom face of the member, as shown in figure 33(b).

In summary, the work of Broms indicated that the assumption of a uniform linear stress distribution in cracked reinforced concrete may lead to large errors when the calculated crack spacing of the main cracks approaches twice the thickness of the concrete cover or the spacing of the individual reinforcing bars. The absolute minimum visible cracking spacing was found to be equal to the thickness of the concrete cover. The average minimum crack spacing will be larger than the absolute minimum crack spacing given above.

$$t < \text{crack spacing} < 2t \quad (27)$$

where t is the thickness of the concrete cover as indicated in figures 29 and 33.

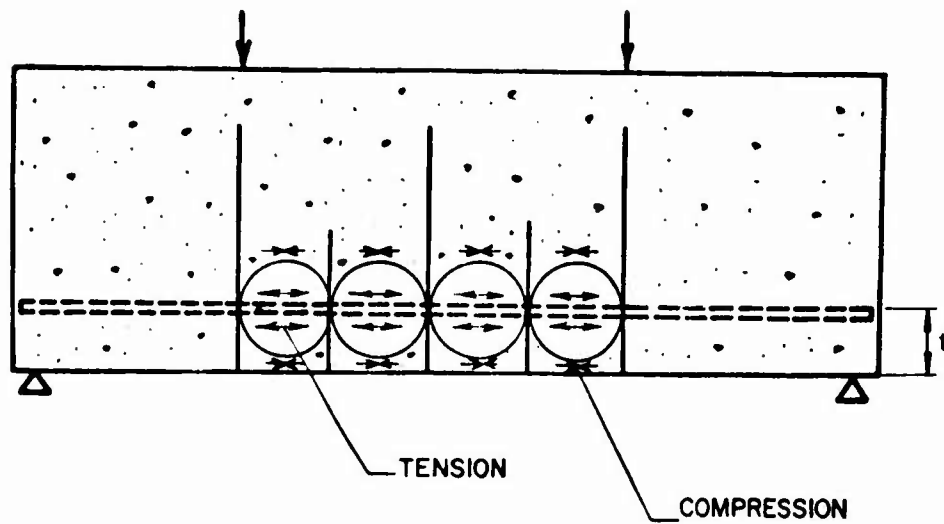
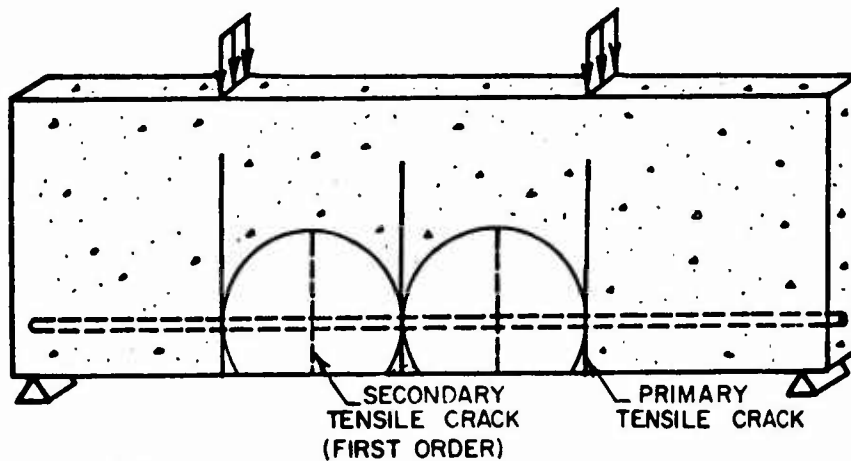
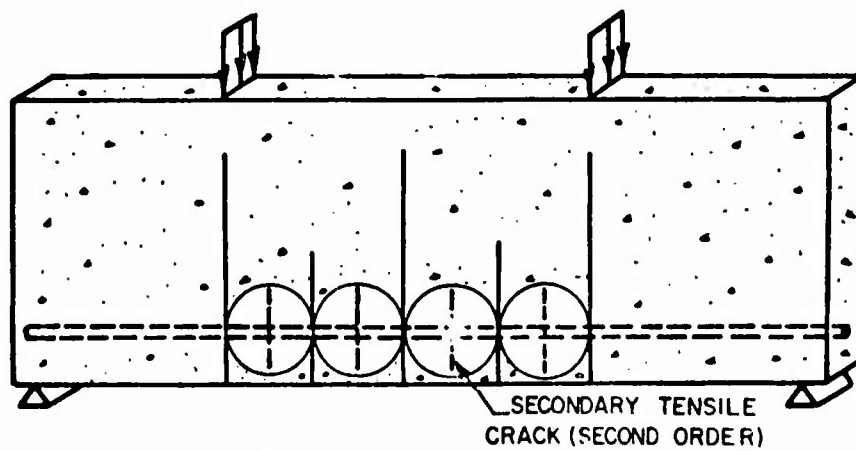


Figure 32. Flexural Members (Ref. 33)



(a) Primary and first order second cracks



(b) Second order secondary cracks

Figure 33. Mechanism of Tension Cracking (flexural members) (Ref. 33)

From the observation that the average crack spacing is approximately equal to twice the distance t , Broms (Ref. 33), found that the crack width can be calculated from the relation

$$W_{ave} = 2t\epsilon_s \quad (28)$$

where W_{ave} = crack width or the total elongation of the reinforcement, and ϵ_s = is the average steel strain.

SECTION IV

FORMULATION OF PRESENT MODEL

This section summarizes the formulation of the mathematical model developed under the present contract. This model is essentially an improved version of the Isenberg/Adham model (Ref. 1). The improvements were based on pertinent studies reviewed at the beginning of this contract. No new development of fracture theory or nonlinear constitutive relations was included in the scope of the present work. Some features of the model such as concrete stress/strain relations were discussed in previous sections, and their details are not repeated. Other features such as the dowel action formula are modified and their details are included in this section. The model was formulated in steps that are summarized below.

1. STEPS TO FORMULATION OF THE MODEL

The model was formulated in two basic steps:

STEP 1. This step establishes the stress/strain relations in principal directions of orthotropy. The formulation includes the following elements:

- Concrete stress/strain relations
- Steel properties
- Bond-slip relations
- Dowel action
- Composite moduli of reinforced concrete

The principal directions of orthotropy were defined as follows:

- Before cracking, principal directions of orthotropy are defined by principal directions of stress.
- Direction of first cracking defines principal direction of orthotropy, while second and third principal direction of orthotropy are directions of the largest stress and its normal.

STEP 2. This step involved matrix transformation from principal directions of orthotropy to global directions and included:

- Formulation of matrix of direction cosines
- Assembling global stiffness matrix

The details of the above steps are given below.

2. CONCRETE STRESS/STRAIN RELATIONS

Two types of plain concrete stress/strain models are incorporated in the present analysis:

a. Variable Modulus Model

The virgin loading stress/strain relations incorporated in this model are those suggested by Liu, Nilson, and Slate in reference 53 and described in detail in Section III. In these relations the moduli of elasticity in orthogonal directions are expressed as functions of strain and confinement. The moduli E_i and G_{ij} are functions of the state of stress and strain and take the form

$$E_i = \frac{E_c \left[1 - \left(\frac{\epsilon_i}{\epsilon_p} \right)^2 \right]}{\left[1 + \left(\frac{1}{1 - \nu \beta_i} \frac{E_c}{E_{sc}} - 2 \right) \left(\frac{\epsilon_i}{\epsilon_p} \right) + \left(\frac{\epsilon_i}{\epsilon_p} \right)^2 \right]^2} \quad (29)$$

$$G_{ij} = \frac{E_i E_j}{(1 + \nu)(E_i + E_j)} \quad (30)$$

where

- E_i, E_j = Tangent moduli in directions i, j
- E_c = Initial tangent modulus for concrete
- ϵ_i = Strain in direction i
- ϵ_p = Strain at peak stress
- ν = Poisson's ratio for concrete
- E_{sc} = Secant modulus based on ultimate stress and corresponding strain
- β_i = Ratio of principal stress in the direction normal to the principal stress in the direction considered

The peak stress, σ_p , due to combined loading is expressed as

$$\left. \begin{aligned} \sigma_p &= \left[1 + \frac{\beta_i}{1.2 - \beta_i} \right] \sigma_o && \text{for } \beta_i < 0.2 \\ &= 1.2 \sigma_o && \text{for } 1.0 \geq \beta_i \geq 0.2 \\ &= \frac{1.2 \sigma_o}{\beta_i} && \text{for } 5.0 \geq \beta_i \geq 1.0 \\ &= \frac{\sigma_o}{\beta_i} \left(1 + \frac{1}{1.2\beta_i - 1} \right) && \text{for } \beta_i > 5.0 \end{aligned} \right\} \quad (31)$$

where

$\sigma_o = f'_c$, the unconfined cylinder strength in uniaxial compression.

The strain ϵ_p corresponding to σ_p is given as

$$\left. \begin{aligned} \epsilon_p &= 0.0025 \text{ for } \beta_i < 1 \\ &= (-500 + 0.55 \sigma_p) \times 10^{-6} \\ &= (-500 + 7.98 \times 10^{-6} \sigma_p) \times 10^{-6} \end{aligned} \right\} \text{ for } \beta_i > 1 \quad \begin{array}{l} \sigma_p \text{ in psi} \\ \sigma_p \text{ in N/m}^2 \end{array} \quad (32)$$

The secant modulus E_{sc} is given by

$$E_{sc} = \frac{\sigma_p}{\epsilon_p} \quad (33)$$

The corresponding incremental stress/strain relation is given by

$$d\sigma_i = \frac{E_i d\epsilon_i}{1 - \nu\beta_i} \quad (34)$$

where $d\sigma_i$ is the stress increment in the i -direction, and ν has a constant value equal to Poisson's ratio for concrete.

The descending branch of the stress/strain relation in unconfined compression is not represented in the model. The reason is that including it would require either a negative tangent modulus or complete reliance on the load vector correction method of integrating equations of motion. The first requirement leads to numerical difficulties with implicit integration methods, while the second demands the capability for iteration in the main code. Since this would tend to produce a material property package which works for some codes and not for others, the falling branch is excluded. However, to include the descending branch of the stress/strain relation in a future model, general equations 12, and/or 29, should be modified to incorporate the point of failure or some other point on the descending branch of the curve.

Observation of the numerator in equation 29 reveals that the tangent modulus E_i becomes negative whenever ϵ_i exceeds ϵ_p . When this occurs, an arbitrary small value is substituted for E_i . This value is usually taken as 1 percent of E_c .

Another difficulty encountered is that equation 29 becomes singular when

$$\nu\beta_i = 1 \quad (35)$$

This problem was avoided by arbitrarily setting $\beta_1 = 0.0$. Therefore, in the present model no additional strength is obtained under biaxial or triaxial confinement.

Figure 34 shows the stress/strain relations for plain concrete in compression, used in the present model. Unloading and reloading occurs with the initial tangent modulus E_c .

The concrete modulus in tension is assumed to have a constant value E_c until cracking. However, after cracking, the concrete modulus is set to a minimal value to avoid numerical difficulties, while the stress is set to zero (Fig. 35). In the event of rebonding, after tension cracks have been closed, the modulus is set to a user specified fraction α of the initial modulus, E_c . Rebonding is indicated in the code by a change of the sign of strain.

The cracking strength of concrete f'_t in any given direction was discussed in Section II and can be expressed as

$$f'_t = f_t - n\sigma_2 \quad (36a)$$

for $\sigma_2 < 0$ and $\sigma_2 < \sigma_3$

where

f_t = Unconfined tensile strength of concrete

n = Slope of cracking envelope to be determined experimentally (Fig. 15)

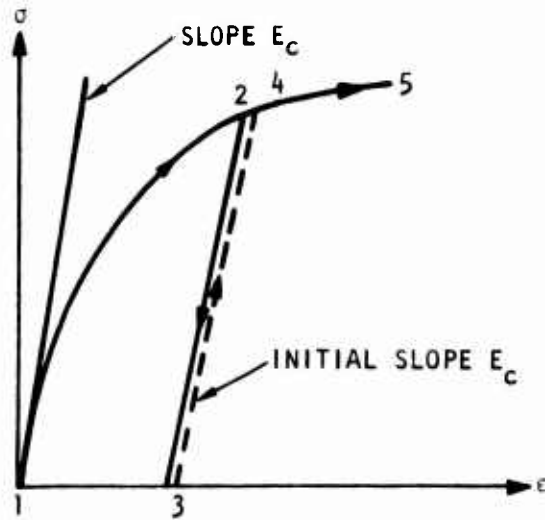
σ_2 = Minimum nonpositive stress normal to the tension axis

Equation 36(a) can also be expressed as

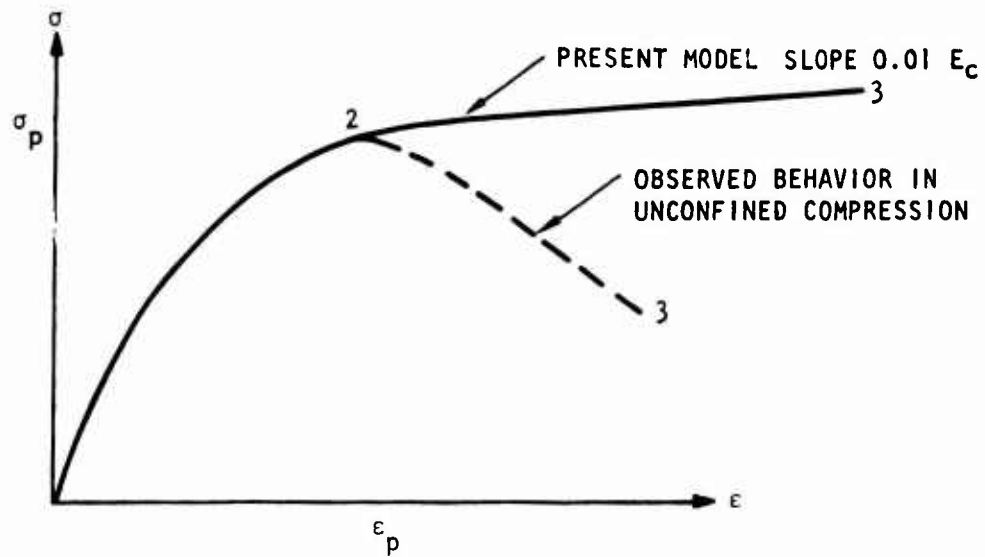
$$f'_t = f_t \left[1 - \frac{\sigma_2}{f'_c} \right] \quad (36b)$$

where

f'_c = Unconfined compressive cylinder strength of concrete



(a) Virgin loading and unloading/reloading behavior of present model



(b) Present model does not account for descending portion of compressive stress/strain relations for plain concrete

Figure 34. Stress/Strain Relations for Plain Concrete

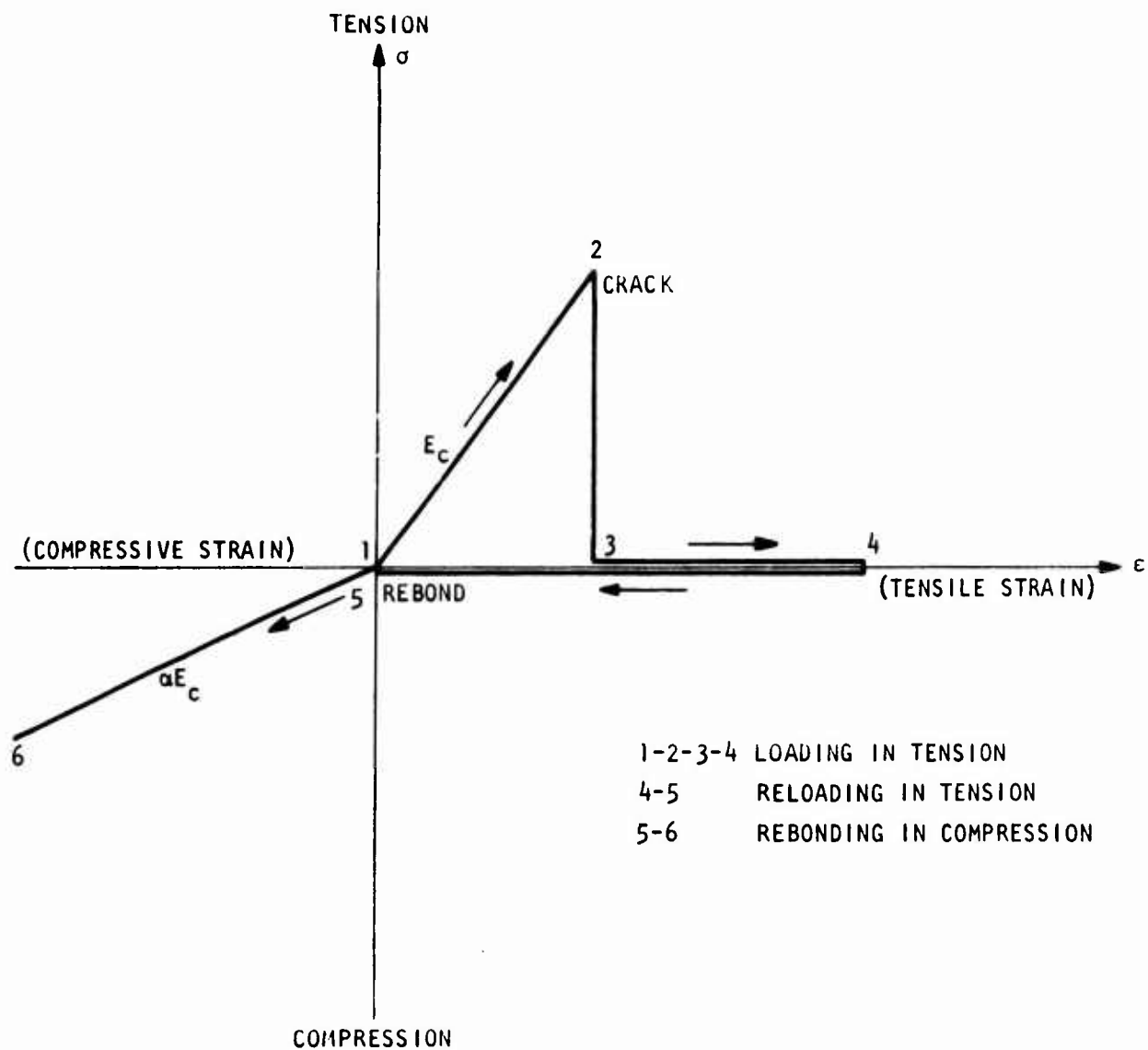


Figure 35. Stress/Strain Relations for Plain Concrete in Tension for Variable Modulus Model

Equation 36(a, b) accounts for the change in ultimate tensile strength due to the interaction of perpendicular stress components.

b. Plastic Capped Model

In addition to the variable moduli stress/strain relations used in composite moduli, stress/strain relations based on a strain-hardening theory of plasticity are also included in the computer program. These relations were developed under a separate project (Ref. 51). This type of mathematical formulation does not readily lend itself to the composite moduli approach, and hence it is included in the computer program for those cases where steel is absent or can be neglected.

The yield surface consists of two segments F_1 and F_2 defined as

$$\left. \begin{aligned} F_1 &= \sqrt{J_2'} + g_1(J_1) - K_1 \\ F_2 &= \frac{(\sqrt{J_2'})^2}{2} + r^2 \frac{J_1^2}{2} - K_2 \end{aligned} \right\} \quad (37)$$

where

- J_1 = First invariant of stress
- J_2' = Second invariant of stress deviator
- $g_1(J_1)$ = A function of J_1
- r = Constant
- K_1, K_2 = Hardening parameters

The hardening laws used are

$$d\sqrt{I_2'}^p = h_1 dF_1 \quad (38)$$

and

$$d\epsilon_{vc}^p = h_2 dF_2 \quad (39)$$

where I_2^P is the second invariant of the plastic-strain deviator and ϵ_{vc}^P is the volumetric plastic strain associated with compaction only. The variables h_1 and h_2 are functions of the stress and plastic strain invariants.

The yield surface and the hardening rules are used in conjunction with the flow rule

$$d\epsilon_{ij}^P = \frac{\partial F_1}{\partial \sigma_{ij}} \lambda_1 + \frac{\partial F_2}{\partial \sigma_{ij}} \lambda_2 \quad (40)$$

where $d\epsilon_{ij}^P$ are the plastic strain increments and λ_1 and λ_2 are factors to be determined.

The incremental stress/strain relations are expressed as

$$d\sigma_{ij} = C_{ijkl}(d\epsilon_{kl} - d\epsilon_{kl}^P) \quad (41)$$

where $d\sigma_{ij}$ is the stress increment, $d\epsilon_{kl}$ is the total strain increment and C_{ijkl} are the coefficients of the elastic stress/strain relationship.

Substitution of equation 40 in equation 41 yields

$$d\sigma_{ij} = C_{ijkl} d\epsilon_{kl} - C_{ijkl} \left(\frac{\partial F_1}{\partial \sigma_{kl}} \lambda_1 + \frac{\partial F_2}{\partial \sigma_{kl}} \lambda_2 \right) \quad (42)$$

The values of λ_1 and λ_2 are obtained from the solution of

$$A_1 \lambda_1 + A_2 \lambda_2 = A_3 \quad (43)$$

$$A_4 \lambda_1 + A_5 \lambda_2 = A_6 \quad (44)$$

where

$$A_1 = \frac{\partial F_1}{\partial \sqrt{J_2}} + 2 h_1 \frac{\partial F_1}{\partial \sigma_{ij}} C_{ijkl} \frac{\partial F_1}{\partial \sigma_{kl}} \quad (45)$$

$$A_2 = \frac{\partial F_2}{\partial \sqrt{J_2}} + 2 h_1 \frac{\partial F_1}{\partial \sigma_{ij}} C_{ijkl} \frac{\partial F_2}{\partial \sigma_{kl}} \quad (46)$$

$$A_3 = 2 h_1 \frac{\partial F_1}{\partial \sigma_{ij}} C_{ijkl} d\epsilon_{kl} \quad (47)$$

$$A_4 = \frac{h_2}{3} \frac{\partial F_2}{\partial \sigma_{ij}} C_{ijkl} \frac{\partial F_1}{\partial \sigma_{kl}} \quad (48)$$

$$A_5 = \frac{\partial F_2}{\partial J_1} + \frac{h_2}{3} \frac{\partial F_2}{\partial \sigma_{ij}} C_{ijkl} \frac{\partial F_2}{\partial \sigma_{kl}} \quad (49)$$

and

$$A_6 = \frac{h_2}{3} \frac{\partial F_2}{\partial \sigma_{ij}} C_{ijkl} d\epsilon_{kl} \quad (50)$$

It should be noted however that if yielding occurs on F_1 only

$$A_2 = A_4 = A_5 = A_6 = 0$$

Consequently

$$\lambda_2 = 0$$

$$\lambda_1 = A_3/A_1$$

Similarly, if yielding occurs on F_2 only

$$A_1 = A_2 = A_3 = A_4 = 0$$

and

$$\lambda_1 = 0$$

$$\lambda_2 = A_6/A_5$$

For an elastic case both λ_1 and λ_2 are zero.

Substitution of λ_1 and λ_2 in equation 41 results in the desired incremental stress/strain relations.

The actual functions used by the authors of reference 51 are detailed below, where the constants (y_1 through y_{25}) are model fitting parameters.

The yield surfaces are given by

$$F_1 = \sqrt{J_2'} - y_1 \left[y_2 - y_3 e^{J_1/y_4} \right] \left[1 - e^{(J_1 - y_5)/y_6} \right] - K_1 \quad (51)$$

and

$$F_2 = \frac{J_2'}{2} + y_7 J_1^2 - K_2 \quad (52)$$

with initial values of K_1 and K_2 given by

$$K_1 = y_{24} \quad (53)$$

$$K_2 = y_{25} \quad (54)$$

The hardening rules are given by

$$h_1 = y_8 \frac{1 - y_9 \left[1 - e^{y_{10} \sqrt{I_2'} p / (1 - y_{11} J_1)} \right]}{y_{12} - y_{13} J_1} \quad (55)$$

and

$$h_2 = \gamma_{14} \sqrt{J_2'} \left[1 - e^{-\sqrt{J_2'}/\gamma_{15}} \right] \frac{e^{-J_2'^2/\gamma_{16}}}{1 - \gamma_{20} J_1} - \gamma_{17} \left[1 - e^{J_1/\gamma_{18}} \right] e^{J_1/\gamma_{19}} \quad (56)$$

where

γ_1 through γ_{25} = Constants

Coefficients γ_1 through γ_{23} were assumed in reference 51 to have the following values (Table II)

Table II. Values of Coefficients Used in Model (Ref. 51)*

$\gamma_1 = 1000$	$\gamma_2 = 12.2$	$\gamma_3 = 11.0$	$\gamma_4 = 40,000$	$\gamma_5 = 800$
$\gamma_6 = 700$	$\gamma_7 = 2$	$\gamma_8 = 0.1 \times 10^{-5}$	$\gamma_9 = 100$	$\gamma_{10} = 120$
$\gamma_{11} = 0.0001$	$\gamma_{12} = 2$	$\gamma_{13} = 0.7 \times 10^{-4}$	$\gamma_{14} = -0.24 \times 10^{-3}$	$\gamma_{15} = 4000$
$\gamma_{16} = 0.8 \times 10^{16}$	$\gamma_{17} = -0.70^2 \times 10^{-11}$	$\gamma_{18} = 0.135 \times 10^{-14}$	$\gamma_{19} = 60,000$	$\gamma_{20} = 0.0005$
$\gamma_{21} = 0$	$\gamma_{22} = 0$	$\gamma_{23} = 0$		

These values were based on concrete cylinder strength $f_c' = 7000$ psi (483×10^5 N/m²). Guidelines for selecting these coefficients for different concrete strengths are included in Appendix I.

3. STEEL PROPERTIES

The main properties of reinforcing steel treated are the area fraction of the steel and its stress/strain relationship. Both of these are originally prescribed by the user in so-called "steel coordinates"; that is, in directions parallel to the axes of the bars. These areas are then transformed to the principal directions of orthotropy where they are incorporated together with the tangent moduli of the steel into the composite moduli.

*Valid only for English units.

a. Steel Transformation

The number of reinforcing "steel sets" for the structure being analyzed is specified by input to the computer program. Each "steel set" is defined as steel reinforcement in any three directions that are not necessarily orthogonal; the three directions are specified by direction cosines with respect to the global coordinates. The actual steel ratios involved in the three directions are left as variables from element to element. Thus, the steel reinforcement in an element is completely specified by specifying the steel set involved. The steel set defines the reinforcement directions and the three steel ratios that set the amounts of reinforcement in the three directions.

In each load step, for each individual element the code determines coordinates based on the principal directions of orthotropy. The code then determines the equivalent amount of steel reinforcement in the principal directions of orthotropy by transforming the actual steel ratios.

Let X , Y , and Z represent the global coordinates for the entire structure; x , y , and z the local coordinates for the element; x_s , y_s and z_s the actual steel directions; and p_{xs} , p_{ys} , and p_{zs} the accompanying steel ratios as shown in figure 36.

Further, let the direction cosines of coordinate directions x , y , and z with respect to coordinates X , Y , and Z be represented by the array

$$\begin{bmatrix} \cos \alpha_x & \cos \beta_x & \cos \gamma_x \\ \cos \alpha_y & \cos \beta_y & \cos \gamma_y \\ \cos \alpha_z & \cos \beta_z & \cos \gamma_z \end{bmatrix} \quad (57)$$

where α_x , β_x , and γ_x , for example, denote the angles between the x -axis and the X -, Y -, and Z -axes, respectively. Similarly, let the array

$$\begin{bmatrix} \cos \alpha_{xs} & \cos \alpha_{ys} & \cos \alpha_{zs} \\ \cos \beta_{xs} & \cos \beta_{ys} & \cos \beta_{zs} \\ \cos \gamma_{xs} & \cos \gamma_{ys} & \cos \gamma_{zs} \end{bmatrix} \quad (58)$$

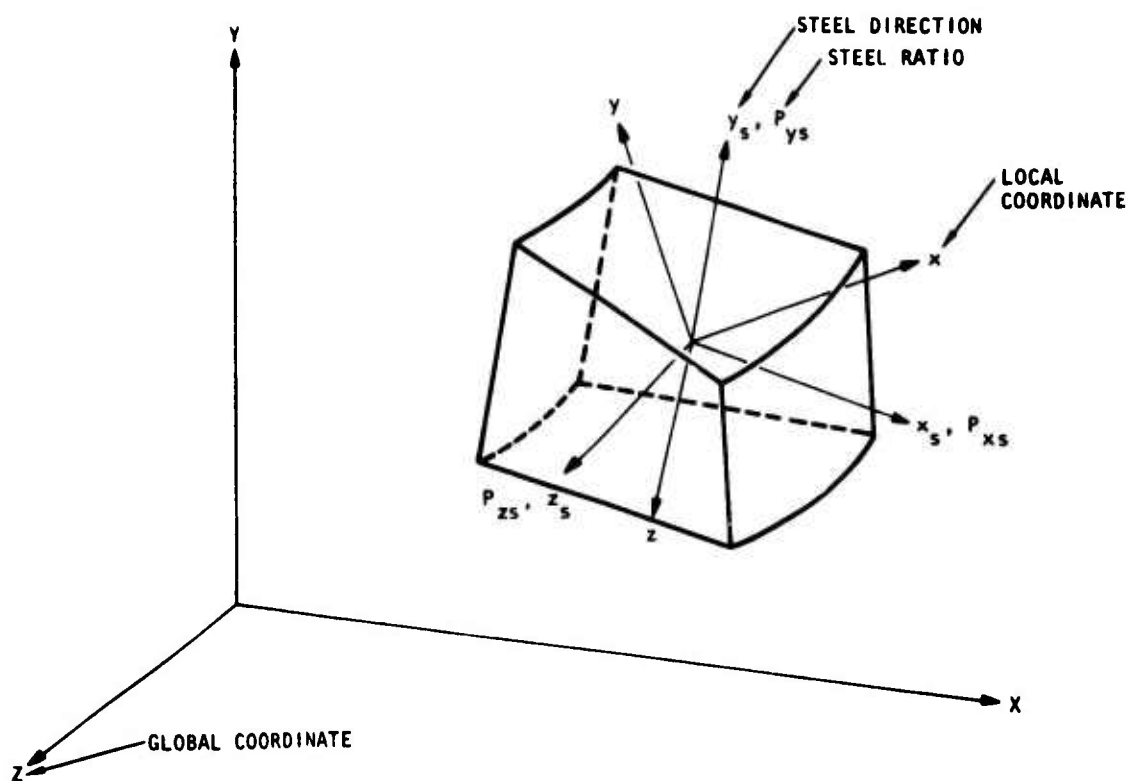


Figure 36. Coordinates System for Transforming Steel Areas

represent the direction cosines of steel directions x_s , y_s , and z_s with respect to the global coordinates X , Y , and Z . The first set of direction cosines as given by equation 57 are calculated internally by the code following the establishment of the local coordinates, while the second set of direction cosines as given by equation 58 are available as part of the input.

The direction cosines of the steel directions x_s , y_s , and z_s with respect to the local coordinates x , y , and z are then easily obtained as the scalar product of equations 57 and 58. Thus,

$$\begin{bmatrix} \cos \bar{\alpha}_{xs} & \cos \bar{\alpha}_{ys} & \cos \bar{\alpha}_{zs} \\ \cos \bar{\beta}_{xs} & \cos \bar{\beta}_{ys} & \cos \bar{\beta}_{zs} \\ \cos \bar{\gamma}_{xs} & \cos \bar{\gamma}_{ys} & \cos \bar{\gamma}_{zs} \end{bmatrix} = \begin{bmatrix} \cos \alpha_x & \cos \beta_x & \cos \gamma_x \\ \cos \alpha_y & \cos \beta_y & \cos \gamma_y \\ \cos \alpha_z & \cos \beta_z & \cos \gamma_z \end{bmatrix} \begin{bmatrix} \cos \alpha_{xs} & \cos \alpha_{ys} & \cos \alpha_{zs} \\ \cos \beta_{xs} & \cos \beta_{ys} & \cos \beta_{zs} \\ \cos \gamma_{xs} & \cos \gamma_{ys} & \cos \gamma_{zs} \end{bmatrix} \quad (59)$$

where $\bar{\alpha}_{xs}$, $\bar{\beta}_{xs}$ and $\bar{\gamma}_{xs}$, for example, represent the angles between the x_s -axis and the x -, y -, and z -axes, respectively.

In FEDRC* code, the original steel ratios specified by the user in the steel directions, x_s , y_s , and z_s , are transformed into equivalent steel ratios in the local coordinate directions x , y , and z , using the direction cosines obtained in equation 59 as follows:

$$\begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{bmatrix} \cos^2 \alpha_{xs} & \cos^2 \alpha_{ys} & \cos^2 \alpha_{zs} \\ \cos^2 \beta_{xs} & \cos^2 \beta_{ys} & \cos^2 \beta_{zs} \\ \cos^2 \gamma_{xs} & \cos^2 \gamma_{ys} & \cos^2 \gamma_{zs} \end{bmatrix} \begin{Bmatrix} p_{xs} \\ p_{ys} \\ p_{zs} \end{Bmatrix} \quad (60)$$

The justification for this transformation is that it does not give rise to negative steel areas and that it conserves the total steel area. The latter follows from the fact that the sum of squares of a set of direction cosines is unity.

*FEDRC is a modified version of Agabian Associates FEDIA code, which is a dynamic, inelastic, two-dimensional continuum finite element code. The modification consisted of incorporating the present material package in FEDIA code.

b. Steel Stress/Strain Relations

The tensile and compressive stress/strain relations are linearly elastic, perfectly plastic as shown in figure 37. Instead of assuming perfect plasticity, the user of the present computer program may specify hardening by means of a nonzero slope.

4. BOND-SLIP RELATION

The bond-slip is accounted for by the quantity λ in equation 26, which is assumed to vary from 1.0 to 0 as the strain in the steel reaches the yield strain. The variation of λ in the present model is shown in figure 38. This relation is based on data from previous experiments. (These are compared to the experiments conducted as part of the present study in Sec. V). Further refinements in this relation, including taking into account the spacing between cracks, the effect of cyclic loading, and the strength of concrete used, are probably desirable from a theoretical point of view. However, in the author's opinion, they were not justified in view of other uncertainties that were encountered at the beginning of this study.

5. DOWEL ACTION

Dowel action was discussed in Section III and refers to the shear stiffness afforded to cracked sections by the reinforcing bars that span the cracks. Dowel action is represented by a variable shear modulus that relates incremental shear stress to incremental shear strain in the principal directions of orthotropy as follows:

$$d\tau_o = G_o d\gamma_o \quad (61)$$

The formula for the shear modulus, used in the present computer program, is similar to that suggested by Dulacska (Ref. 65) and summarized in Section III in equations 24 and 25. However, the present formulation is simplified by the assumption that the angle between the stirrup and the crack direction is assumed to be 90° . This assumption follows directly from transforming the steel from its natural coordinates to the principal directions of orthotropy.

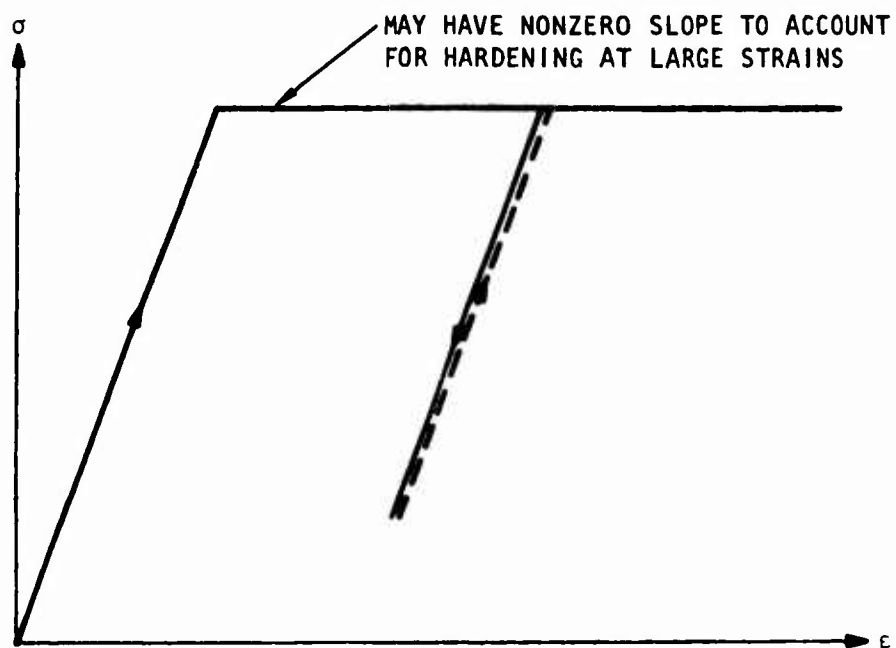


Figure 37. Stress/Strain Relations for Reinforcing Steel

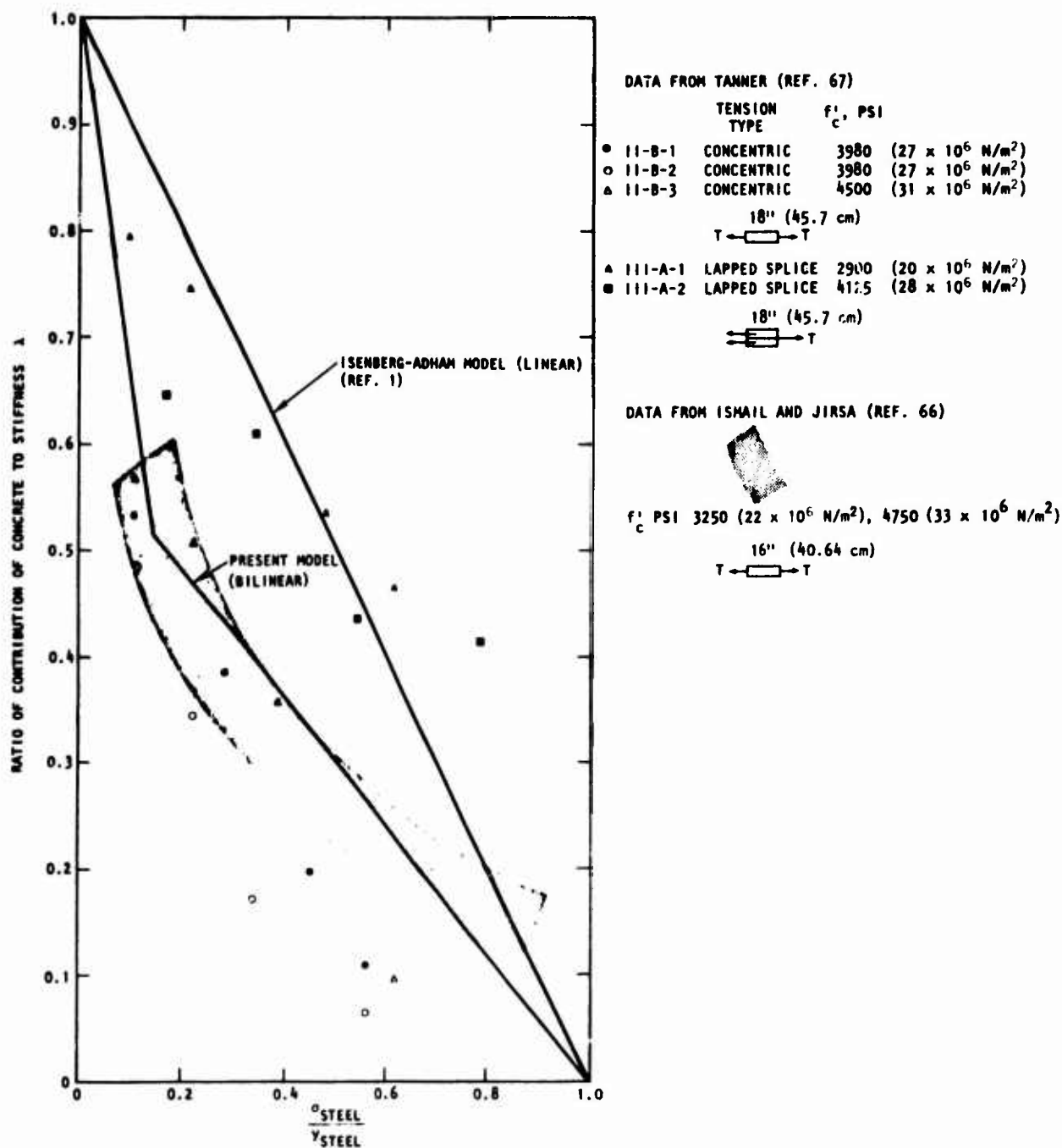


Figure 38. Experimental Bond-Slip Results (Refs. 66, 67) Compared with Two Sets of Analytical Results (Ref. 1 and present study)

For $\delta = 0$, Dulacska's relationship, equation 24, reduces to the simple form

$$T_f = \phi^2 \sqrt{\frac{\sigma_c \rho \sigma_y n}{3}} \quad (62)$$

where

T_f = Failure force

ϕ = Bar size

σ_c = Cube strength of concrete

σ_y = Tension yield stress of bar

n = Coefficient of local compression of concrete

$$\rho = 1 - \frac{N^2}{N_y^2}$$

N = Axial tension force of bar

N_y = Axial tensile force inducing yield in pure tension

and the maximum shear stress τ_f that the bar is capable of withstanding is as follows:

$$\tau_f = \frac{4}{\pi} \sqrt{\frac{\sigma_c \rho \sigma_y n}{3}} \quad (63)$$

The relation between the dowel shear force T and the displacement Δ given by equation 25 was also simplified to the form

$$\frac{T}{T_f} = 1 - e^{-\alpha \Delta} \quad (64)$$

where α is a constant.

The validity of the present simplified relation was demonstrated by comparing it with Dulacska's relation, which agrees well with experimental data. Five different cases involving different stirrup angles, diameters and yield strengths were performed as shown in table III. The value of the parameter α in equation 64 was selected in order to match the relation of Dulacska. The results obtained by both relations are compared in figure 39.

Equation 64 can be written in terms of stresses in the form

$$\frac{\tau}{\tau_f} = 1 - e^{-\bar{\alpha}\gamma_{\text{shear}}} \quad (65)$$

where

$$\bar{\alpha} = L \alpha$$

$$L = \text{Length of element}$$

$$\gamma_{\text{shear}} = \text{Dowel shear strain}$$

The dowel shear modulus can be calculated from equation 65 by differentiation

$$\frac{1}{\tau_f} \frac{d\tau}{d\gamma} = \bar{\alpha} e^{-\bar{\alpha}\gamma_{\text{shear}}} \quad (66)$$

or

$$G = \tau_f \bar{\alpha} e^{-\bar{\alpha}\gamma_{\text{shear}}} \quad (67)$$

and for

$$\gamma_{\text{shear}} = 0 \quad G = G_o \quad (68)$$

Substituting equation 68 into equation 67, the constant $\bar{\alpha}$ can be calculated from

$$\bar{\alpha} = \frac{G_o}{\tau_f} \quad (69a)$$

Table III. Dulacska's Cases Used to Match Present Simplified Relationship

Case No.	Stirrup Angle δ degrees	Stirrup Diameter \dagger in. (cm)	Stirrup Yield σ_y ksi (10^3 N/m ²)	Concrete Strength σ_c ksi (N/m ²)	Theoretical Strength T_{\max} lb (N)	α
1	10	0.39 (0.99)	41.89 (288831)	4.54 (31303)	2249 (10004)	146.5
2	20	0.2535 (0.64)	35.07 (241807)	4.26 (29373)	794 (3532)	261.3
3	20	0.39 (0.99)	41.89 (288831)	1.42 (9791)	1014 (4510)	181.7
4	20	0.39 (0.99)	41.89 (288831)	4.54 (31303)	2072 (9216)	159.0
5	20	0.546 (1.39)	36.49 (251598)	4.26 (29373)	3682 (16378)	121.4

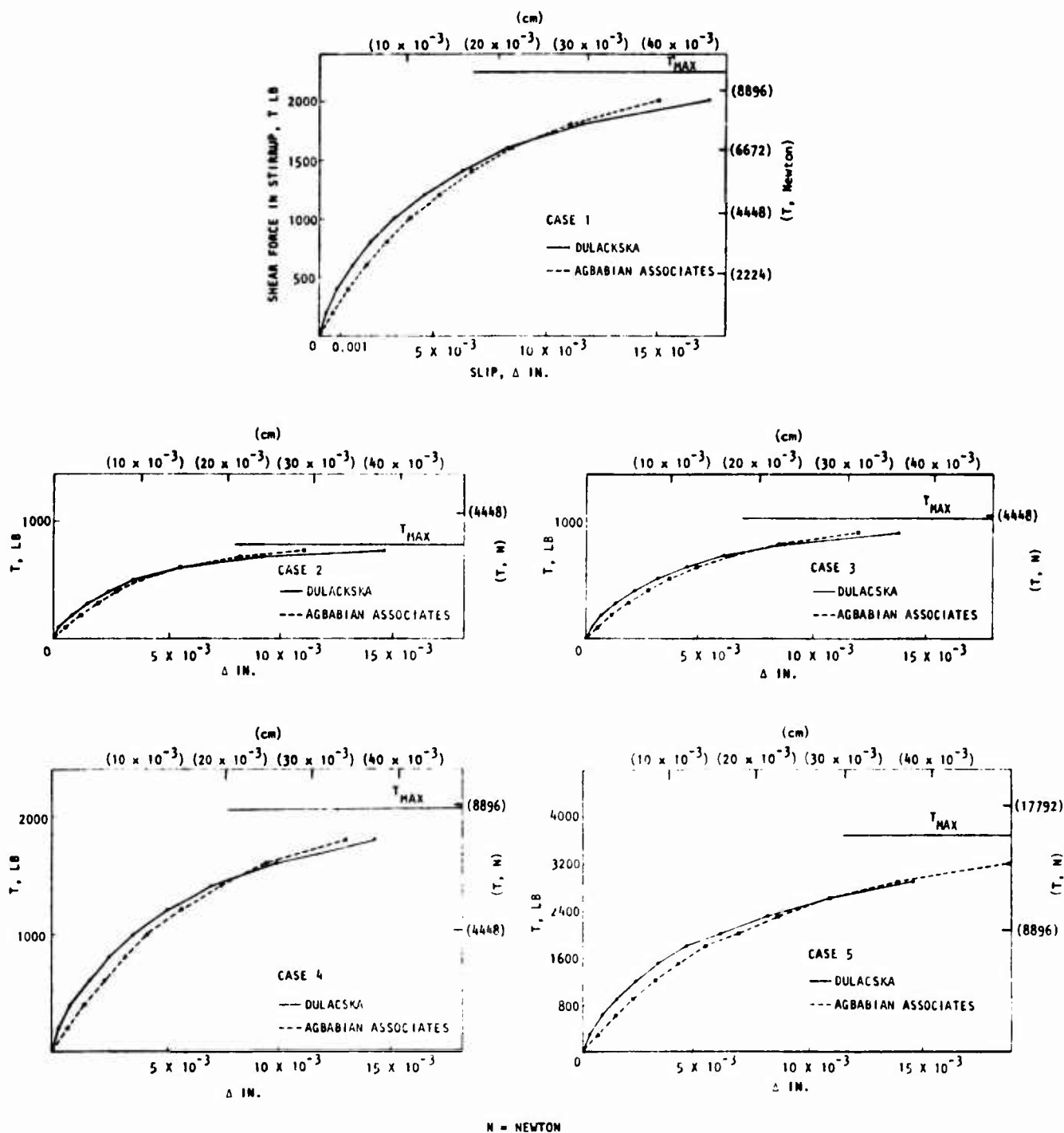


Figure 39. Comparison of Dowel Action Relationships

Thus equation 67 can be used to calculate the dowel shear modulus for various stages of loading. Substituting equation 69(a) in equation 67, we get

$$G = G_o e^{-\bar{\alpha} |\gamma_{\text{shear}}|} \quad (69b)$$

6. COMPOSITE MODULI OF REINFORCED CONCRETE

The purpose of this subsection is to introduce a method that incorporates, in a composite model, the variations in steel and concrete properties discussed above. The composite model accounts for

- The combined properties of steel and concrete
- The change in principal directions of stresses and the orientation of cracks

The combined properties of steel and concrete at various phases of loading will be discussed in the next paragraphs, while the matrix transformation from principal and orthotropic directions to global directions will be given in the next subsection.

a. Composite Moduli for Uncracked Reinforced Concrete

The composite moduli for the uncracked reinforced concrete elements are given by

$$\bar{E}_i = E_i^c A_i^c + E_{st} A_i^s \quad (70)$$

where

$$\bar{E}_i = \text{Effective composite moduli}$$

$$E_i^c = \begin{cases} E_i & \text{(from equation 29) for loading in compression} \\ E_c & \text{for unloading or reloading in compression} \\ E_c & \text{for loading in tension} \end{cases}$$

E_{st} = Modulus of steel

A_i^s = Effective steel ratio in direction i (area fraction)

A_i^c = $1 - A_i^s$ = effective concrete ratio in direction i

The composite tangent moduli matrix is 6×6 . For convenience it is partitioned in 3×3 blocks as follows:

$$C = \begin{bmatrix} X & Y \\ Y & Z \end{bmatrix} \quad (71)$$

In the principal orthotropic directions there is no coupling between normal and shear components. Hence

$$Y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (72)$$

and X can be defined as follows:

$$X = \frac{1}{D} \begin{bmatrix} \left[\bar{E}_1 (1 - \nu_{23}\nu_{32}) A_1^c \right] & \left[\bar{E}_1 (\nu_{12} + \nu_{13}\nu_{32}) A_1^c \right] & \left[\bar{E}_1 (\nu_{13} + \nu_{12}\nu_{23}) A_1^c \right] \\ \left[\bar{E}_2 (1 - \nu_{13}\nu_{31}) A_2^c \right] & \left[\bar{E}_2 (\nu_{23} + \nu_{21}\nu_{13}) A_2^c \right] & \left[\bar{E}_2 (\nu_{21} + \nu_{23}\nu_{12}) A_2^c \right] \\ \left[\bar{E}_3 (1 - \nu_{12}\nu_{21}) A_3^c \right] & \left[\bar{E}_3 (\nu_{31} + \nu_{32}\nu_{12}) A_3^c \right] & \left[\bar{E}_3 (\nu_{32} + \nu_{31}\nu_{23}) A_3^c \right] \end{bmatrix} \quad (73)$$

where

$$D = 1 - \nu_{23}\nu_{32} - \nu_{12}\nu_{21} - \nu_{13}\nu_{31} - \nu_{12}\nu_{23}\nu_{31} - \nu_{13}\nu_{21}\nu_{32}$$

ν_{ij} = Poisson's ratio of concrete taken as a constant ν in this study

and

$$[Z] = [Z_c + Z_s] \quad (74)$$

where

$$Z_s = \begin{bmatrix} G_s A_1^s & 0 & 0 \\ 0 & G_s A_2^s & 0 \\ 0 & 0 & G_s A_3^s \end{bmatrix} \quad (75)$$

where

G_s = Shear modulus of steel

A_1^s, A_2^s, A_3^s = Projected steel ratios in the principal stress directions

and

$$Z_c = \begin{bmatrix} G_{12} A_1^c & 0 & 0 \\ 0 & G_{23} A_2^c & 0 \\ 0 & 0 & G_{31} A_3^c \end{bmatrix} \quad (76)$$

where G_{12}, G_{23}, G_{31} have already been defined by equation 30, and $A_i^c = 1 - A_i^s$.

The possibility of yielding of steel in uncracked concrete is not included in the present model.

b. Onset of Cracking

In order to check for cracking, it is necessary to calculate the stress carried by concrete. This is done by the following equation:

$$\sigma_i^c = \frac{E_c \sigma_i}{E_c A_i^c + E_{st} A_i^s} \quad (77)$$

where

σ_i = Effective average stress

σ_i^c = Stress in concrete

Cracking occurs whenever

$$\sigma_i^c > f_t' \quad (78)$$

The value of f_t' can be calculated from equation 36.

In order to simulate the sudden increase in strain without additional loading at the onset of cracking, the stress σ_i is reduced by the amount $\sigma_i^c A_i^c$ at that point. This implies that the force in the element at this instant is carried by steel alone.

c. Composite Tensile Modulus After Cracking

The composite modulus after cracking is given by

$$\bar{E}_i = \frac{(E_c A_i^c + E_{st} A_i^s) E_{st} A_i^s}{E_{st} A_i^s + (1 - \lambda) E_c A_i^c} \quad (79)$$

where λ defines the bond-slip relation shown in figure 38 and can be expressed in the form

$$\lambda = \begin{cases} 1 - \frac{10x}{3} & \text{for } x \leq 0.15 \\ \frac{10(1-x)}{17} & \text{for } 0.15 < x \leq 1.0 \end{cases} \quad (80)$$

where

$$x = \frac{\text{tensile stress in steel}}{\text{yield stress of steel}}$$

For a crack in the 1-direction (Fig. 20) the first row and column of combined tangent moduli X and Z of equation 71 are altered to

$$X = \left[\begin{array}{c|cc} \bar{E}_1 & 0 & 0 \\ \hline 0 & & \\ 0 & & \text{unaltered} \end{array} \right] \quad (81)$$

$$Z = \left[\begin{array}{c|cc} G A_1^s & 0 & 0 \\ \hline 0 & & \\ 0 & & \text{unaltered} \end{array} \right] \quad (82)$$

For cracks in both the 1- and 2-directions, the alterations in X are:

$$X = \left[\begin{array}{cc|c} \bar{E}_1 & 0 & 0 \\ 0 & \bar{E}_2 & 0 \\ \hline 0 & 0 & \text{unaltered} \end{array} \right] \quad (83)$$

For cracks in all 3 directions

$$X = \left[\begin{array}{ccc} \bar{E}_1 & 0 & 0 \\ 0 & \bar{E}_2 & 0 \\ 0 & 0 & \bar{E}_3 \end{array} \right] \quad (84)$$

where \bar{E}_1 is calculated from equation 79 and G is the dowel shear modulus calculated from equation 67.

For unloading/reloading in uncracked direction

$$E_i^c = E_c \quad (85)$$

For unloading/reloading for cracked element in cracked directions

$$\bar{E}_i = E_{st} A_i^s + 0.01 E_c A_i^c \quad \text{for} \quad \epsilon_i \geq 0 \quad (86)$$

d. Composite Modulus for Postyielding of Steel

When steel yields, \bar{E}_i is set to a nominal minimum value to avoid numerical difficulties, the value presently used is

$$\bar{E}_i = 0.01 E_c A_i^c + 0.05 E_{st} A_i^s \quad (87)$$

However, the stress is not allowed to exceed the yield stress of steel.

For unloading/reloading, the steel modulus is used. Thus

$$\bar{E}_i = E_{st} A_i^s \quad (88)$$

e. Composite Moduli for Rebonding Behavior

If unloading causes the strain to be negative (i.e., compression), rebonding occurs. During rebonding a user-specified fraction of the concrete stiffness is used in the composite modulus. Thus

$$\bar{E}_i = \alpha E_c A_i^c + E_{st} A_i^s \quad (89)$$

where α is a user-specified quantity. A value not to exceed 0.5 is recommended by the authors. The rebonding was illustrated in figure 35 by the portion 5-6 of the stress/strain path. The rebonding relations included in the code are based on adjusting the stiffness of the element to avoid any instability in the solution.

7. COORDINATE TRANSFORMATION PRINCIPLES

Various coordinate transformations are carried out in the program. The procedure and principles of these transformations are discussed in this section.

a. Definition of Direction Cosines

Let (x, y, z) be one orthogonal system of coordinates and (x', y', z') be another system of coordinates obtained by rotating the system (x, y, z) . The direction cosine table for the system is defined in table IV, where l_1, m_1, n_1 are the cosines of the angles between the positive x' -axis and the positive x, y and z axes respectively. Similarly, l_2, m_2, n_2 are the direction cosines of y' with respect to x, y , and z and l_3, m_3, n_3 are the direction cosines of z' with respect to x, y , and z .

Table IV. Direction Cosine for Rotation of Axes

	x'	y'	z'
x	l_1	l_2	l_3
y	m_1	m_2	m_3
z	n_1	n_2	n_3

b. Stress and Strain Transformation

Let σ and σ' be the stress components in the (x, y, z) and (x', y', z') coordinate systems, respectively. In expanded form these are

$$\sigma = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad \sigma' = \begin{Bmatrix} \sigma'_x \\ \sigma'_y \\ \sigma'_z \\ \tau'_{xy} \\ \tau'_{yz} \\ \tau'_{zx} \end{Bmatrix}$$

Since stress is a tensor quantity, the transformation due to rotation of axes is represented by

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \quad (90)$$

or in expanded form

$$\begin{Bmatrix} \sigma'_x \\ \sigma'_y \\ \sigma'_z \\ \tau'_{xy} \\ \tau'_{yz} \\ \tau'_{zx} \end{Bmatrix} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2 l_1 m_1 & 2 n_1 m_1 & 2 l_1 n_1 \\ l_2^2 & m_2^2 & n_2^2 & 2 l_2 m_2 & 2 n_2 m_2 & 2 l_2 n_2 \\ l_3^2 & m_3^2 & n_3^2 & 2 l_3 m_3 & 2 n_3 m_3 & 2 l_3 n_3 \\ l_1 l_2 & m_1 m_2 & n_1 n_2 & m_1 l_2 + l_1 m_2 & n_1 m_2 + m_1 n_2 & l_1 n_2 + n_1 l_2 \\ l_2 l_3 & m_2 m_3 & n_2 n_3 & m_2 l_3 + l_2 m_3 & n_2 m_3 + m_2 n_3 & l_2 n_3 + n_2 l_3 \\ l_1 l_3 & m_1 m_3 & n_1 n_3 & m_1 l_3 + l_1 m_3 & n_1 m_3 + m_1 n_3 & l_1 n_3 + n_1 l_3 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad (91)$$

Symbolically, equation 91 is written as

$$\sigma' = A \sigma \quad (92)$$

where A is the transformation matrix in equation 91.

In a similar manner, the transformation from σ' to σ is obtained as

$$\sigma = B \sigma' \quad (93)$$

where

$$B = \begin{bmatrix} \ell_1^2 & \ell_2^2 & \ell_3^2 & 2 \ell_1 \ell_2 & 2 \ell_2 \ell_3 & 2 \ell_1 \ell_3 \\ m_1^2 & m_2^2 & m_3^2 & 2 m_1 m_2 & 2 m_2 m_3 & 2 m_1 m_3 \\ n_1^2 & n_2^2 & n_3^2 & 2 n_1 n_2 & 2 n_2 n_3 & 2 n_1 n_3 \\ \ell_1 m_1 & \ell_2 m_2 & \ell_3 m_3 & \ell_2 m_1 + m_2 \ell_1 & \ell_3 m_2 + m_3 \ell_2 & \ell_1 m_3 + m_1 \ell_3 \\ m_1 n_1 & m_2 n_2 & m_3 n_3 & m_1 n_2 + n_1 m_2 & m_2 n_3 + n_2 m_3 & m_1 n_3 + m_1 n_3 \\ \ell_1 n_1 & \ell_2 n_2 & \ell_3 n_3 & \ell_2 n_1 + \ell_1 n_2 & \ell_3 n_2 + n_3 \ell_2 & \ell_1 n_3 + n_1 \ell_3 \end{bmatrix} \quad (94)$$

If the strain components ϵ, ϵ' correspond to σ and σ' respectively, the strain transformations can be obtained from the conservation of energy.

$$\sigma'^T \epsilon' = \sigma^T \epsilon \quad (95)$$

Substituting equation 93 for σ into equation 95 results in

$$\sigma'^T \epsilon' = \sigma'^T B^T \epsilon$$

Hence

$$\epsilon' = B^T \epsilon \quad (96)$$

or substituting equation 92 for σ' into equation 95 results in

$$\sigma^T A^T \epsilon' = \sigma^T \epsilon$$

Hence

$$\epsilon = A^T \epsilon' \quad (97)$$

c. Transformation of Matrix of Stress-Strain Relations

In order to transform the matrix relating stress and strain the following procedure is followed. C and C' interrelate stress with strain in the respective coordinate systems. Thus

$$\sigma = C\epsilon \quad (98)$$

and

$$\sigma' = C'\epsilon' \quad (99)$$

Premultiplying equation 99 by B and substituting for ϵ' from equation 96 results in

$$B\sigma' = BC'B^T\epsilon \quad (100)$$

Substituting equation 93 into equation 100 results in

$$\sigma = BC'B^T\epsilon \quad (101)$$

Comparison of equation 98 and 101 results in the designed transformation of the stress-strain relationship matrix. Thus

$$C = BC'B^T \quad (102)$$

Similarly, it can be shown that

$$C' = ACA^T \quad (103)$$

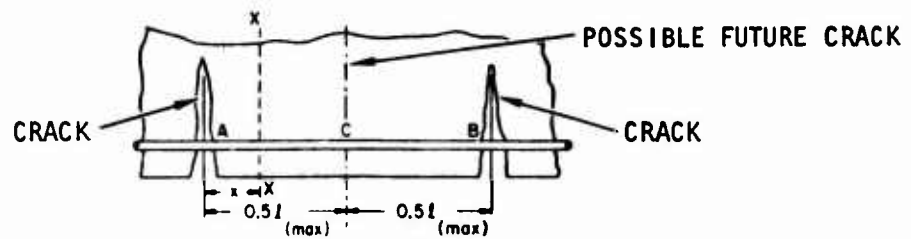
8. WIDTH OF FINITE ELEMENTS IN CRACKING ZONE

When a reinforced concrete member is subjected to tensile stresses, the first cracks (primary cracks) will be developed when the stresses exceed the allowable tensile stresses of the concrete. When a crack forms, the tensile stress in the concrete immediately adjacent to it must drop to zero (Refs. 3, 84, and 85), as illustrated in figure 40. However, the reinforcement, which is bonded to the concrete, retains its tensile stress and therefore strain. Thus tensile stress must continue to exist in the concrete along the reinforcement and will spread throughout the portion between cracks. At the same time, the concrete stress f_o , along the bars, must increase gradually from zero at the crack to a maximum value $f_o(\max)$ (Fig. 40).

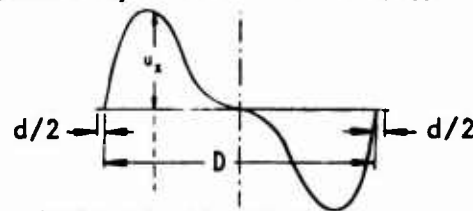
When the forces on the reinforced concrete member are increased above those causing the primary cracks, the steel stresses will increase; and the portion of bond remaining after slip will cause the longitudinal concrete stresses to also increase. The maximum tensile stress in the concrete, $f_o(\max)$, may then somewhere well exceed the tensile strength of the concrete, and a new crack will be started. The same mechanism may result in several cycles of cracks. Broms (Ref. 33) indicated that crack spacing is usually stabilized when the crack spacing is approximately equivalent to the thickness of the cover. The bond-slip relation used in the model is based on initial perfect bond. This perfect bond requires enough length to develop. Therefore, the width of the finite element in the expected cracking regions must be enough to develop this bond. However, the element size in areas of stress concentration should be small enough to provide reasonable approximation of the exact stress gradient. Therefore the analyst should select the element size based on these two requirements and should interpret the results on the basis of this selection. This subject is addressed further in section V.

9. ASSUMPTIONS AND LIMITATIONS OF THE MODEL

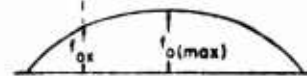
- a. The deformations (displacements, rotations, and strains) are assumed to be small and, accordingly, the analysis utilizes the initial, undeformed geometry of the system.



(a) Cracked Segment of a Reinforced Concrete Beam.



(b) Distribution of Bond Stress



(c) Distribution of Stress in Concrete
Adjacent to Steel.



(d) Distribution of Stress in Steel

U_x = Bond stress at distance x from crack

f_{ox} = Stress in concrete at distance x from crack

f_{sx} = Stress in steel at distance x from crack

Figure 40. Stress Distributions in Cracked Segment
(Refs. 3, 84, 85)

- b. The geometry can be two- or three-dimensional.
- c. The continuum can be approximated by a system of two- or three-dimensional finite elements.
- d. The material properties are homogeneous within each element.
- e. The directions of first cracks in an element define the directions of orthotropy in this element.
- f. The continuum is divided into elements that are long enough to provide full bond at the beginning of loading in tension.
- g. The bond is perfect between steel and concrete in compression.
- h. The mechanical properties of reinforced concrete elements can be approximated by composite variable moduli.
- i. The steel bars are allowed to yield only in tension.
- j. The rebonding capability is based on adjusting the stiffness properties of the element.
- k. The creep effects are neglected.
- l. The value of Poisson's ratio in uncracked elements is assumed constant throughout the solution.
- m. Poisson's ratio is assumed zero after cracking.
- n. Confining effects were not activated in the present model.
- o. The strains in a cracked element represent the average value of the strains in the bonded and debonded zones of the element.

SECTION V

EXPERIMENTAL STUDIES

1. INTRODUCTION

Experimental studies of bond slip were performed as part of the present project to add to the data that are presented by Mains (Ref. 87). These data indicate the average strength contributed by the concrete to a composite section subjected to uniaxial tensile stress. The concrete contribution is expressed as a fraction λ varying from 1.0 to zero. The value of λ is known to be a function of several variables:

- a. Strain in the steel
- b. Length of uncracked section
- c. Concrete strength, bar deformations, and other material parameters

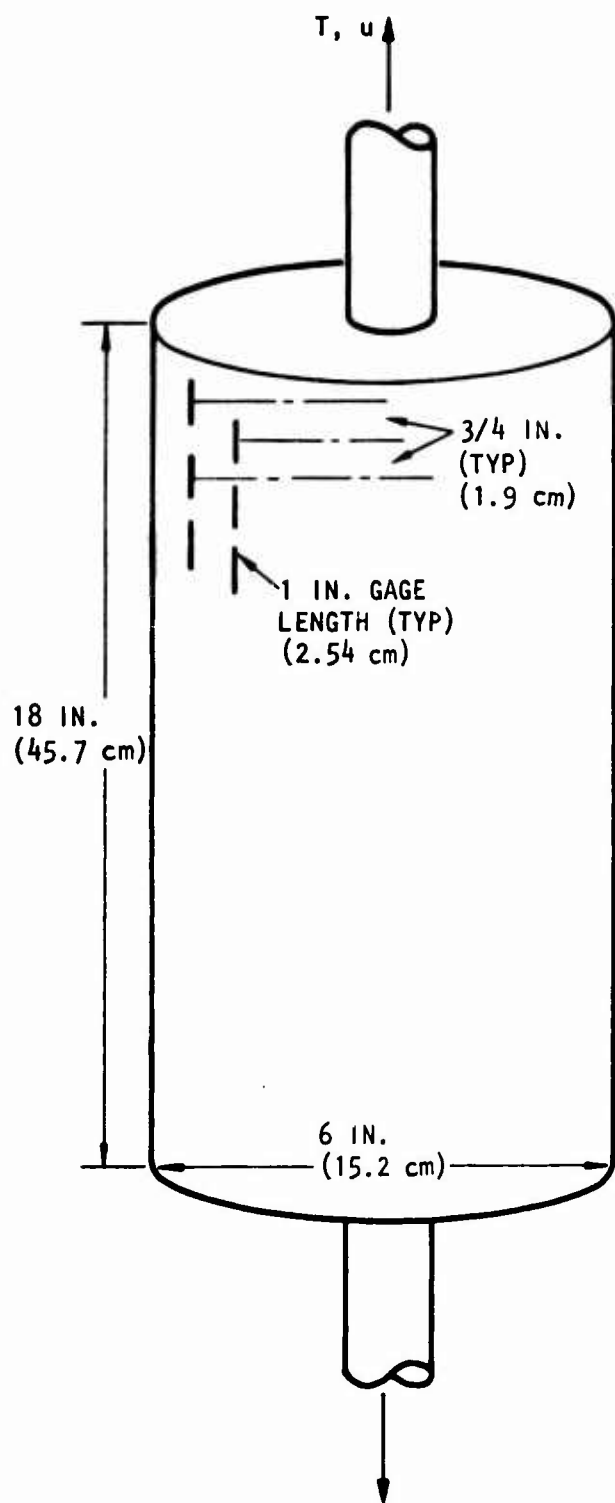
In the present experiments, factors b and c are held constant.

In the composite model explained in Section IV the function of λ is to facilitate forming a composite modulus from the concrete and steel moduli, as follows:

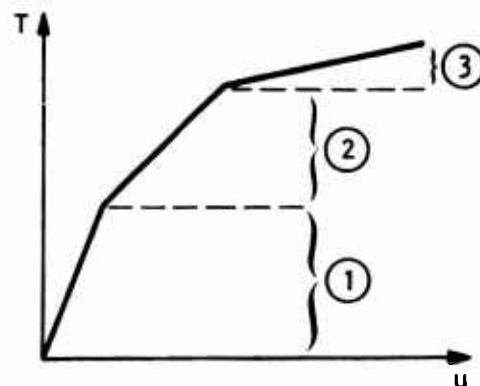
$$E = \frac{1}{\frac{1-\lambda}{E_s A_s} + \frac{\lambda}{E_s A_s + E_c A_c}} \quad (104)$$

Experimental data by Ismail and Jirsa (Ref. 66) and Tanner (Ref. 67) were shown in figure 38 and were reduced to a form that expresses λ as a function of average steel strain in a finite element. In addition to exploring the variation of λ in tension, the present experiments also investigated bond slip in compression.

Cylindrical specimens with a coaxial piece of reinforcing bar were subjected to tension and compression, as illustrated in figure 41. Strain gages were attached to the reinforcing bar as shown in figure 42. Strains were also measured on the surface of the concrete. The approximate load/deflection properties of the specimens are shown in figure 41.



(a) Test specimens

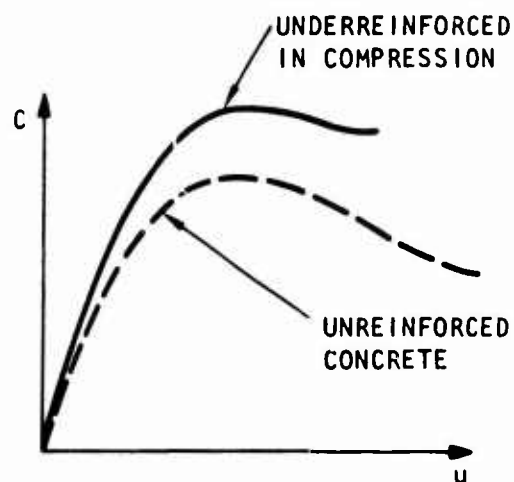


REGION

EVENT

- | | |
|---|--|
| ① | STEEL IN ELASTIC CONCRETE IS NEARLY ELASTIC. BOND IS BROKEN ONLY AT THE ENDS OF THE SPECIMEN |
| ② | STEEL IS ELASTIC. CONCRETE IS CRACKED AT SEVERAL POINTS. BOND IS BROKEN OVER AN APPRECIABLE LENGTH |
| ③ | STEEL YIELDS. BOND MAY BE BROKEN OVER A LARGE PERCENT OF LENGTH OF SPECIMEN |

(b) Tension test



(c) Compression test

Figure 41. Test Specimen Configuration and Loading

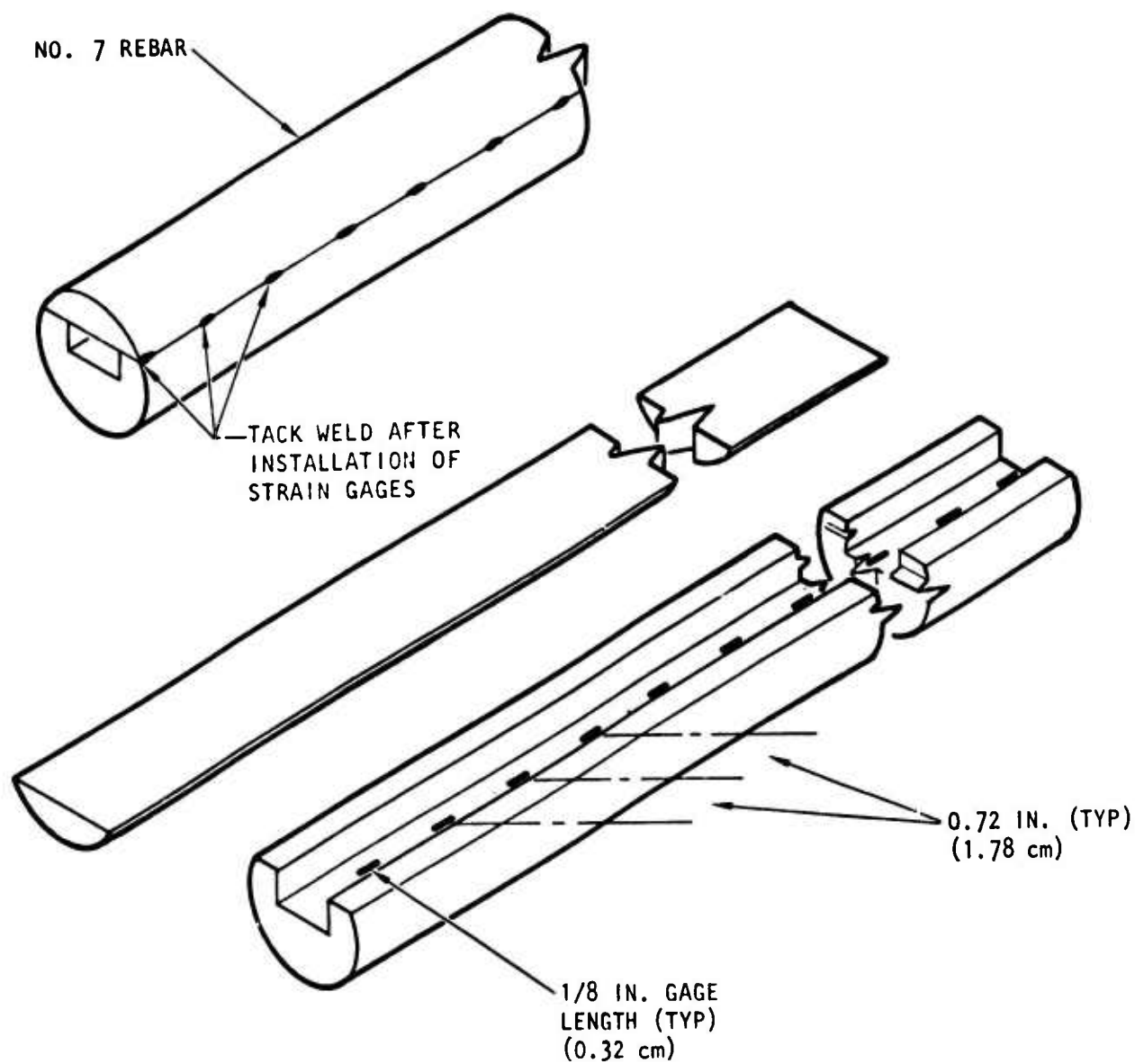


Figure 42. Instrumented Rebar Details

The specimens were tested in unconfined compression and tension in a universal testing machine. The tension specimens were tested by applying tensile forces to the ends of the reinforcing bar (Fig. 41(a)). The compression specimens were tested by applying compressive stress to the concrete only; one end of the bar was cut slightly above the level of the bottom end of the concrete cylinder, and the other end passed through a hole in the crosshead of the testing machine (Fig. 43).

Automatic recording of data from the strain gages and load cells facilitated the processing of results.

2. MANUFACTURE OF SPECIMENS

a. Strain Gaging of the Specimen

The major task in preparing specimens was the gaging of the reinforcing bars. To increase survivability, the strain gages were installed inside the rebar by machining a ground-finish flat surface on the 18-in. (81.28 cm) long pieces of rebar in order to combine them as a pair for one specimen. When the two flat surfaces were matched, the pair formed a rebar with the same diameter as in the unmodified condition. Figure 4 shows the cross section of the specimen rebar.

Prior to welding the two parts together, a slot with a ground-surface bottom was first machined into the flat side of the larger piece of rebar. Foil strain gages (Micro-Measurements, Type SA-06-125-DP-350) were installed with AE-15 adhesive in the bottom of the slot at a spacing of 0.72 in. (1.78 cm) center to center, for a total of 25 gages over the 18-in. (45.7 cm) concrete specimen length. The gages were cured two hours at 150°F. Hook-up wires were No. 30 AWG magnet wire with double Formvar insulation. Gages were wired for the 3-wire hook-up method (3 wires to each gage). When all the gages had been installed and wired with the leads extending out one end of the slot, a wash coat of epoxy was used as waterproofing over the gages.

The next step in the processing was to weld the two pieces of rebar together to form a single piece again. The steel was chilled in dry ice and

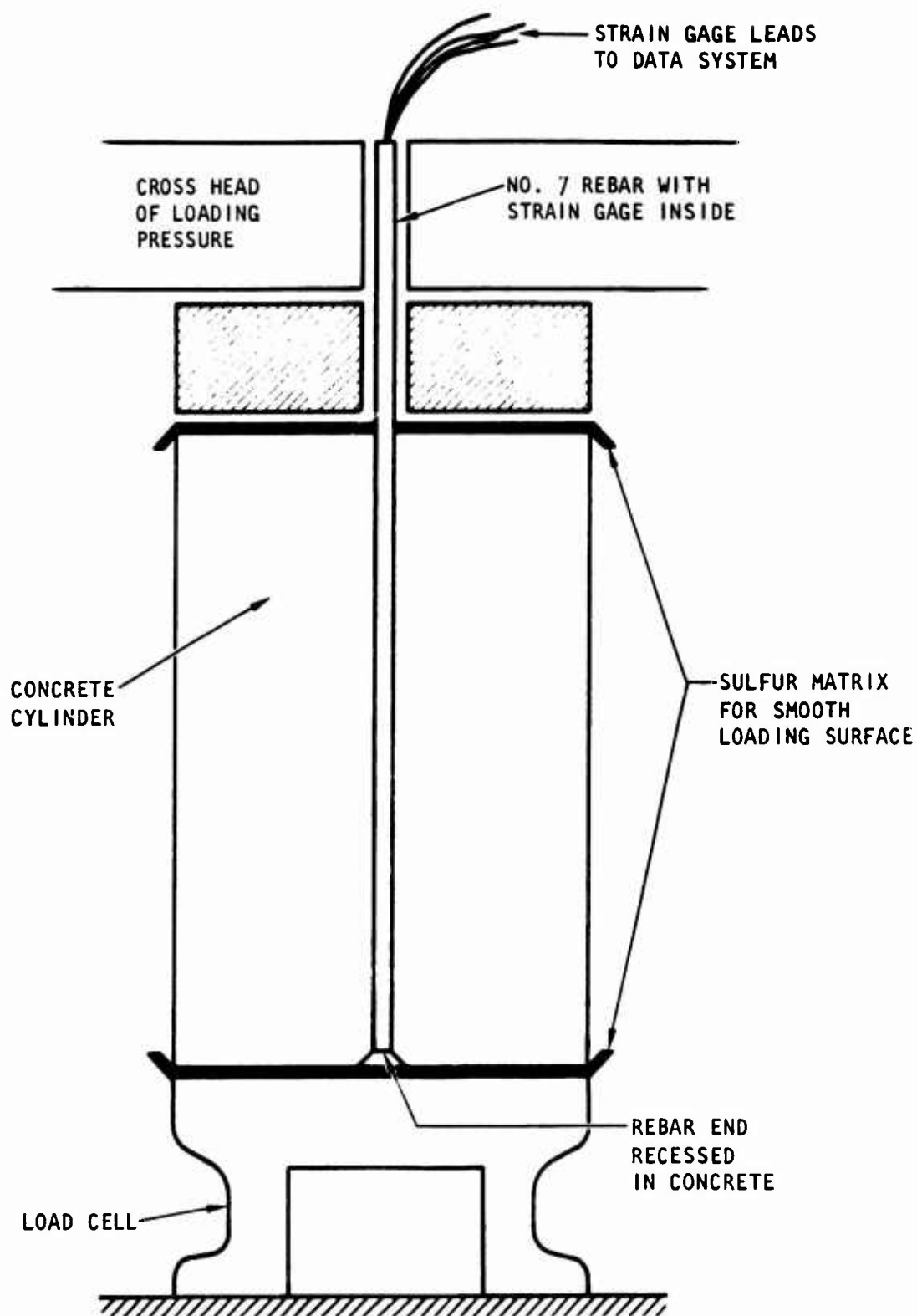


Figure 43. Configuration of Compression Test Specimen

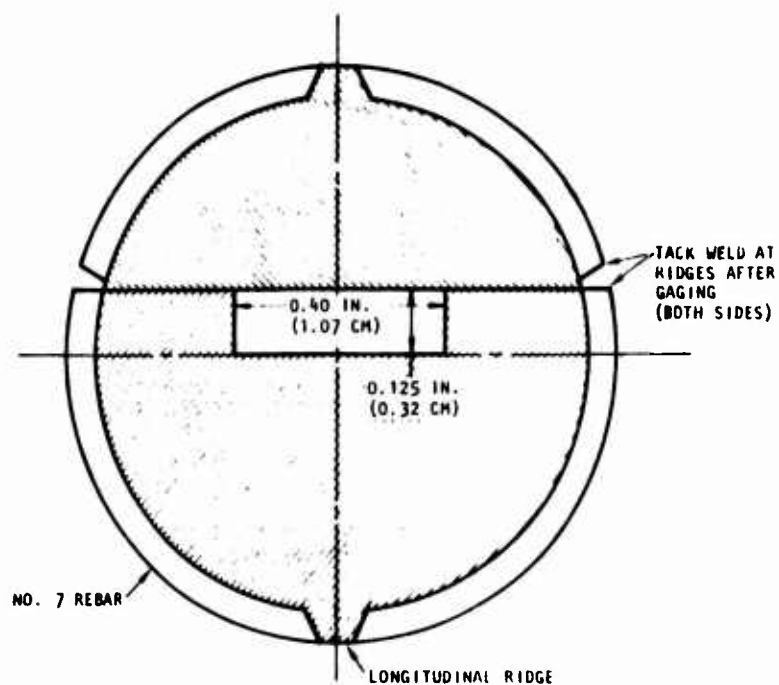


Figure 44. Cross Section of Strain-Gaged Rebar

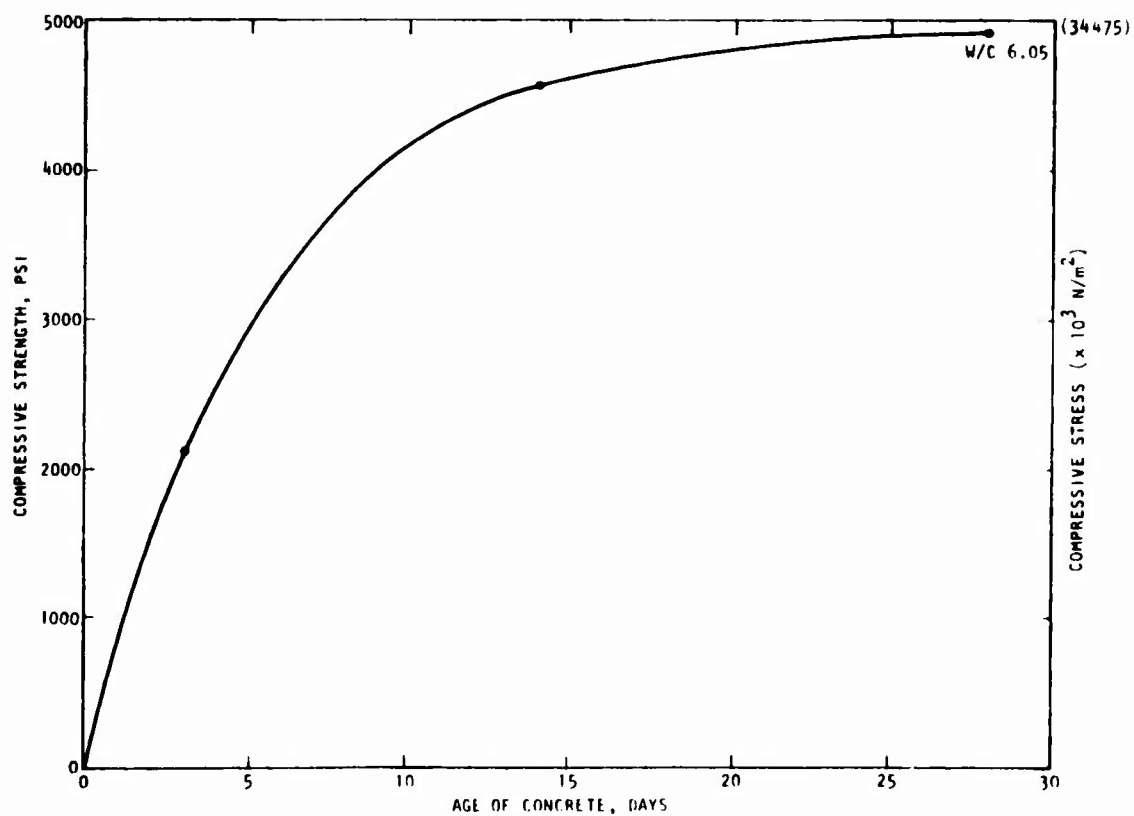


Figure 45. Concrete Strength versus Cure Time

spot welded along the length. The prechilling successfully prevented heat damage to the installed gages since the temperature of the bars remained well below room ambient temperature. Finally, the central slot was filled with epoxy to preclude concrete entry.

When all four rebars had been welded, they were installed in a testing machine and exercised three times to 60 percent of yield to minimize "first cycle off-set" that is common in strain gage work. At this time the load cell built into the tensile specimens just above the "concrete line" was calibrated. After the exercisings, the bottom portions of the two compression specimen rebars were cut off adjacent to the last strain-gage. (The compression specimen rebar was originally the same length as the tensile specimen rebars, to facilitate the exercising after gage installation.)

The next step was to cast the specimens, and after the concrete was cured a minimum of 35 days, the specimens were taken out of cure and dried sufficiently to install the surface gages. The concrete surface was sanded lightly in the gage installation area and the vicinity neutralized with phosphoric acid. Washed and redried, the area was coated with A-10 epoxy to provide a smooth surface for gage installation. After the surface was cured overnight, sanded and cleaned, the gages were installed with Eastman 910 adhesive. These outside gages (except for end gages) had a gage length of one inch to attempt to average around any aggregate inclusions near the surface. The center-to-center spacing of these gages was the same as the spacing on the rebar. In order to accommodate this spacing, two changes were required:

1. The gages were staggered in two lines the length of the specimen.
2. The end gages used were the 1/8-in. (0.32 cm) length used on the rebar, in order to get the required center spacing close to the end. The external gages were postyield foil gages, Micro-Measurements type EP-08-10CBE-120.

The outside gages were also wired in the 3-wire method using No. 30 AWG magnet wire. The finished installation was given a coating of AE-15 adhesive as waterproofing.

b. Pouring Concrete Specimens

Pouring of the specimens was done after the reinforcing bars were gaged and waterproofed. In pouring the specimens, the reinforcing bars were centered in molds of polyvinyl tubing. Concrete was poured in three lifts, and each lift was vibrated for about 10 sec. In the tension specimens the bars were centered by drilling a center hole in the bottom cap, which aligned the bottom of the bar. The top was aligned using a jig with a central hole. In the compression specimens, the bottom of the bar rested on a cardboard plug centered in the bottom of the mold and covered with masking tape, which held the bar centered while concrete was poured and vibrated. The top of the bar was positioned with a jig. The cardboard plug caused the bar to be recessed about 1/4 in. (0.63 cm) relative to the concrete surface. This recess prevented the rebar from shunting compressive load around the concrete and directly into the testing machine base.

3. MIX PROPORTIONS OF CONCRETE

The mix proportions and aggregate grading are shown in table V. The strengths of 6-x-12-in. (15.2-x-30.4 cm) cylinders, cured according to ASTM standards, are shown in figure 45. These samples were poured as a trial batch. When the specimens with reinforcing bars were cast, companion 6-x-12-in. (15.2-x-30.4 cm) cylinders and 6-x-18-in. (15.2-x-45.6 cm) cylinders were also cast. They were tested on the same day as the reinforced specimens.

4. STEEL PROPERTIES

The reinforcing bars used in this investigation are No. 7 deformed bars, ASTM Grade 60. They conformed to standard specifications for "Deformed Billet-Steel Bars for Concrete Reinforcement," ASTM Designation A615-68 (Ref. 88). The minimum yield strength is 60,000 psi ($4137 \times 10^5 \text{ N/m}^2$).

5. SPECIMEN TESTING AND DATA RECORDING

After 35 days of curing in a 100-percent-humidity environment, the four instrumented test specimens and the five noninstrumented specimens were removed from the curing room. The exterior gages were applied to the four test specimen as detailed in Section 1(a). The data recording system was readied and the testing began.

Table V. Mix Proportions and Aggregate Grading of Concrete
Used for Test Specimens



Osborne

PHYSICAL CHEMICAL
RESEARCH NON DESTRUCTIVE
SOILS ENGINEERING

EST. 1907

LABORATORIES, INC.

2636 SOUTH GRAND AVENUE
LOS ANGELES, CALIF. 90007
PHONE: (213) 749-3111

Testing and Inspection Services

1601 SO. SINCLAIR ST. BLDG. E
ANAHEIM, CALIF. 92006
PHONE: (714) 639-9793

STATEMENT OF MIX DESIGN FOR CONCRETE

<input type="checkbox"/> PROJECT <input type="checkbox"/> ADDRESS <input type="checkbox"/> ARCHITECT <input type="checkbox"/> ENGINEER <input type="checkbox"/> GEN. CONTR. <input type="checkbox"/> CONC. SUB-CONTR. <input type="checkbox"/> CONC. SUPPLIER	Agabian Associates	MIX DESIGN NO.	OT 3719-1
	250 North Nash St., El Segundo	DATE	
		DESIGN STRENGTH	Trial Batch
		MAX. AGGR. SIZE	3/4"
		MAX. SLUMP	
	WATER / CEMENT	6.45	
	DESIGN METHOD		
		WSD	USD

AVERAGE COMPRESSIVE STRENGTH OF PREVIOUSLY
EVALUATED CYLINDERS psi

DESIGN FOR ONE CUBIC YARD OF CONCRETE SATURATED, SURFACE DRY

MATERIAL			BATCH WEIGHT	SP. GR.	ABSOLUTE VOLUME
CEMENT	TYPE II	5.4b sks	513	3.15	2.61
AGGREGATE 1.	V C Sand		1300	2.64	7.88
AGGREGATE 2.	Gravel No. 4		265	2.65	1.60
AGGREGATE 3.	Gravel No. 3 (regraded)		1695	2.65	10.23
AGGREGATE 4.					
WATER	35.2	gals	293	1.0	4.68
ADMIXTURE	None				

GRADING ANALYSIS PERCENT PASSING U.S. STANDARD SIEVE

	%	1 1/2	1	3/4	1/2	3/8	4	8	16	30	50	100	200	F.M.
AGGR. 1.	40					100	99	86	66	48	22	7		2.70
AGGR. 2.	8				100	92	13	3	2					5.89
AGGR. 3.	52			100	56	21	6							6.73
AGGR. 4.														
COMBINED				100	77	58	44	34	26	19	9	3		5.07
SPECIFIED LIMITS														

CEMENT BRAND Victor TYPE II

AGGREGATE SOURCE Conrock - San Gabriel

RESPECTFULLY SUBMITTED,
OSBORNE LABORATORIES, INC.

R.C.E. NO

a. Plain Specimen Testing

The nongaged sample specimens were tested prior to the gaged specimens to determine basic strength of the concrete, and to determine the fracture characteristics of samples with rebar included, when tested in the same manner as that to be used on the strain-gaged specimens. The five tests were all run the same day (October 5, 1973). The samples were 53 days old at that time. The results are summarized below:

- Sample 1: Split cylinder test on a 6-in. dia. by 12-in.-long (15.2 x 30.4 cm) cylinder (compressive loading on side of cylinder, roller bearing fashion). Ultimate load: 75,750 lb (337×10^3 N). Tensile strength: 715 psi (4930×10^3 N/m²).
- Sample 2: Compressive test, end loading of 6 in. dia. by 12-in. (15.2 x 30.4 cm) long cylinder. Ultimate load: 183,000 lb (814×10^3 N). Compressive strength: 6470 psi (4461×10^4 N/m²).
- Sample 3: Compressive test, end loading of 6-in. dia. by 18-in.-long (15.2 x 45.6 cm) cylinder. Ultimate load: 168,000 lb (747×10^3 N). Compressive strength: 5960 psi (4109×10^4 N/m²).
- Sample 4: Compressive test, 6-in. dia. by 18-in.-long (15.2 x 45.6 cm) cylinder with coaxial No. 7 rebar protruding 11 in. (27.94 cm) from top of specimen only. Capping compound partially removed from lower end of rebar to prevent load transfer to rebar. Ultimate load: 135,000 lb (600×10^3 N).
- Sample 5: Tensile test, 6-in. dia. by 18-in.-long (15.2 x 45.6 cm) cylinder with coaxial No. 7 rebar through specimen and protruding from each end. Tensile load applied to rebar. Ultimate load: 34,000 lb (151×10^3 N) to first circumferential crack, continued to 46,000 lb (205×10^3 N), by which time cracking was extensive over entire specimen.

b. Instrumented Specimen Testing

Instrumented tests took longer to set up than did tests of the plain samples. The four specimens were tested on the following schedule:

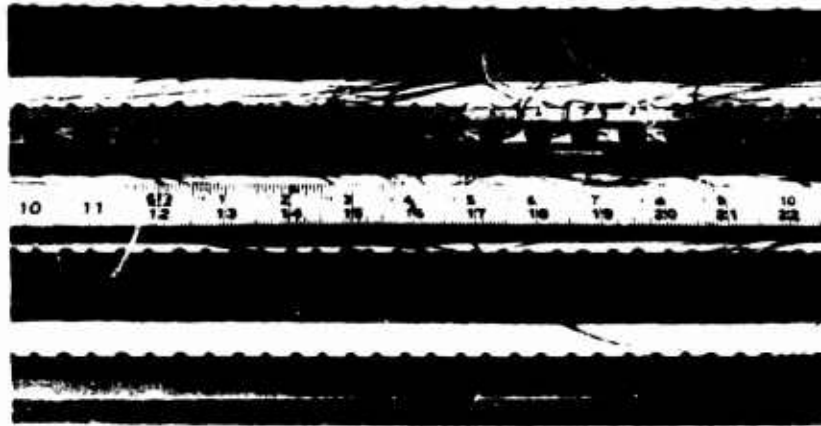
- Compressive Test (Sample No. 2) tested 10/18/73 to a load of 154,500 lb (687×10^3 N). Sample was 66 days old.
- Tensile Test (Sample No. 3) tested 10/24/73 to a load of 44,000 lb (196×10^3 N). Sample was 72 days old.
- Compressive Test (Sample No. 1) tested 10/29/73 to a load of 160,000 lb (712×10^3 N). Sample was 77 days old.
- Tensile Test (Sample No. 4) tested 10/29/73 to a load of 43,000 lb (191×10^3 N). Sample was 77 days old.

Physically, the evidence of failure of these specimens looked similar to the failure modes of the nongaged samples. Photographs of all samples are shown in figures 46 and 47.

c. Data Logging System

The data from the strain gages were sequentially sampled and routed to an analog-to-digital converter (ADC) and then recorded on 7-track magnetic computer tape. The tape was directly computer-compatible, which greatly facilitated data processing.

The data system consisted of 50 channels of bridge completion and balancing system, bridge excitation, a 50-channel reed-relay scanner, a 12-bit ADC and a digital magnetic-tape recorder, all interconnected as indicated on the block diagram shown in figure 48. This system scanned through all 50 channels in 1/4 sec. These scans were conducted every 5 to 20 sec, as indicated in

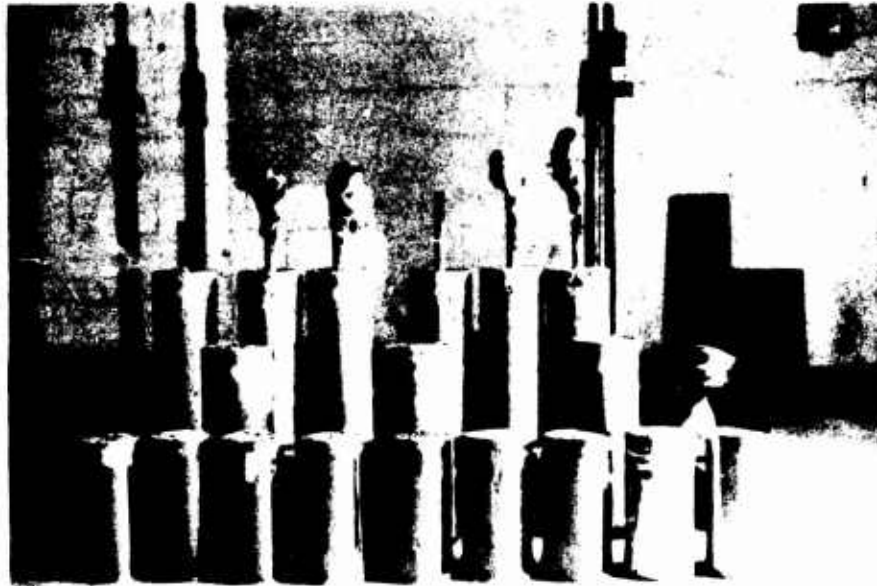


- (a) A rebar preparation. Bottom: Internal groove.
 Next to bottom: Strain gages installed in groove.
 Next to top: Leads attached to gages.
 Top: Rebar welded together.



- (b) A view showing external strain gages
 prior to testing

Figure 46. Specimen Preparation



(c) Test specimens and batch test samples
prior to curing



(d) Same as above, top view

Figure 46. Specimen Preparation (concluded)



(a) Specimen No. 2 during compression testing



(b) Specimen No. 3 during tension testing. Note cracks.

Figure 47. Specimen Testing and Results



(c) 12 in. x 6 in. (30.48 x 15.24 cm) diameter split test specimen. Failure at 75,750 (337×10^3 Newton) lb equal to tensile strength of 715 psi (4930×10^3 N/m²)



(d) Split test specimen showing interior

Figure 47. Specimen Testing and Results (continued)



(e) 18 in. x 6 in. (45.6 x 15.24 cm)
diameter specimen with non-
instrumented rebar, after
compression test



(f) 18 in. x 6 in. (45.6 x 15.24 cm)
diameter batch sample after
compression test

Figure 47. Specimen Testing and Results (continued)

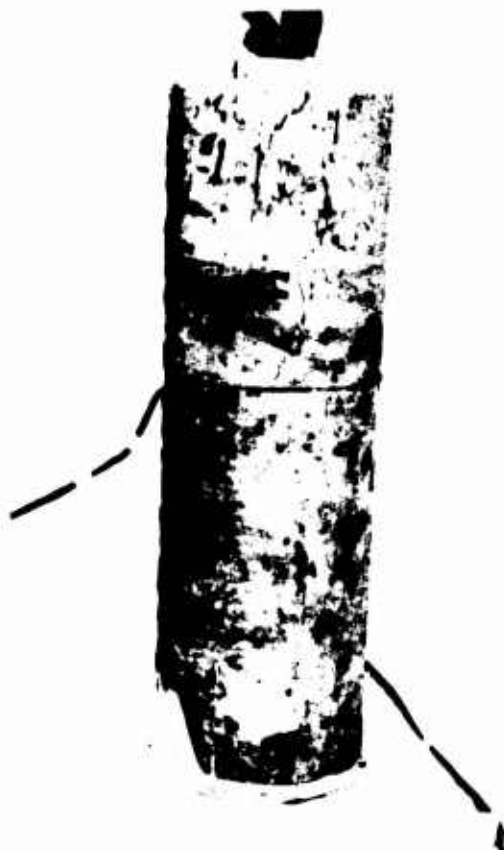


(g) Noninstrumented specimen with rebar, after tension test. Note cracking pattern.



(h) Back side of above specimen

Figure 47. Specimen Testing and Results (continued)

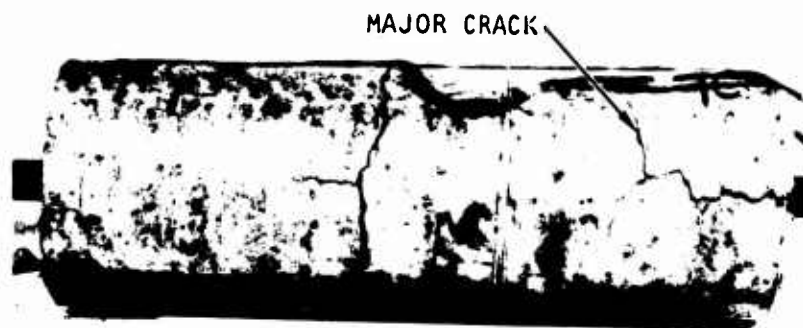


(i) Instrumented specimen No. 1, after compression test. Cracking at upper end

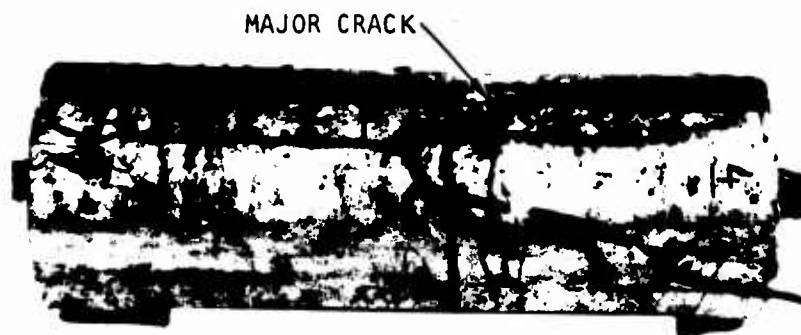


(j) Instrumented specimen No. 2, after compression test

Figure 47. Specimen Testing and Results (continued)



(k) Instrumented specimen No. 3, after tension test, showing cracking pattern



(l) Instrumented specimen No. 4, after tension test, showing cracking pattern

Figure 47. Specimen Testing and Results (concluded)

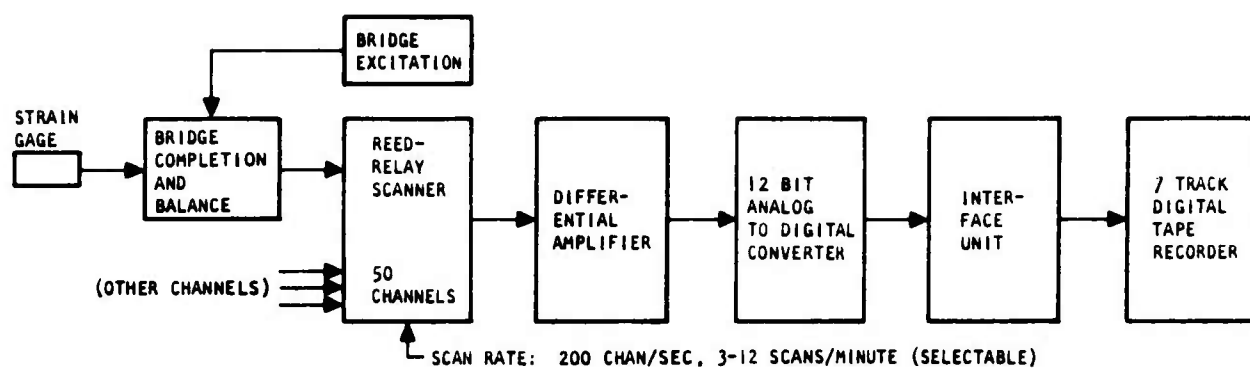


Figure 48. Block Diagram of Data Logging System

the testing summary above. Because of the essentially steady-state, bandwidth-limited nature of the data, no antialiasing filters* were used ahead of the scanner.

The specimens were hooked up in pairs, a compressive specimen with a tensile specimen, each serving in turn as temperature-compensating dummy to the other. The gage factor desensitization effect of the resistance of the long leads of small gage wire was compensated mathematically. When a step-function increase in the strain level of some channels was found much later in processing, the effect was confirmed and quantified by ECAP† computer analysis of the circuit; one of the double lead wires was broken. Thus, correction factors were incorporated as needed.

6. EXPERIMENTAL RESULTS

For the tensile tests, specimens of the type shown in figure 41(a) were subjected to tensile forces applied directly to the reinforcing bar, as shown. An idealization of the load/deflection curve to be expected from such a test is shown in figure 41(b), where the three regions of differing stiffnesses are explainable in terms of cracking of the concrete and yielding of the steel. Before cracking occurs, the tensile forces are resisted by the composite stiffness of the concrete/steel specimen. As the bond between the concrete and steel deteriorates, through cracking of the concrete, the participation of the concrete in resisting the tensile forces diminishes, eventually leaving only the steel to carry the load. The variable composite Young's modulus resulting from such a model is expressed through equation 104, in which λ is the bond strength parameter. Ismail and Jirsa (Ref. 66) have defined λ for a test specimen similar to that of figure 41(a), loaded in tension, according to the formula

*Sampled data systems usually require antialiasing filter ahead of the scanner to prevent errors caused by the sampling (scanning) process. However, the bandwidth limited nature of the essentially steady state data collected on this test precluded the need for such filtering.

†Electronic Circuit Analyses Program, developed by IBM.

$$\lambda = 1 - \frac{\int_0^L \epsilon_s dx}{\frac{TL}{A_s E_s}} \quad (105a)$$

where

ϵ_s = The steel strain

T = The tensile force

A_s = The steel cross-sectional area

E_s = Young's modulus for steel

L = The length of the specimen

Before cracking $\lambda = 1$, which indicates strong bond between concrete and steel and results in full participation of concrete stiffness in equation 104. However, after cracks occur, the first term in the denominator of equation 104 increases with the decreases of λ , while the second term decreases with the decrease of λ . When bond becomes entirely broken, $\lambda = 0$, and the second term disappears while the first term equals the stiffness of the steel bars. Continuous change of λ thus accounts for the continuous loss of bond due to increased loads and provides a simple way for updating the composite properties of the element.

Equation 105(a) can also be expressed in the form

$$\lambda = 1 - \frac{\int_0^L \epsilon_s dx}{\bar{\epsilon}_s L} \quad (105b)$$

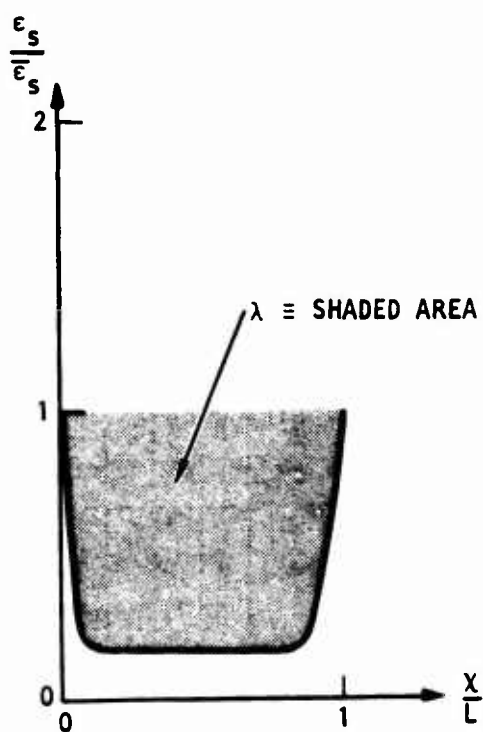
where $\epsilon_s/\bar{\epsilon}_s$ for a unit length of the specimen can be regarded as the ratio of the strain ϵ_s in a steel bar totally or partially bonded to concrete and the strain $\bar{\epsilon}_s$ in a steel bar free of any bond with concrete. It should be noted that

$$\frac{\epsilon_s}{\epsilon_s} = \frac{\epsilon_s A_s E_s}{T} = 1 \quad (106)$$

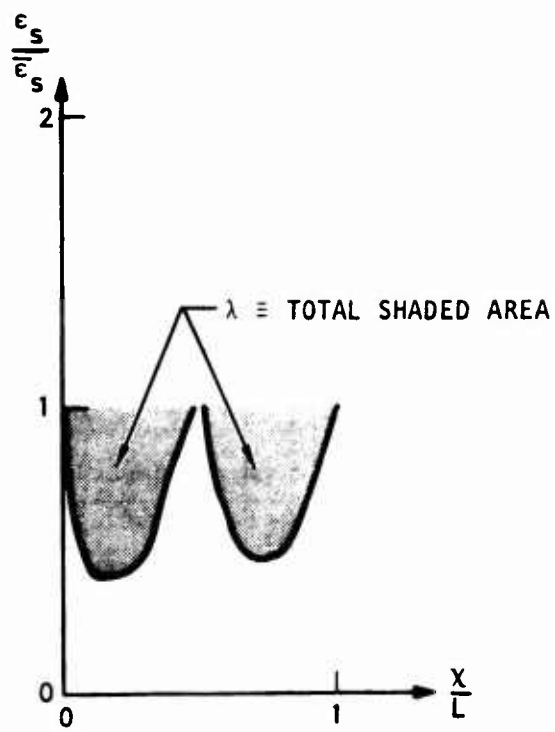
for the case where bond between concrete and steel bar is completely broken.

Figure 49 shows an idealization of the strain distribution to be expected in the reinforcing bar at various stages of loading of the specimen. The area of the shaded portion of each figure is equal to λ . Figure 49(a) shows the strain distribution at an early stage of loading, before any major cracks have developed. Throughout most of the length of the reinforcing bar, the strain is but a small fraction of the strain which would prevail if there were no concrete. At the ends of the specimen, however, since the load beyond the concrete portion of the specimen is carried entirely by the reinforcing bar, the strain in the reinforcing bar approaches the value it would have if there were no concrete. Thus, local bond deterioration near each end of the specimen has reduced the role of the concrete in supporting the tensile load. Figure 49(b) shows the strain distribution in the reinforcing bar after a crack has developed. The value of the strain at the site of the crack is the "no-concrete" strain value. Comparison with figure 49(a) shows that the formation of a crack results in a large reduction in the value of λ . Figure 49(c) shows the strain distribution in the reinforcing bar after the steel/concrete bond has completely deteriorated. Note that the strain is the no-concrete strain throughout the specimen and that the value of λ is zero.

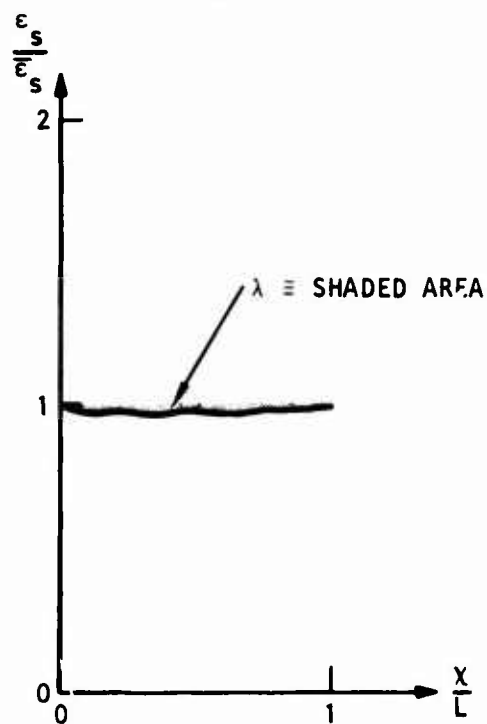
Test specimens 3 and 4 were tested in tension. Strain gages distributed along the reinforcing bar were used to obtain plots of the strain distribution. Figure 49 shows plots of the strain distribution in the reinforcing bar of specimen 4 during each of the three stages illustrated in figure 49. The strains at each end of the reinforcing bar are higher than the no-concrete



(a) At an early stage of loading



(b) After a crack has developed



(c) Bond completely broken

Figure 49. Idealization of the Strain Distribution in a Tensile Specimen at Three Stages of Loading

strain. As described in subsection 2, the reinforcing bars were split in order to install strain gages. An error was caused by the local failure of the weld joining the two parts of the reinforcing bar. Therefore, strain measurements near each end of the specimen were disregarded in computing λ . Curve (a) of figure 50 shows the strain distribution in specimen 4 during the load stage corresponding to (a) of figure 49. Curve (b) of figure 50 shows the strain distribution after cracks had developed. Unlike (b) of figure 49, cracking in specimen 4 resulted in the total loss of bond strength over almost half the length of the specimen. Curve (c) of figure 50 shows the strain distribution in specimen 4 during the final stages of loading. The strain throughout the specimen is approximately the no-concrete strain.

Figure 51 shows the strain distribution in specimen 3 during the three load stages. Curve (a) is the strain distribution prior to cracking, curve (b) shows the postcracking strain distribution, and curve (c) shows the strain distribution after total deterioration of the bond. The strain at one end of the specimen is abnormally high, due to splitting of the reinforcing bar during load stages (a) and (b). By load stage (c), both ends had split.

Equation 105 was used to evaluate λ at each load step of the tensile tests. The close spacing of the strain gages along the reinforcing bar made possible the evaluation of the integral of equation 105 by numerical integration. Due to the fact that a typical load step contained several bad strain gage readings, which had to be ignored in the analysis, an integration routine capable of variable step size was required. For this reason the trapezoidal rule was used. Figure 52(a) shows λ , as obtained from the tensile tests, plotted against fraction of ultimate load. Ultimate load, in the case of tensile specimens, is taken to be the product of the yield stress and the cross-sectional area of the reinforcing bar.

The sudden drop in λ , occurring once for each specimen, signals the onset of cracking. Although in both cases this drop occurred at a load fraction of 0.4, it should be emphasized that the two specimens were identical. Hence, agreement between the specimens on such matters as cracking stress is to be expected.

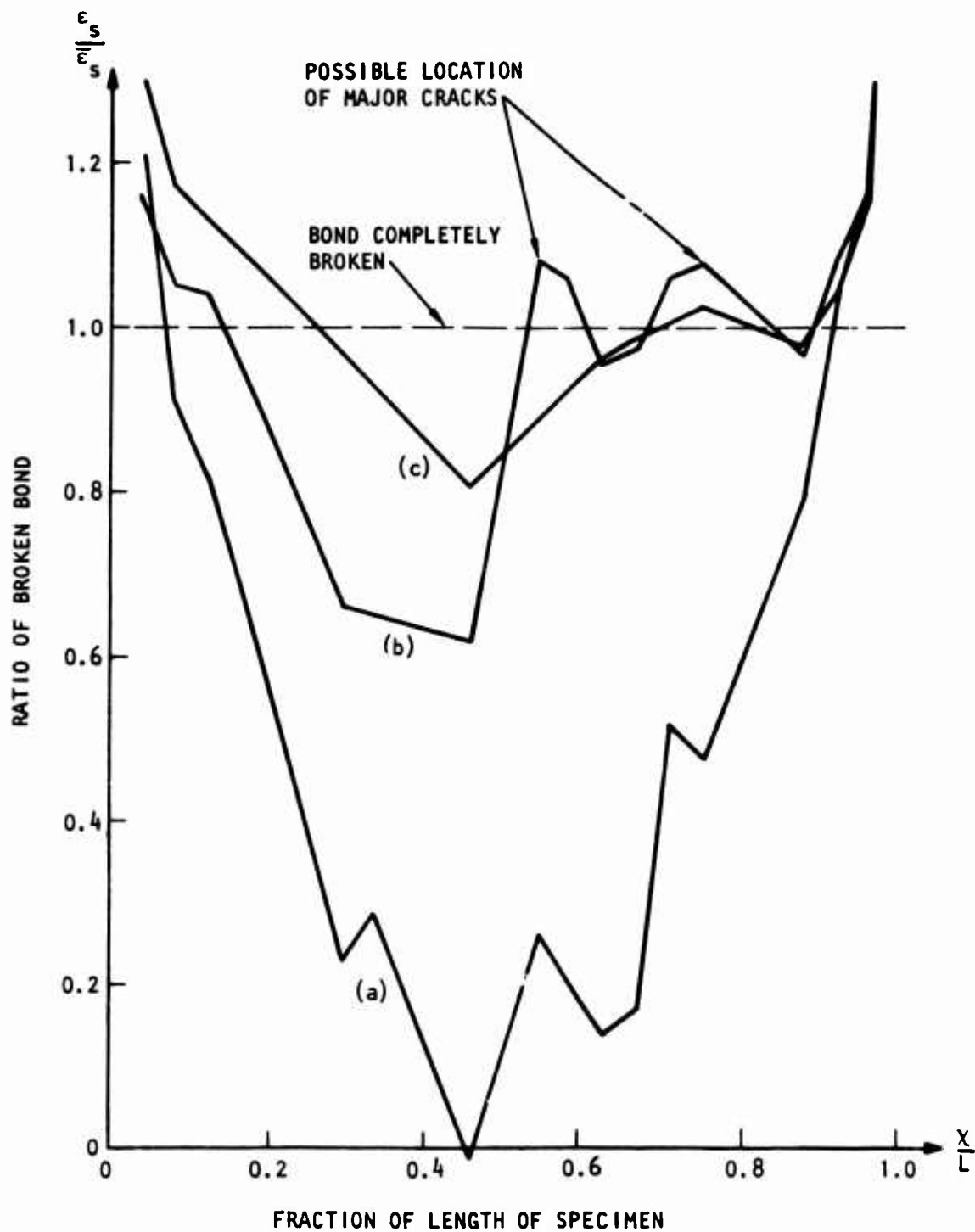


Figure 50. Strain Distribution in the Reinforcing Bar of Specimen 4 for Load Fractions (a) 0.356, (b) 0.533, (c) 0.828

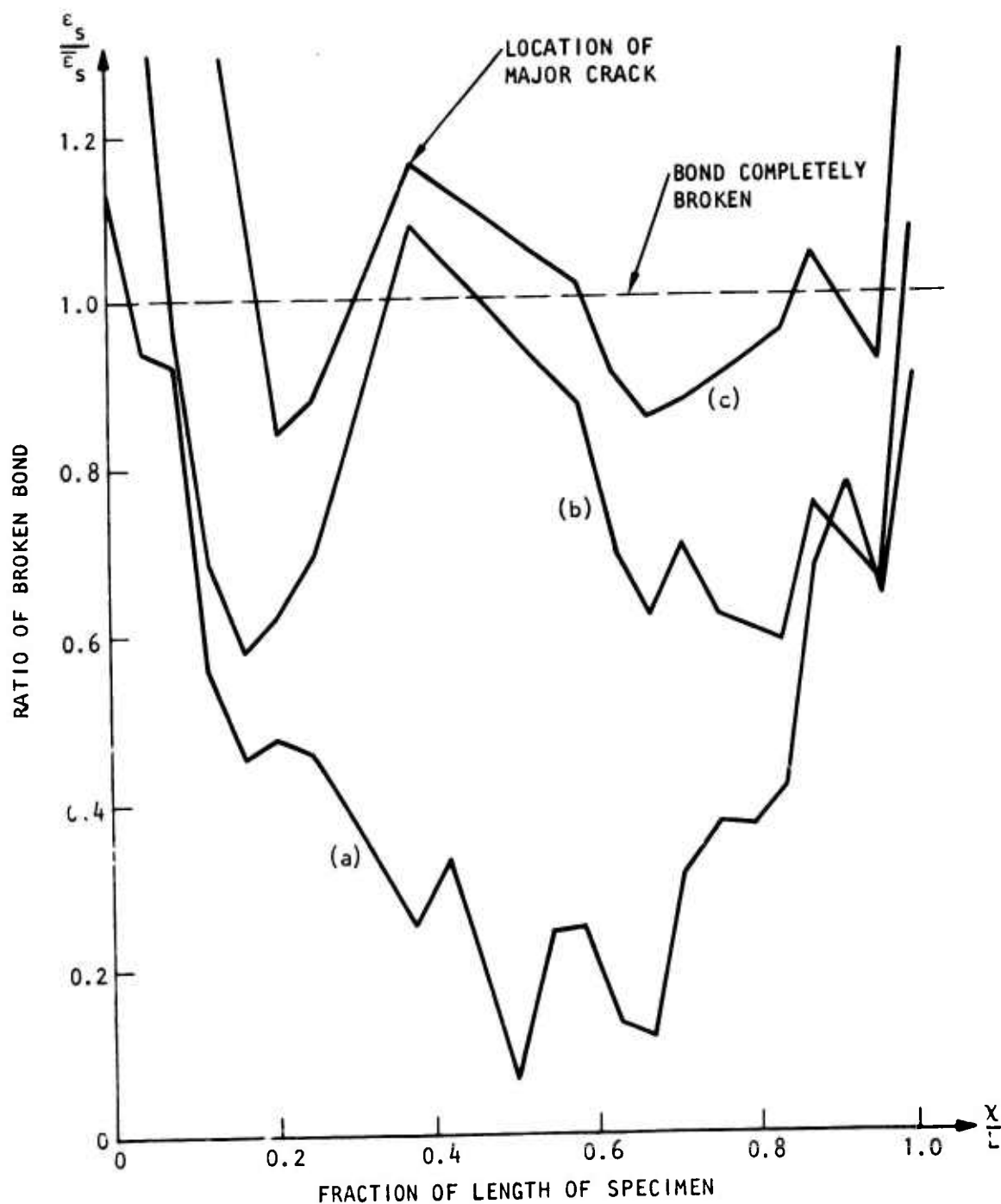
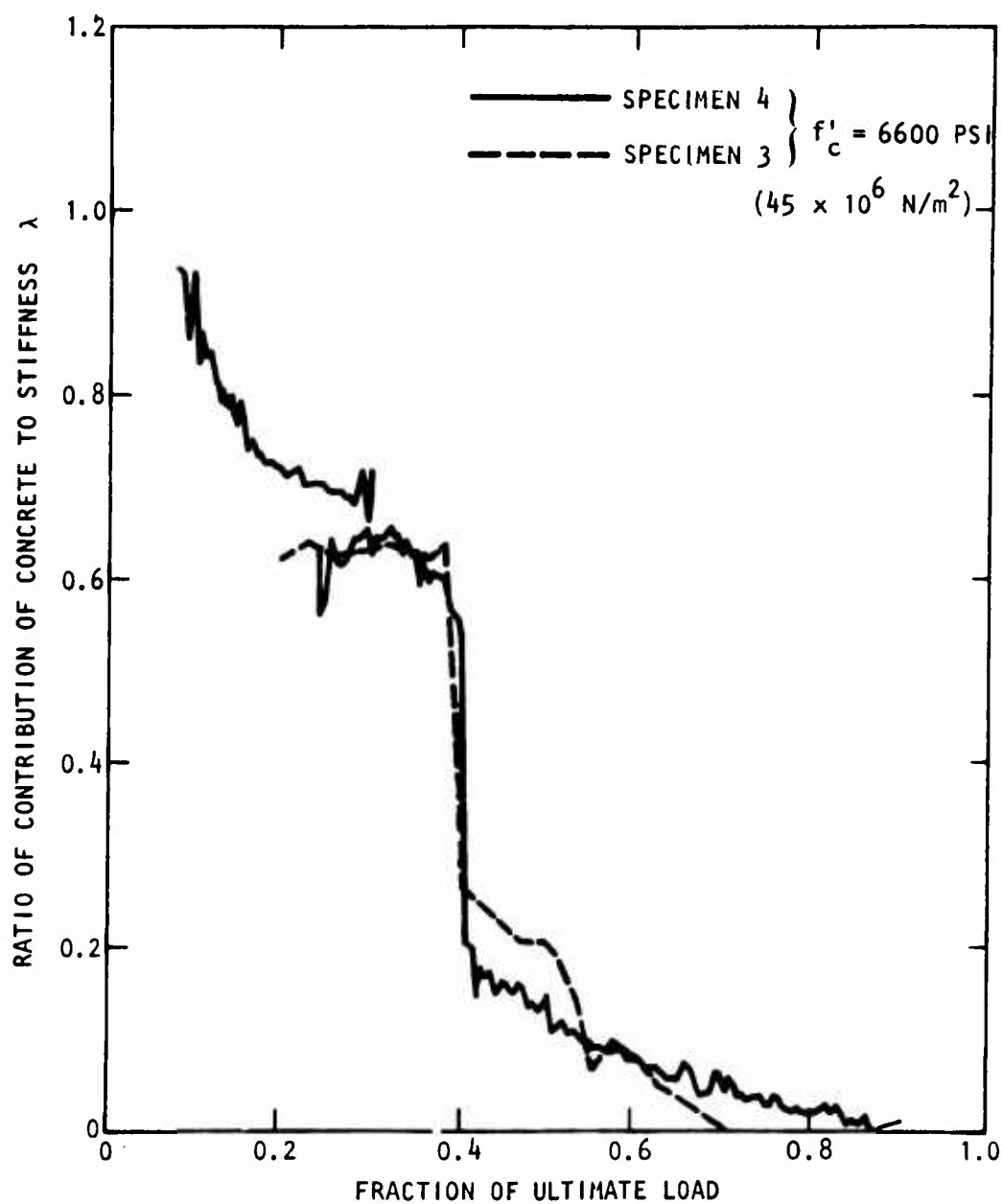
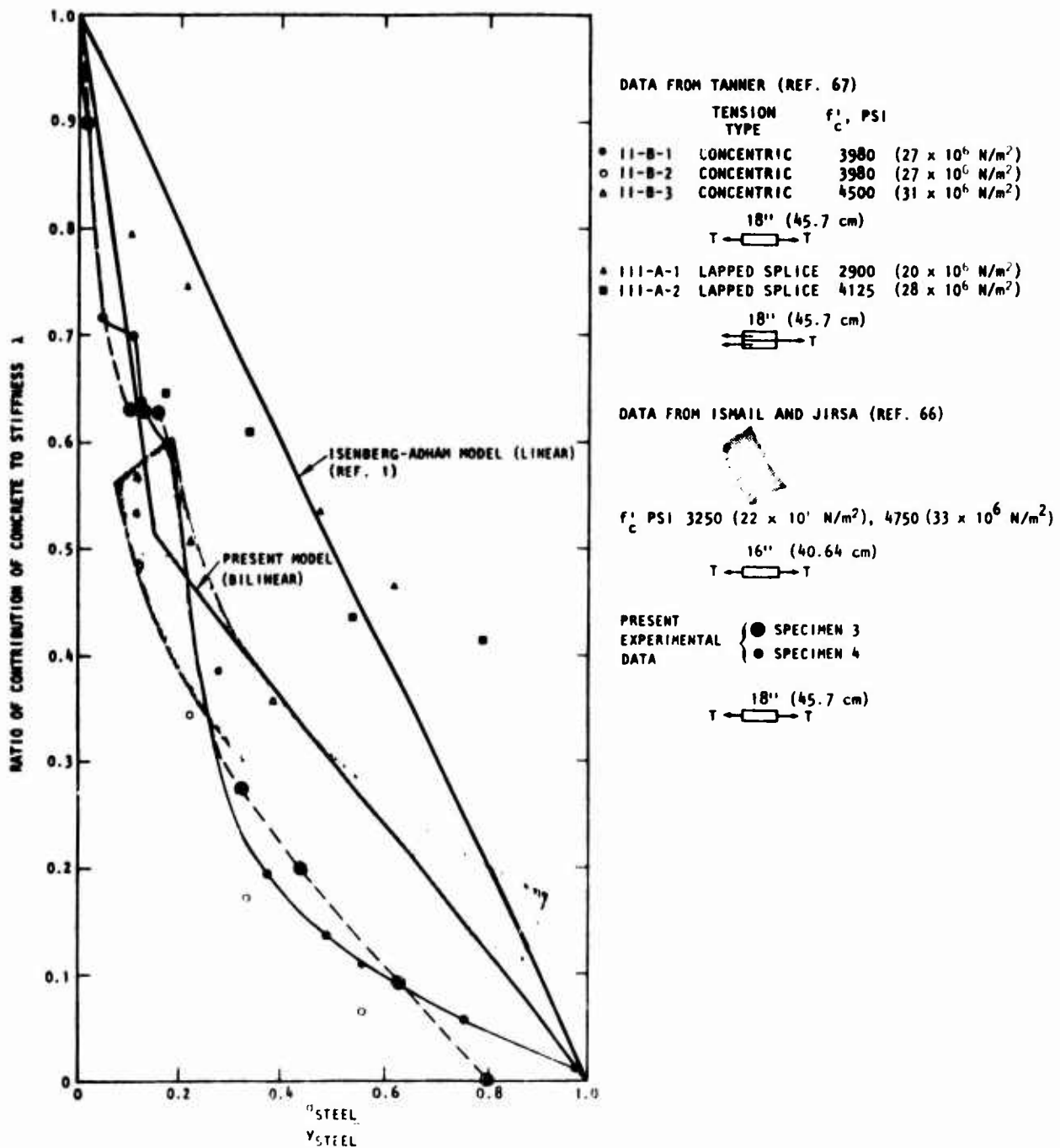


Figure 51. Strain Distribution in the Reinforcing Bar of Specimen 3 for Load Fractions (a) 0.325, (b) 0.455, (c) 0.795



(a) Present experimental results, bond-slip relations in tension

Figure 52. Bond-Slip Relations



(b) Experimental results (Refs. 66, 67, and present study) compared with analytical results (Ref. 1 and present study)

Figure 52. Bond-Slip Relations (concluded)

An interruption in the testing of specimen 4 is evident in figure 52. After the specimen had been loaded to a load fraction of 0.30, a power failure occurred, causing the load fraction to drop to 0.24. The test was resumed from there. The deterioration of the bond that took place prior to the power failure could not be recovered. Therefore, when the test was resumed, λ retained its value from before the power failure. During reloading, further destruction of the bond was minimal until the specimen had been reloaded to a load fraction of 0.30, the value at which the power failure had occurred. At this point, destruction of the bond at the original rate was resumed. A similar interruption of the testing of specimen 3 occurred. This interruption was caused by a failure of the load cell to operate properly. Hence, no data are available for the portion of the test prior to the interruption. The curve shown in figure 52(a) is the portion of the specimen 3 test after the interruption. The left-hand portion of the curve is flat, indicating reloading rather than virgin loading.

Ismail and Jirsa (Ref. 66) conducted similar tests, using tensile specimens with No. 9 reinforcing bars, whereas No. 7 bars were used in the present study. They used concrete of 3.25 ksi (224×10^5 N/m²) and 4.75 ksi (328×10^5 N/m²) ultimate strength, while concrete of 6.6 ksi (455×10^5 N/m²) ultimate strength was used in the present study. The primary effort of the tests of Ismail and Jirsa was to study the bond deterioration under cyclic loading. They obtained the result that unloading and reloading up to the previous maximum load did little damage to the bond. Loading beyond the previous maximum was detrimental to the bond. The path of the λ versus load fraction curve observed during the power failure of specimen 4 in the present study is consistent with this finding.

Other bond-slip data were discussed in sections III and IV. The choice of the bond-slip relation used in this study was based mainly on the experimental data by Ismail and Jirsa (Ref. 66) and Tanner (Ref. 67), as illustrated in figure 38. The experimental results obtained from specimens 3 and 4,

superimposed on bond-slip data of figure 38, are presented in figure 52(b). Several observations on this figure follow:

- There is a good agreement between experimental results of specimens 3 and 4 of the present study and specimens 11-B of Tanner (Ref. 67). Both tests were performed under similar conditions where cracks were allowed to develop arbitrarily during testing.
- The bilinear bond-slip relation used in the present model correlates well with the experimental data of the crack-controlled specimens of Ismail and Jirsa (Ref. 66) and falls in between experimental results of Tanner (Ref. 67) and specimens 3 and 4 of this study.
- The Isenberg-Adham model (Ref. 1) correlates well with the experimental results of specimens 111-A of Tanner (Ref. 67).
- The results of specimens 3 and 4 of this study and specimens 11-B of Tanner, which had a total length less than the 30 bar diameter required to develop full bond, can be considered representative of bond-slip behavior of short samples pulled from both ends. The behavior can be characterized by three stages: (1) gradual deterioration of bond caused by end effects, (2) sudden loss of bond when major cracks occur, (3) some recovery due to aggregate interlock followed by gradual deterioration of bond.
- The results of specimens 111-A of Tanner, where the bar was pulled only from one side, can be considered representative of behavior of samples which are long enough to develop full bond. Behavior of such samples is characterized by deterioration of bond.

Specimens 1 and 2 were tested in compression, the load being applied directly to the concrete, rather than to the steel, as was the case with the tension specimen. A bonding parameter, $\bar{\lambda}$, applicable to these tests, and for which the values 1.0 and 0.0 have meanings similar to those described for concrete in the tension specimens, can be defined for steel in the compression specimens through

$$\bar{\lambda} = \frac{\int_0^L \epsilon_s dx}{\int_0^L \epsilon_c dx} \quad (107)$$

where ϵ_c is the strain in the concrete. If the bond were perfect, the strain in the steel would equal the strain in the concrete. The value of $\bar{\lambda}$ obtained from equation 107 would, therefore, be 1.0. If the bond had partially deteriorated, the strain in the concrete would exceed the strain in the steel, since the load bears directly on the concrete. In this case $\bar{\lambda}$ would be less than 1.0. A value of $\bar{\lambda} = 0.0$ would result if the bond were so ineffective that no strain could be transmitted from the concrete to the steel.

Figure 53 shows the strain distribution in the steel and in the concrete of specimen 1 at various stages of loading. At each stage the strain in the concrete exceeds the strain in the steel. Evident in figure 53 is a trend for the strain in the concrete to exceed the strain in the steel by a proportion that increases with load, although the trend is not rapid. There appears to be no sudden deterioration of the bond as there was in the tensile tests when cracks appeared. Figure 54 shows a similar sequence of strain distributions for specimen 2. These curves invite the same observations made in connection with specimen 1, except that the steel and concrete strains recorded during the early stages of loading are in less agreement than in the case of specimen 1.

Figure 55 shows $\bar{\lambda}$, calculated through numerical integration by trapezoidal rule, plotted against fraction of ultimate load. In the case of the compression specimens, ultimate load was taken to be the experimentally measured load at which failure occurred. The observation from figures 53 and 54 that the bond for small load fractions was weaker in specimen 2 than in specimen 1 is reconfirmed in figure 55. Note that the compression data contain no striking features, such as the sudden deterioration of the bond due to cracking. However, due to bad placing of the bearing cap in the compression test, symmetry of the concrete strain from both ends was not preserved (Figs. 53 and 54) and some irregularities of strains were observed

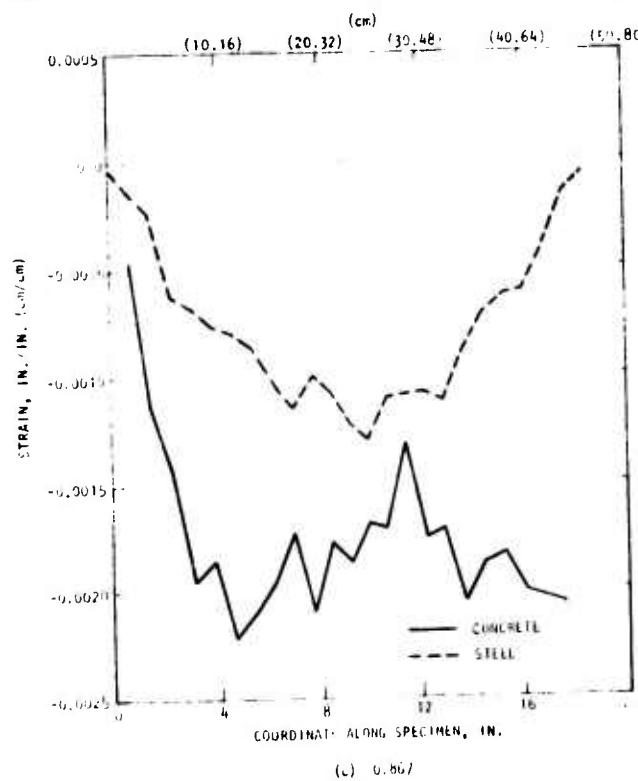
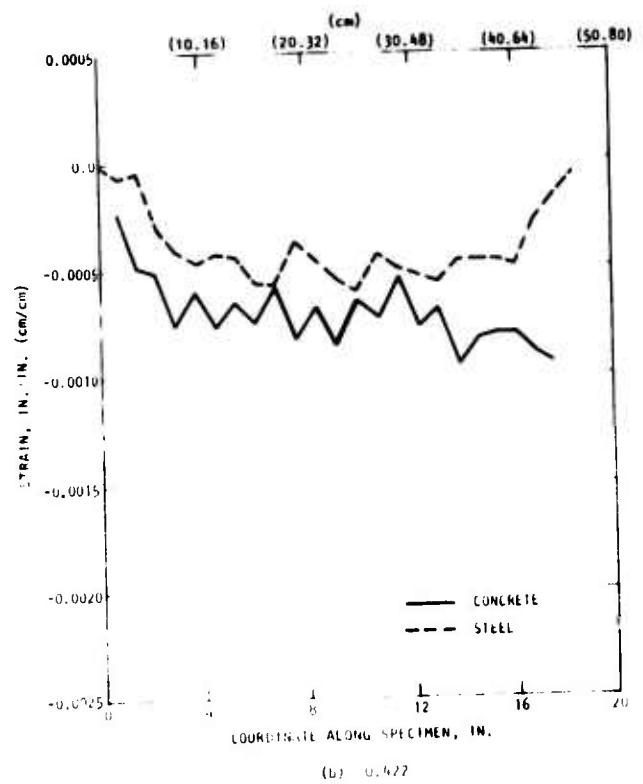
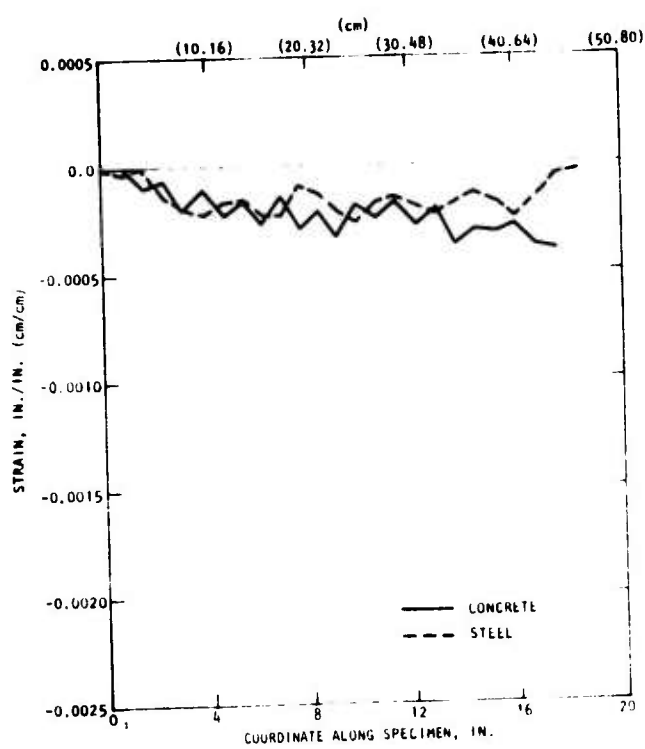


Figure 53. Strain Distribution in Specimen 1 for Load Fractions

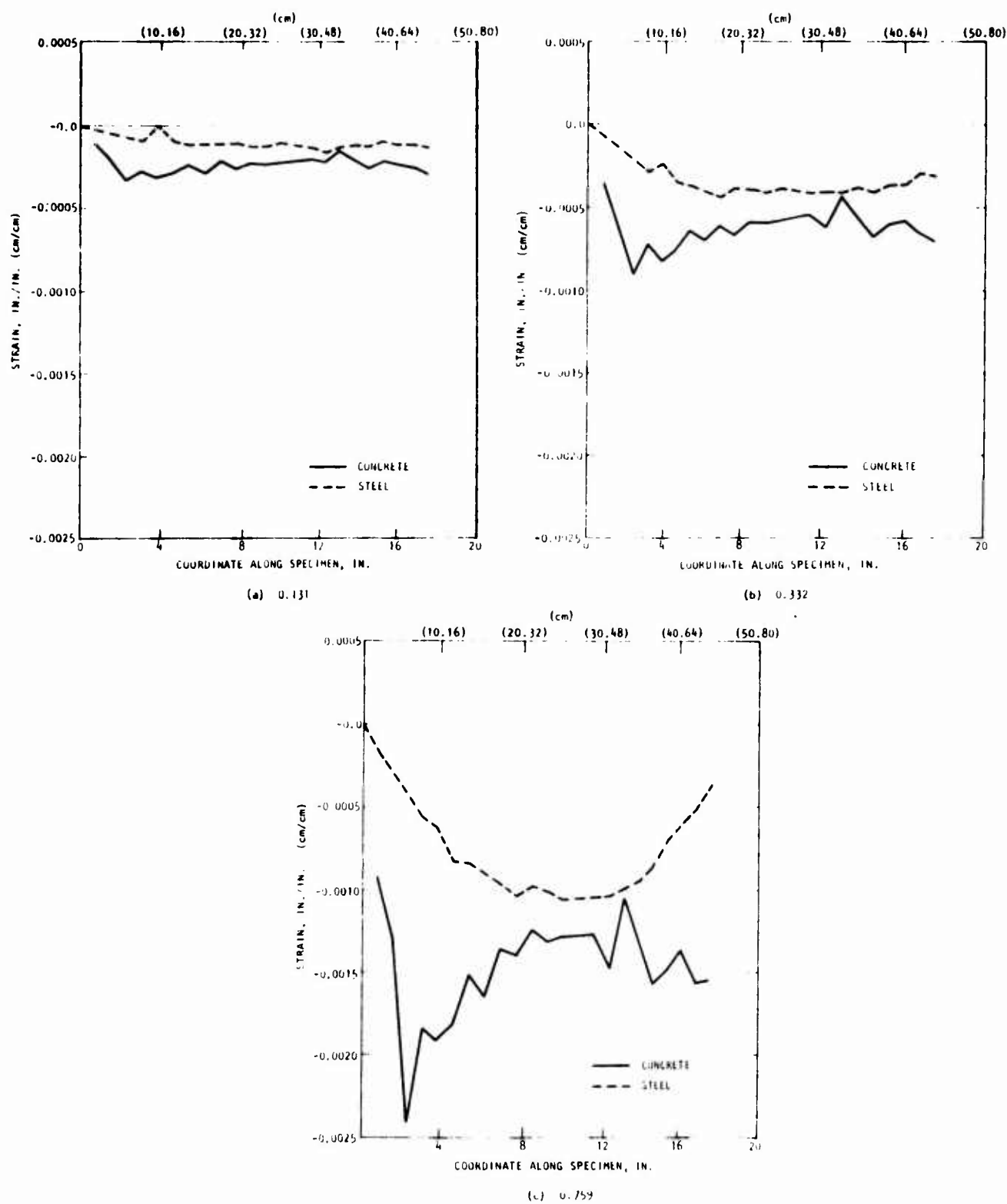


Figure 54. Strain Distribution in Specimen 2 for Load Fractions

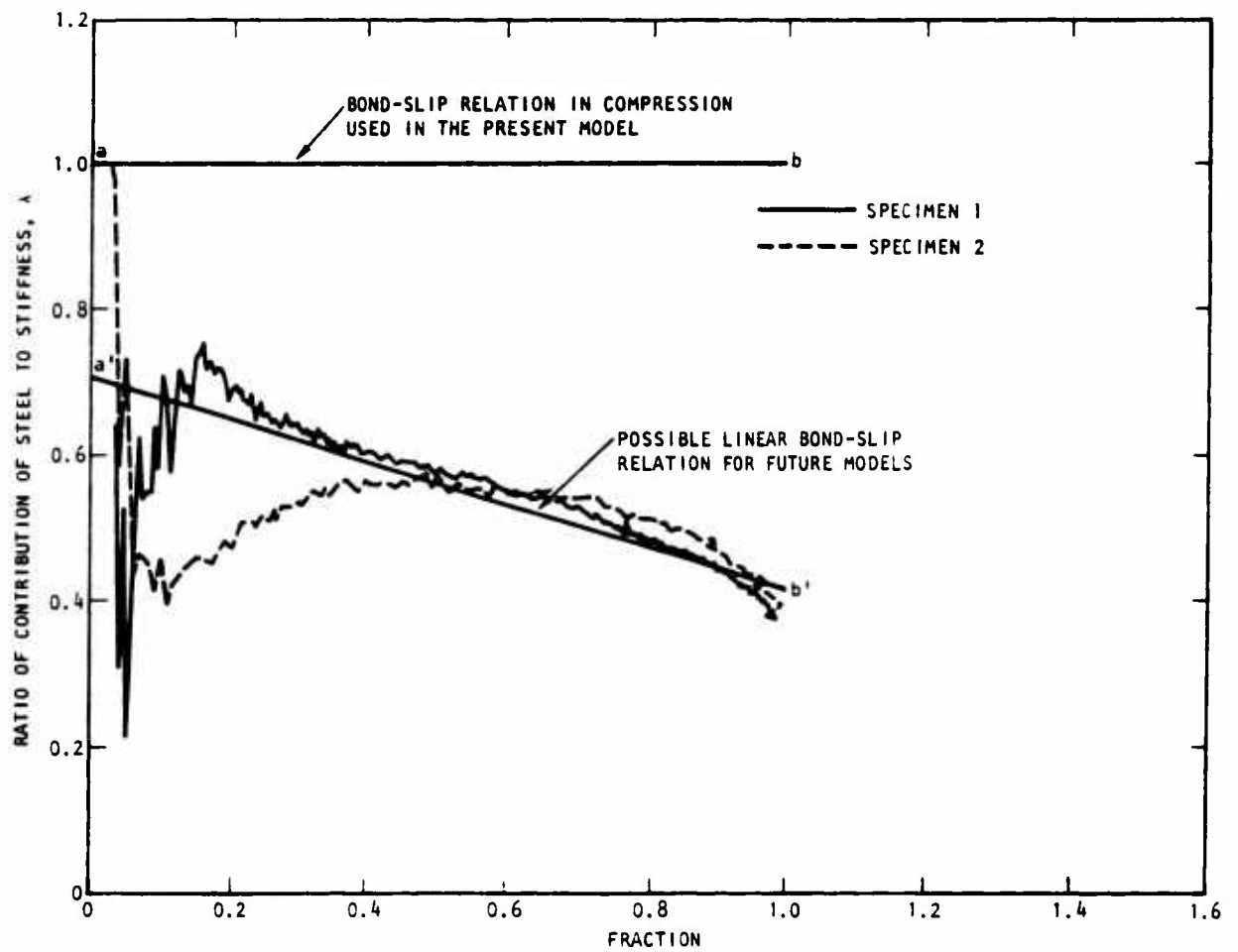


Figure 55. Bond-Slip Relation in Compression

at early stages of the loading. This irregularity has apparently caused the unrealistic drop in bond between steel and concrete at the beginning of the loading (Fig. 55). Therefore a reasonable correction of figure 55 would be to let a' represent the start of the test with full bond, i.e., $\bar{\lambda} = 1$. A possible linear relation for bond slip in compression, $a' - b'$, based on the present experimental data is postulated in figure 55. This relation indicates that the effective stiffness of steel can be reduced by as much as 36 percent by application of the ultimate load in compression.

7. CONCLUSIONS

From the above observations on bond slip in tension, it can be concluded that the bilinear bond-slip relation used in this study follows a course approximately through the middle of the experimental data for both short and long samples shown in figure 52(b). However, the use of very short finite elements may reduce the accuracy of the model. The dependence of bond slip on other parameters, such as concrete strength or number of loading cycles, requires additional study.

In compression, the computer model assumes perfect bonding. Since compressive loads are carried primarily by the concrete, and since figure 55 indicates that the contribution of steel in compression remains high throughout most of the range of loading, this approximation seems reasonable. However, for heavily reinforced concrete structures with a dominant compression mode of behavior, a refinement of the compression bond-slip relation included in the present model may be justified.

SECTION VI

DEMONSTRATION OF CONSTITUTIVE PROPERTIES IN RESPONSE OF STRUCTURAL ELEMENTS

1. INTRODUCTION

This section describes the application of the orthotropic model of reinforced concrete to the response of structural elements such as shallow and deep beams, circular arches, and beam columns. The analyses were performed with an existing two-dimensional axisymmetric finite element computer program, FEDIA,* into which the constitutive equations were inserted as a module. Both static and dynamic calculations were performed. In many of the nine demonstration cases, comparison was made between experimental measurements and the results of analysis. In fact, the reason for performing static analyses was that there were many more published results of static experiments than of dynamic experiments. Hence, there were greater opportunities for validating the model through static than through dynamic calculations. A summary of the demonstration cases is given in table VI.

One of the main difficulties in performing these analyses was that some details of the concrete and steel properties required for the model were not measured. Among these were the Young's modulus, Poisson's ratio, tensile strength, slope of the cracking envelope for concrete, and dowel action parameter n . Where these were not available from experiments, they were assumed on the basis of previous measurements on similar concrete. A summary of the properties used in each of the cases is given in table VII.

2. CASE S-1 (DEEP BEAM 2S1.6-1 OF CRIST, REF. 89)

The static analysis of deep beam 2S1.6-1 was performed using the finite element mesh shown in figure 56. Sizes and locations of the elements were chosen such that the refinement was greatest in regions of high stress gradients (for example, compression zone) so that reinforcing bars were approximately in the center of the elements. The area of tension steel in Elements 1

*FEDIA is a dynamic, inelastic, two-dimensional, continuum finite element code developed by Agbabian Associates.

Table VI. Summary of Example Problems

Case Number	Loading	Type of Structure	Experimental Source Data
S-1	Static	Deep beam	Crist (Ref. 89)
S-2	Static	Deep beam	Crist (Ref. 89)
S-3	Static	Beam column	Lane (Ref. 90)
S-4	Static	Beam column	Lane (Ref. 90)
S-5	Static	Beam column	Lane (Ref. 90)
D-1	Dynamic	Deep beam	Crist (Ref. 89)
D-2	Dynamic	Beam column	Lane (Ref. 90)
D-3	Dynamic	Circular arch	
D-4	Dynamic	Beam	Feldman, et al. (Ref. 91)

Table VII. Summary of Properties Used in Example Problems

Case No.	E_c , 10^6 psi (10^6 N/m ²)	E_s , 10^6 psi (10^6 N/m ²)	f'_c , 10^3 psi (10^3 N/m ²)	f'_s , 10^3 psi (10^3 N/m ²)	ν_c	f_t conc., psi (10^3 N/m ²)	Slope of Cracking Envelope,* η	Dowel Action Parameters n	G_{steel} , 10^6 psi (10^6 N/m ²)
S-1	3.00 (20685)	30.0 (206850)	4.16 (28683)	48.0 (330960)	0.2	365.0 (2517)	-0.083	4.0	12 (82740)
S-2	3.72 (25649)	30.0 (256490)	5.02 (34612)	50.0 (344750)	0.2	365.0 (2517)	-0.083	4.0	12 (82740)
S-3	3.00 (20685)	30.0 (206850)	4.05 (27925)	62.0 (427490)	0.2	320.0 (2206)	-0.080	4.0	12 (82740)
S-4	3.00 (20685)	30.0 (206850)	4.40 (30338)	62.0 (427490)	0.2	350.0 (2413)	-0.080	4.0	12 (82740)
S-5	3.00 (20685)	30.0 (206850)	5.00 (34475)	62.0 (427490)	0.2	400.0 (2758)	-0.080	4.0	12 (82740)
D-1	3.00 (20685)	30.0 (206850)	4.16 (28683)	48.0 (330960)	0.2	365.0 (2517)	-0.083	4.0	12 (82740)
D-2	3.00 (20685)	30.0 (206850)	5.00 (34475)	62.0 (427490)	0.2	400.0 (2758)	-0.080	4.0	12 (82740)
D-3	7.00 (48265)	30.0 (482650)	7.00 (48265)	50.0 (344750)	0.2	500.0 (3447)	-0.083	4.0	12 (82740)
D-4	3.74 (25787)	30.0 (257870)	3.26 (22478)	45.8 (315791)	0.2	450.0 (3103)	-0.08	4.0	12 (82740)

* See equation 36(a).

† n is the empirical dowel shear constant of equation 63.

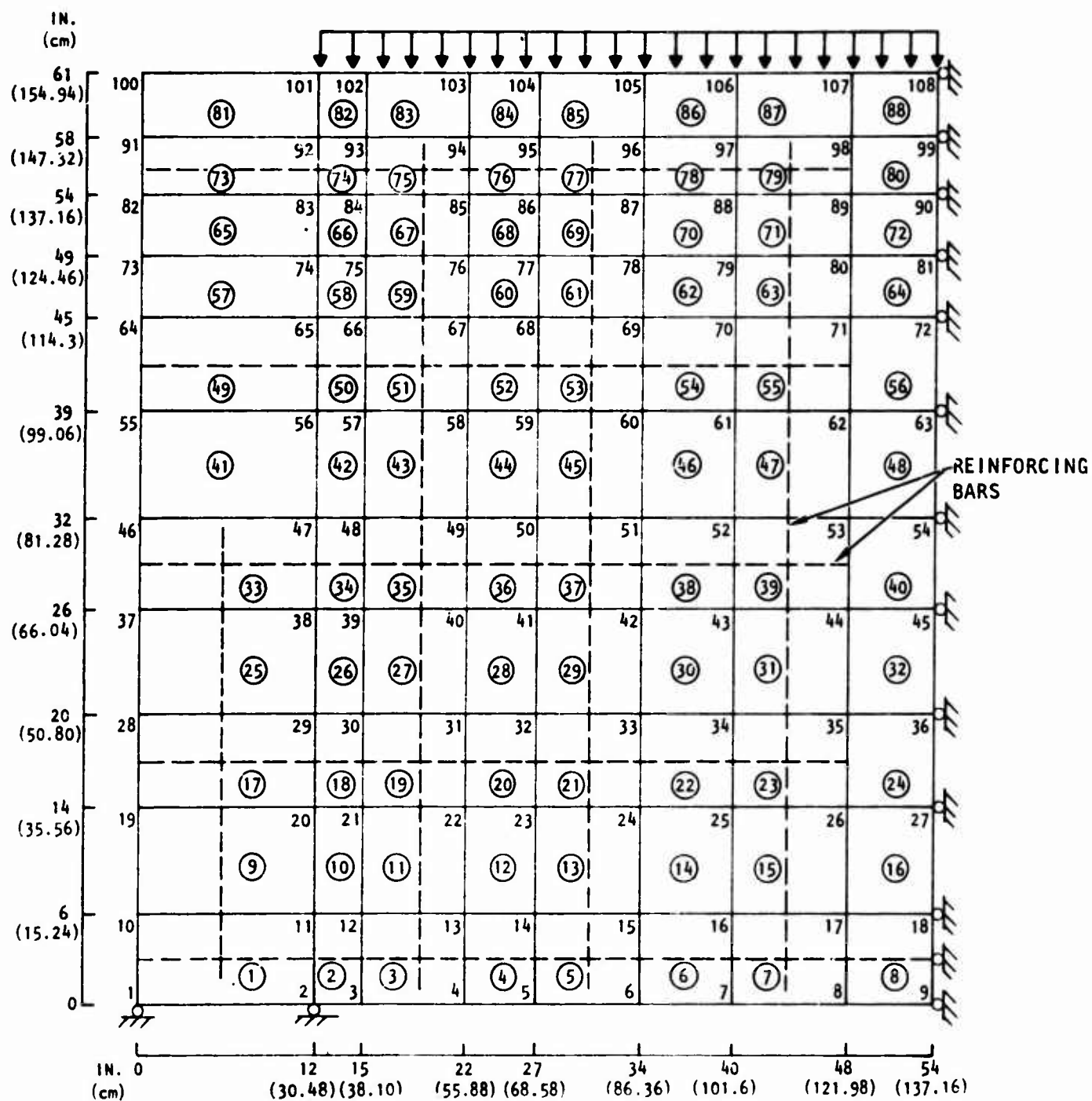


Figure 56. Finite Element Representation of Beams 2S1.6-1 and 2D1.6-1 for Cases S-1 and D-1 (Crist, Ref. 89)

through 8 was about 5 percent of the cross-sectional areas of the elements. Stirrups in other parts of the beam had areas ranging between 0.15 percent to 0.4 percent of the cross-sectional areas of elements in which they were located.

Several different methods of applying uniform pressure loading were investigated. These include:

- a. Applying the full value of the experimentally measured collapse load in one step and then iterating until equilibrium is reached
- b. Applying increments of load, reforming the global stiffness matrix and iterating for equilibrium at every step
- c. Applying increments of load, reforming the global stiffness matrix at every step, and accepting some degree of disequilibrium at each step

The results presented below were obtained with procedure *b*.

The main results of the analysis are shown in figures 57 through 59. Figure 57 shows that the analysis overestimates both stiffness and strength of the beam, as indicated by the curve of external load versus midpoint deflection. The steel yields at 570 kips (2535×10^3 N) external load in the analysis and 470 kips (2091×10^3 N) in the experiment. Figure 58 compares measured and calculated distribution of steel strain and beam displacement at the onset of yielding and at the collapse load. As figure 57 shows, these are not the same loads for analysis and experiment. There is a general agreement between calculated and measured strains and displacements except for the measured displacement at $L/4$, which was not reported in reference 89. The present writers consider that it is more meaningful to compare strains at the onset of similar phenomena than at the same load levels in the event these are different. Figure 59 compares measured and calculated crack patterns at the collapse load. The model results appear to predict most of the cracked regions. However, the absence of cracks from some elements such as 19, 29, 30, 38, 55, and 63 is an indication that the analytical model is stiffer than the experimental model.

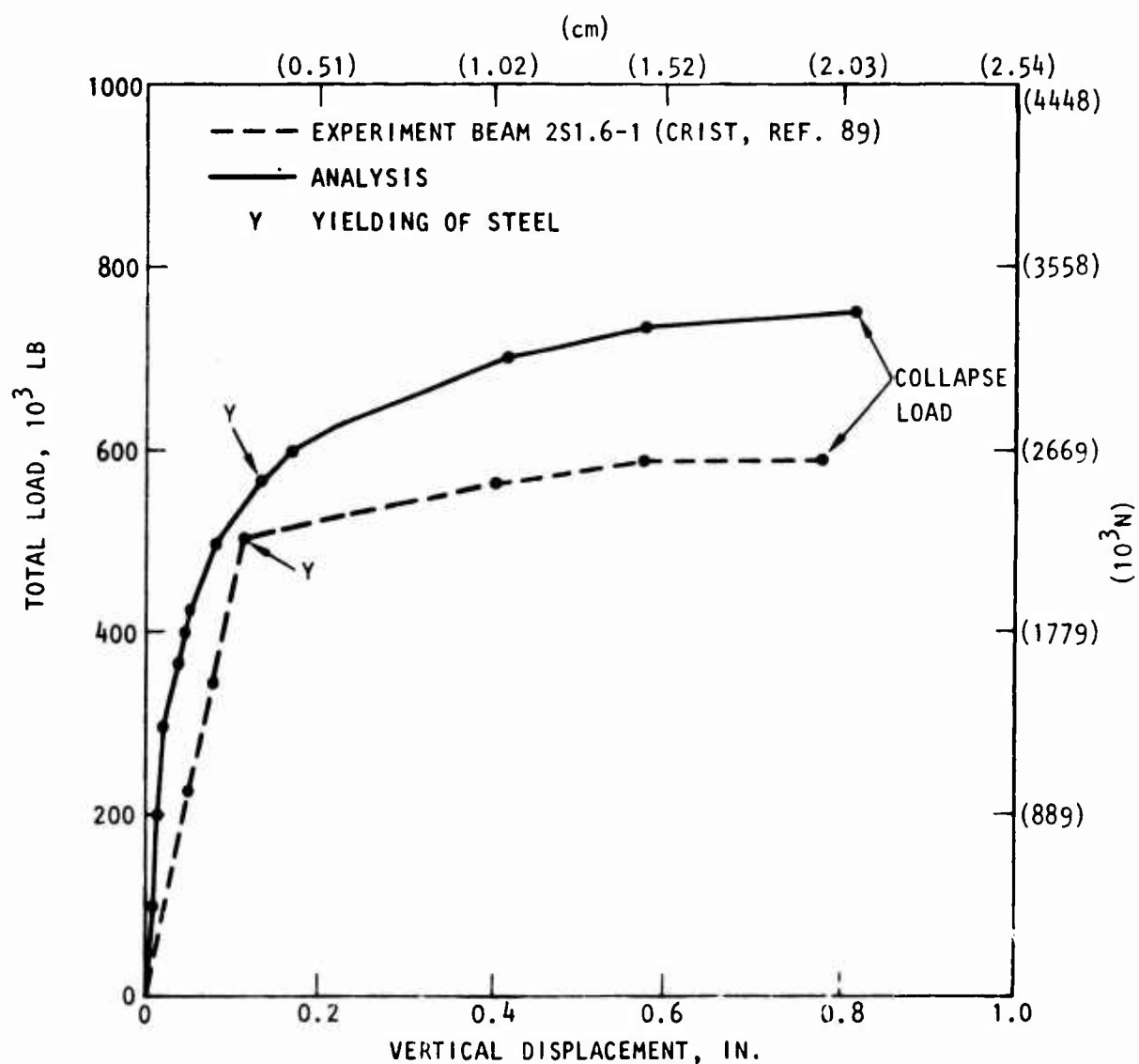
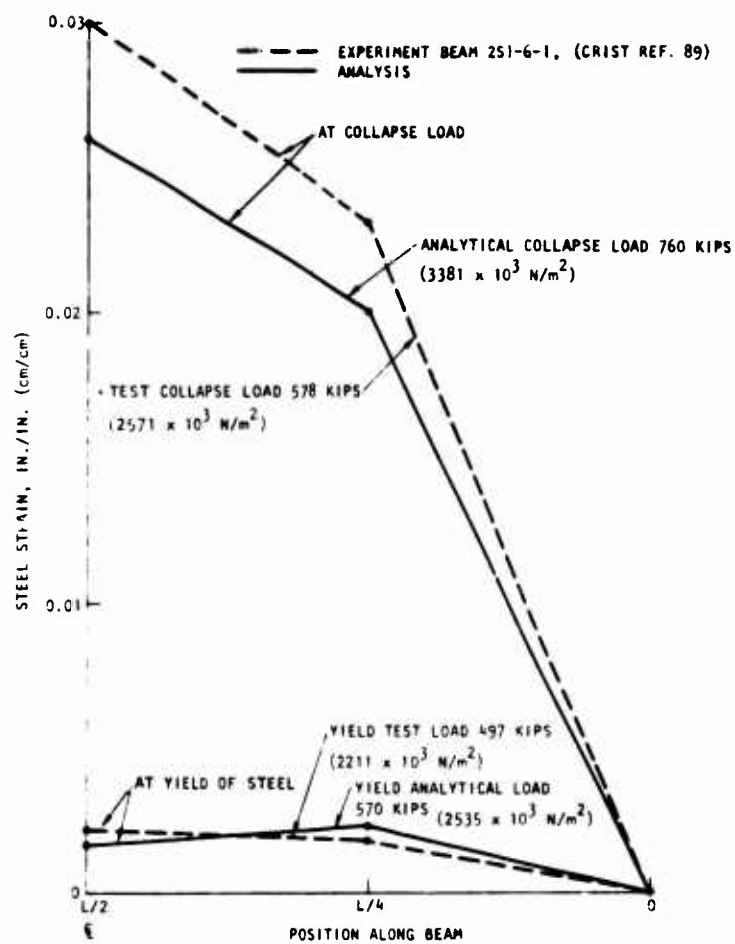
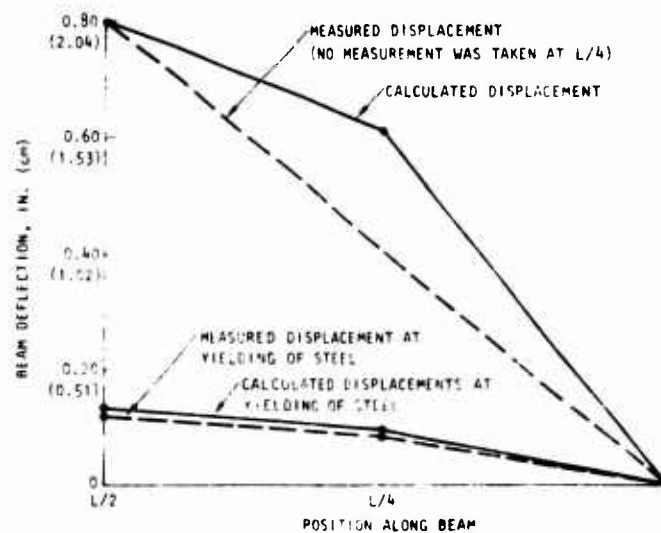


Figure 57. Comparison of Measured and Calculated Vertical Displacements at Middle of Beam 2S1.6-1, Case S-1 (Crist, Ref. 89)



(a) Comparison of measured and calculated strains



(b) Comparison of measured and calculated displacements

Figure 58. Displacements and Strains in Tension Steel, Beam 2S1.6-1, Case S-1 (Crist, Ref. 89)

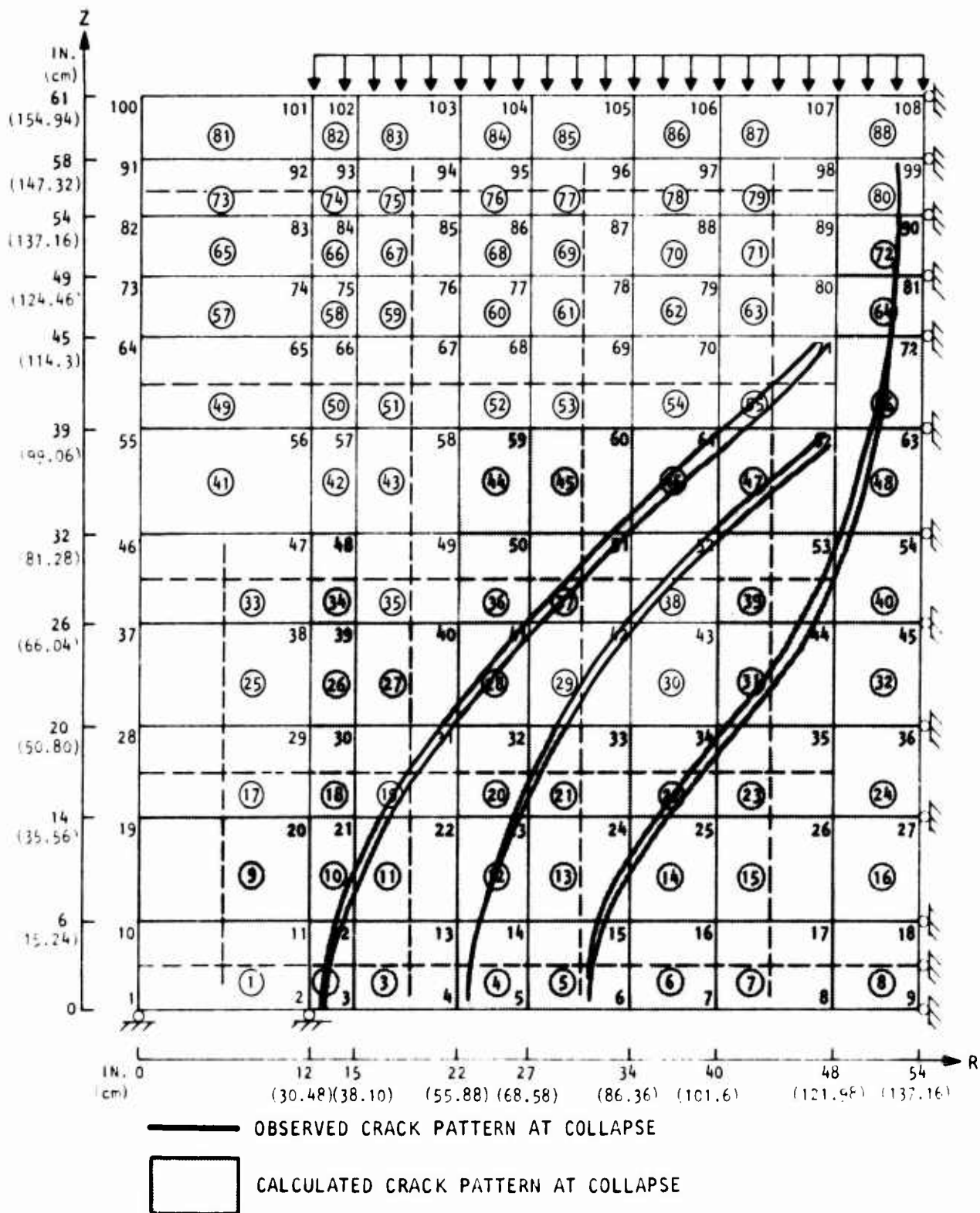


Figure 59. Comparison of Calculated and Observed Crack Patterns at Collapse of Beam 2S1.6-1, Case S-1 (Crist, Ref. 89)

3. CASE S-2 (DEEP BEAM 2S1.6-3 of CRIST, REF. 89)

The static analysis of deep beam 2S1.6-3 was performed using the finite element mesh shown in figure 60. The refinement of the finite elements and their arrangement with respect to the reinforcing is similar to the mesh for Case S-1, except that there are fewer elements in the compression zone in Case S-2. The tension steel is 5 percent of the cross-sectional area of the concrete. The steel percentage elsewhere is about 0.15 percent.

The main results of the analysis are shown in figures 61 through 63. Figure 61 shows that the analysis overestimates the stiffness and collapse load, as was the case for S-1. Reasonable agreement between measurements and calculations of steel strains at yielding of steel and collapse load is shown in figure 62(a). There is a general agreement between calculated and measured displacements as illustrated in figure 62(b). However, since the collapse load is taken as the load at which the solution begins to be unstable, both calculated strains and displacements should be considered as approximate values. As in Case S-1, it is found that nodal point forces are in disequilibrium following yielding of the steel. This contributed to overestimating the collapse load in the analysis. The model appears to predict well the crack patterns in the bottom and center of the beam. However, the absence of cracks from elements such as 27, 35, 36, 43, 44, 45, 51, 52, 53, 59, 60, 61, 62, and 71 indicates that the analytical model is stiffer than the experimental model.

4. STATIC CASES S-3, S-4, S-5 (BEAM COLUMNS 5-3-1, 5-2-1, 5-0-1 OF LANE, REF. 90)

An analysis was performed of beam columns subjected to different ratios of transverse to axial load. The geometry, properties, and loading correspond to those used in physical experiments performed by Lane (Ref. 90). The geometry and arrangement of reinforcing is the same in each case. Three values of the ratio of axial load (P) to lateral load (F) were treated: 3.18, 1.9, and 0 (transverse load only). The finite element representation of the beam column is shown in figure 64. A plane of symmetry was assumed at midspan in order to conserve computer time.

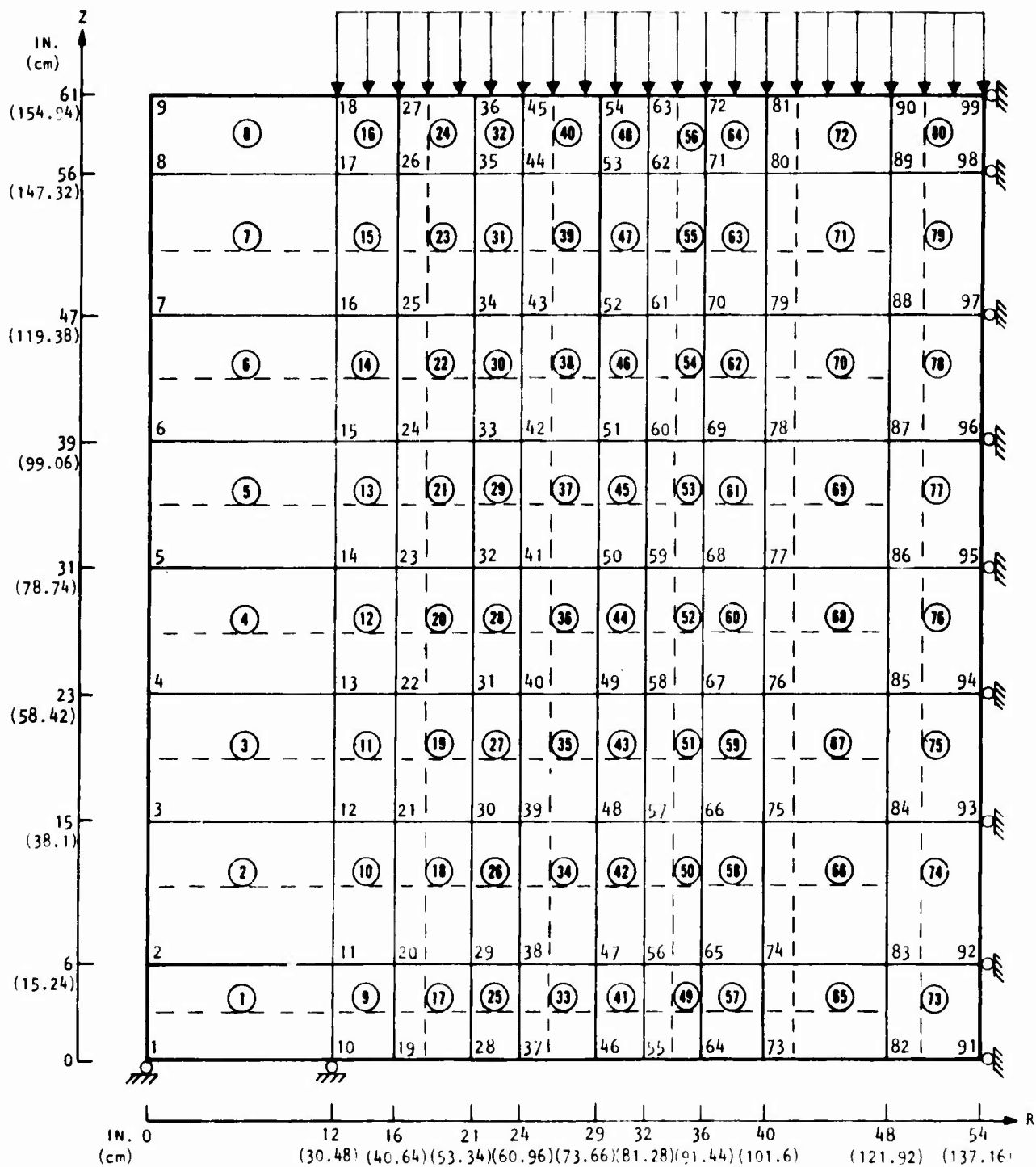


Figure 60. Finite Element Representation of Beam 2S1.6-3, Case S-2
(Crist, Ref. 89)

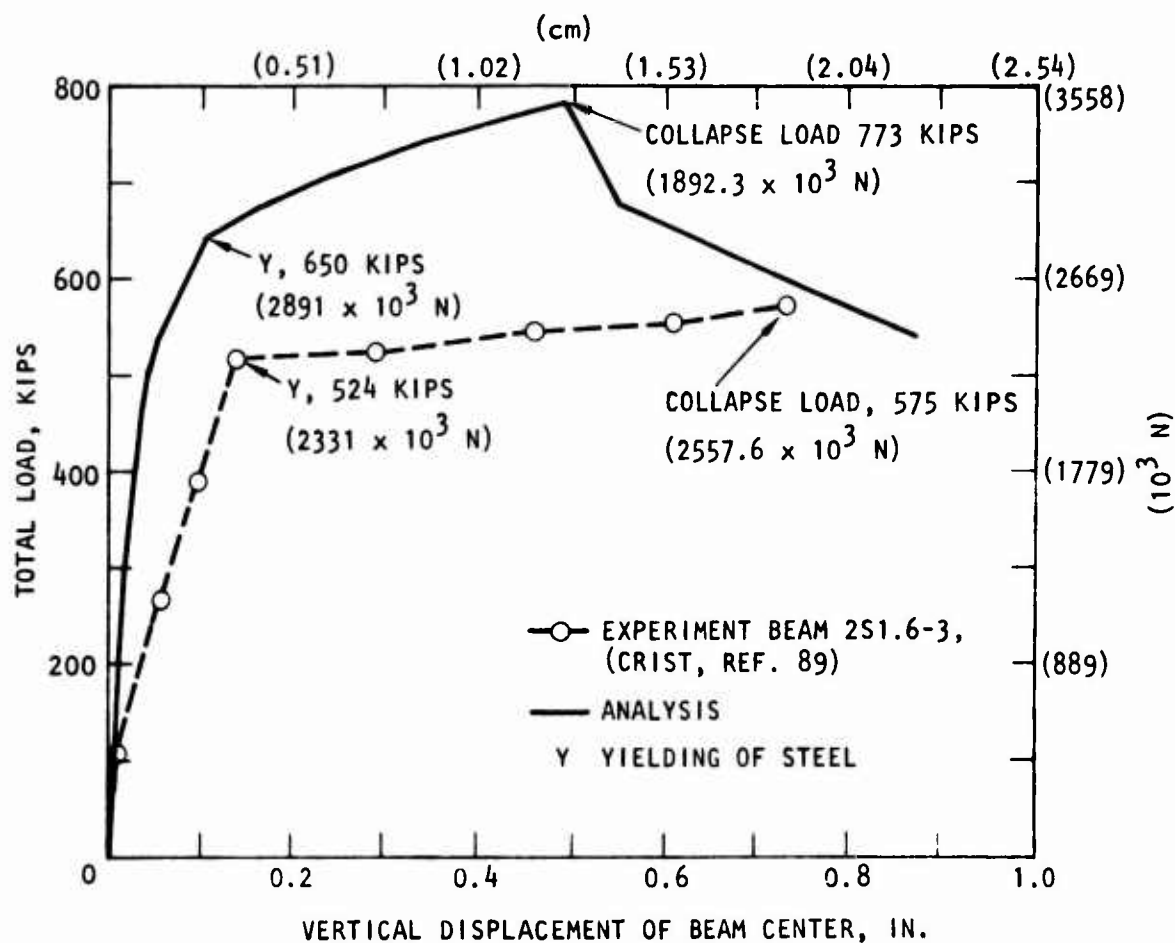
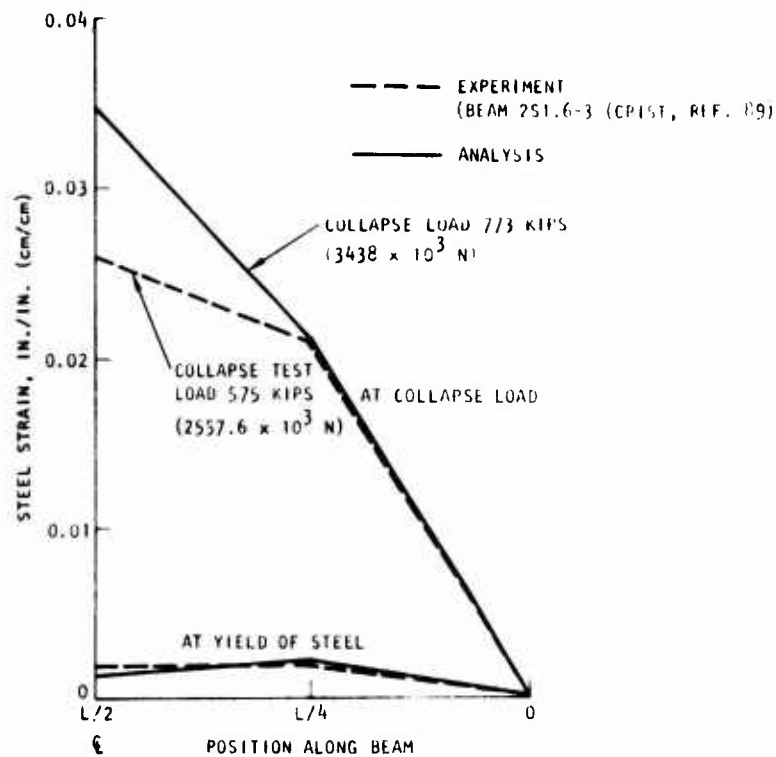
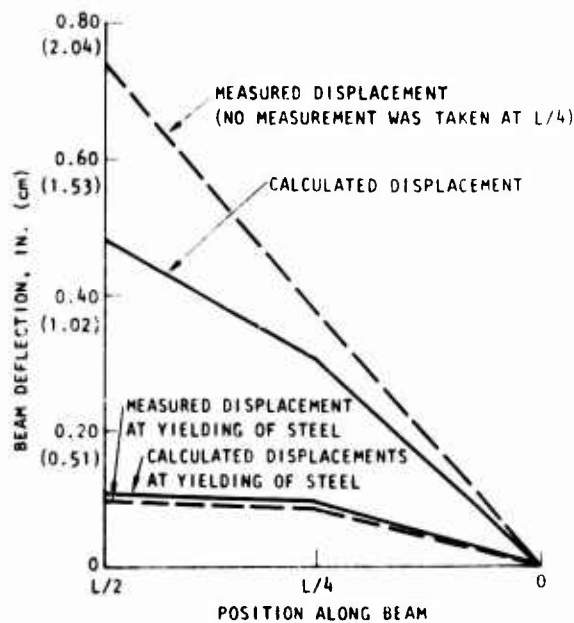


Figure 61. Comparison of Measured and Calculated Vertical Displacements at Middle of Beam 2S1.6-3, Case S-2 (Crist, Ref. 89)



(a) Comparison of measured and calculated strains



(b) Comparison of measured and calculated displacements

Figure 62. Displacements and Strains in Tension Steel, Beam 2S1.6-3, Case S-2 (Crist, Ref. 89)

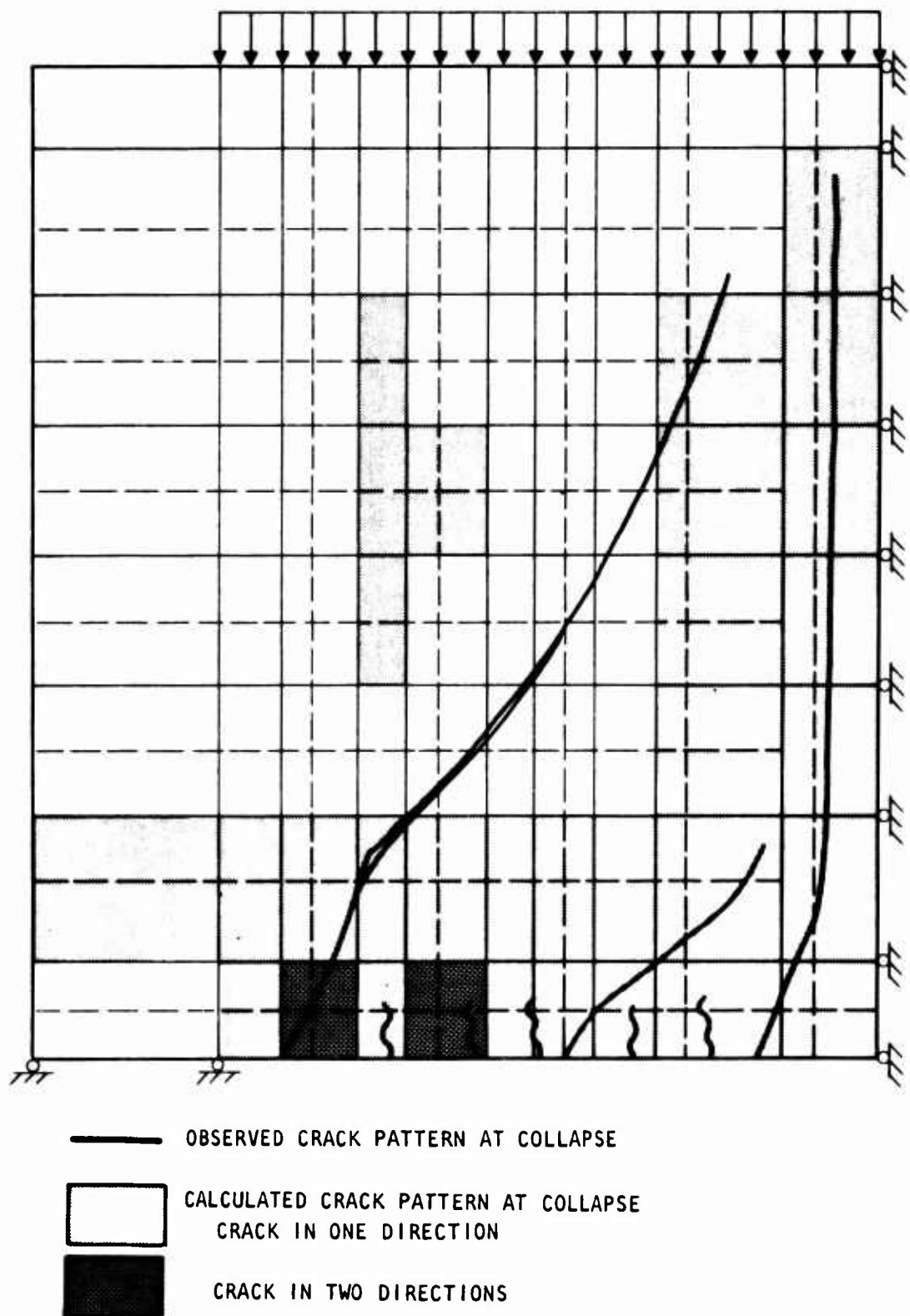


Figure 63. Comparison of Calculated and Observed Crack Patterns at Collapse of Beam 2S1.6-3, Case S-2 (Crist, Ref. 89)

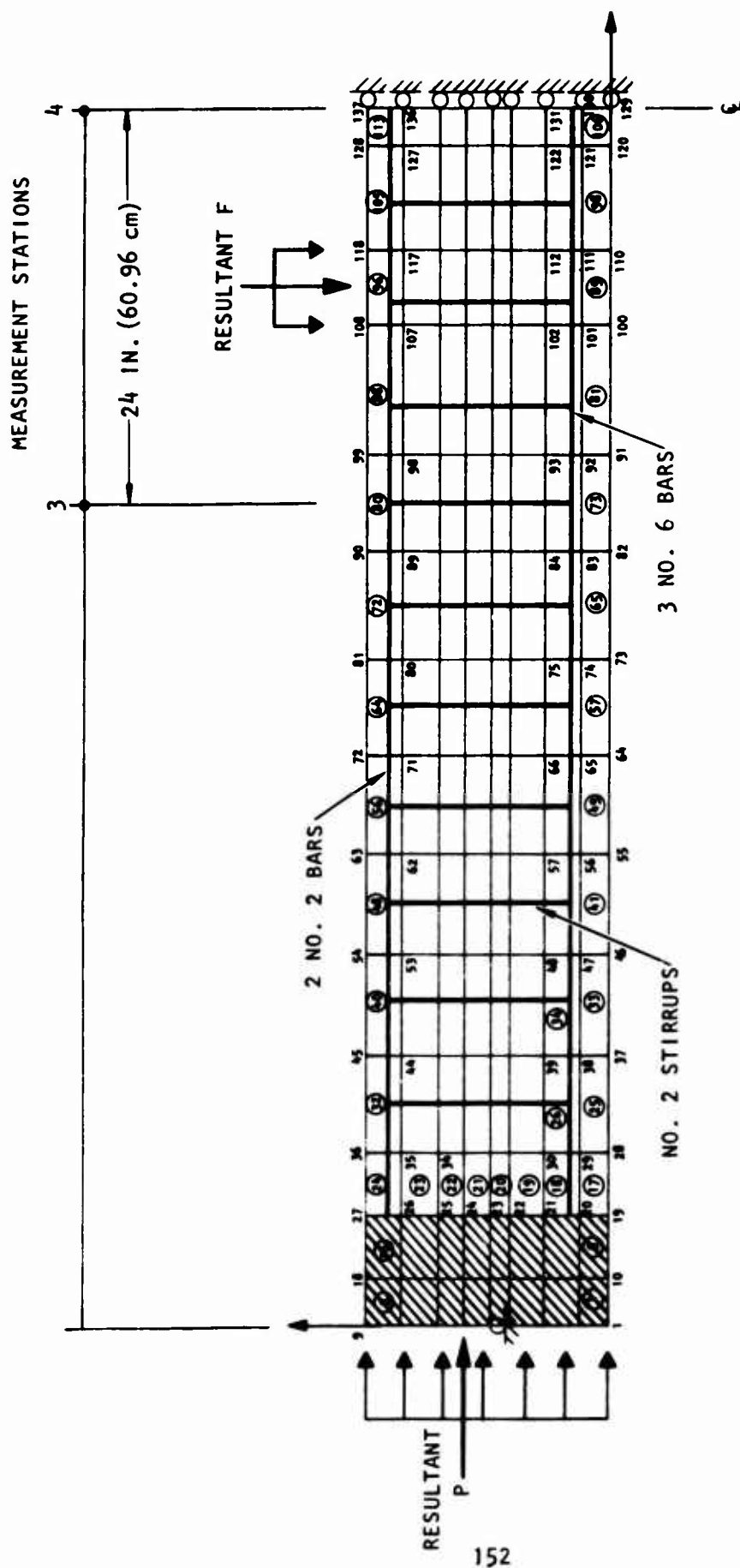


Figure 64. Finite Element Mesh used in Static Analysis of Beam Column of Cases S-3, S-4, S-5 (Lane, Ref. 90) and Dynamic Analysis Case D-2.

As in the case of other static analyses, the collapse load must be approached slowly, with iteration at each load step to assure that nodal point equilibrium is reached at each step. The onset of collapse is detected by observing that displacements increase while external load is held fixed. This signifies that nodal point equilibrium cannot be satisfied, and that collapse eventually will occur if enough iterations are performed. This procedure leads to an upper bound on collapse load. In the following analyses, an attempt was made to find the collapse load through the procedure of incrementing external load in small steps and iterating at each step.

Results of the three beam-column analyses are compared in figures 65 through 70 with experimental measurements. The general indication of the results is that the analytical model overestimates the collapse load of the beam. Figures 65, 67, and 69 indicate that before yielding, the analytical model is stiffer than the experiment. However, after yielding, the analytical model results in displacements that are higher than the measured values. The same trend was observed in the comparison of strains at the bottom of beam 5-3-1 of case S-3 where the calculated strains in Element 107 were compared to the measured steel strain as illustrated in figure 66(a). Since the strains in the element after propagation of cracks are essentially those of the steel embedded in the element, the comparison between element strain and steel strain is justified in this case.

The average strain calculated in Element 80 at top of beam 5-3-1 (Station 3) was compared to strains measured in both concrete and steel (Fig. 66(b)). The results reveal good correlation except in the postyielding regime, where the calculated strains appear to exceed the measured values.

The strain in steel at the bottom of the middle of beam 5-2-1 of case S-4 was compared to the strain in Element 107 at the same location of the beam as illustrated in figure 68(a). The comparison reveals a trend similar to that observed for case S-3 where the analytical model appeared to be stiffer than the experimental beam at the early stages of the loading.

The strain calculated at the top of beam 5-2-1 was compared to the strain measured in both steel and concrete at this location as illustrated in figure 68(b). The comparison reveals that the calculated element strain exceeds

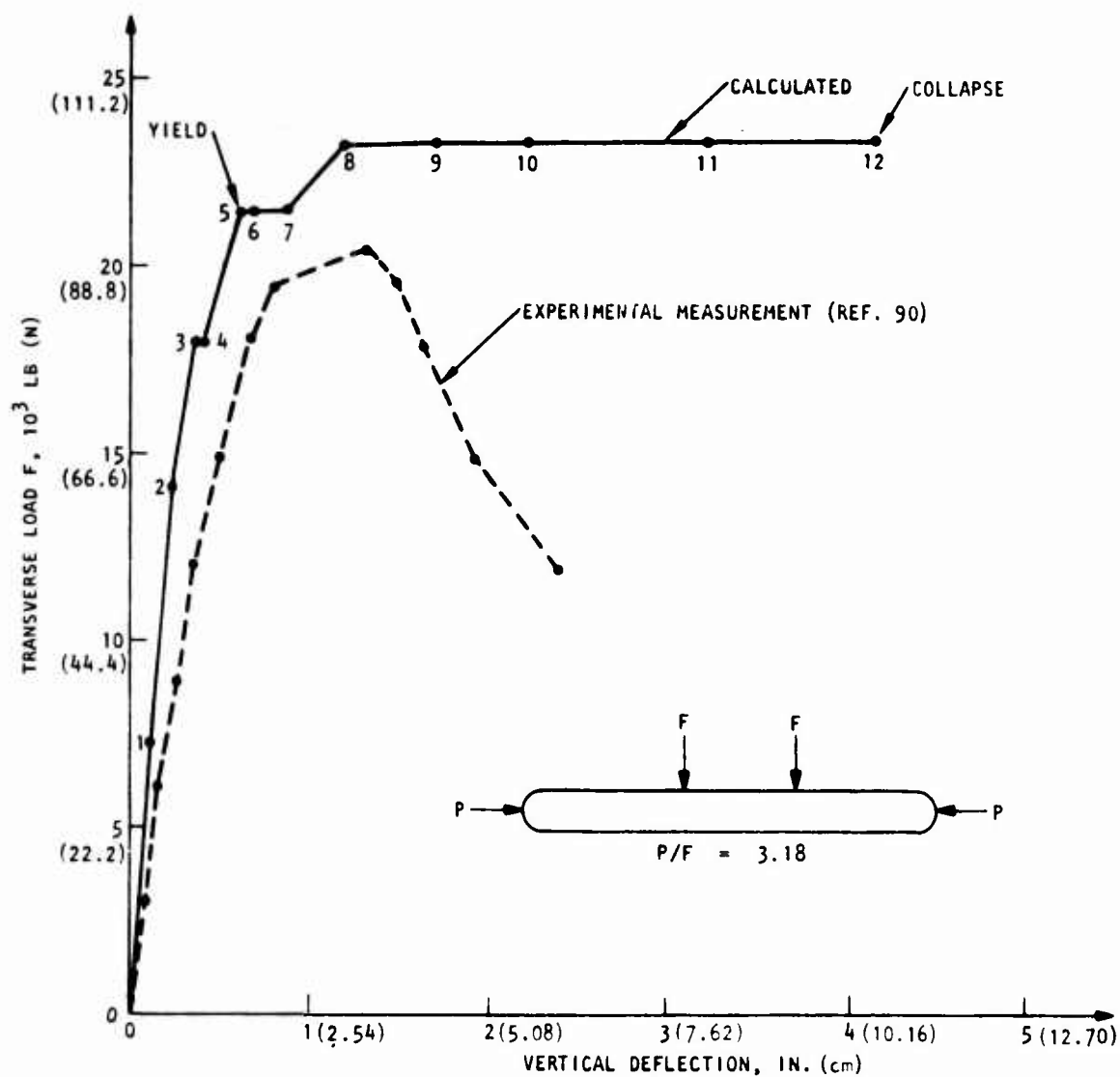
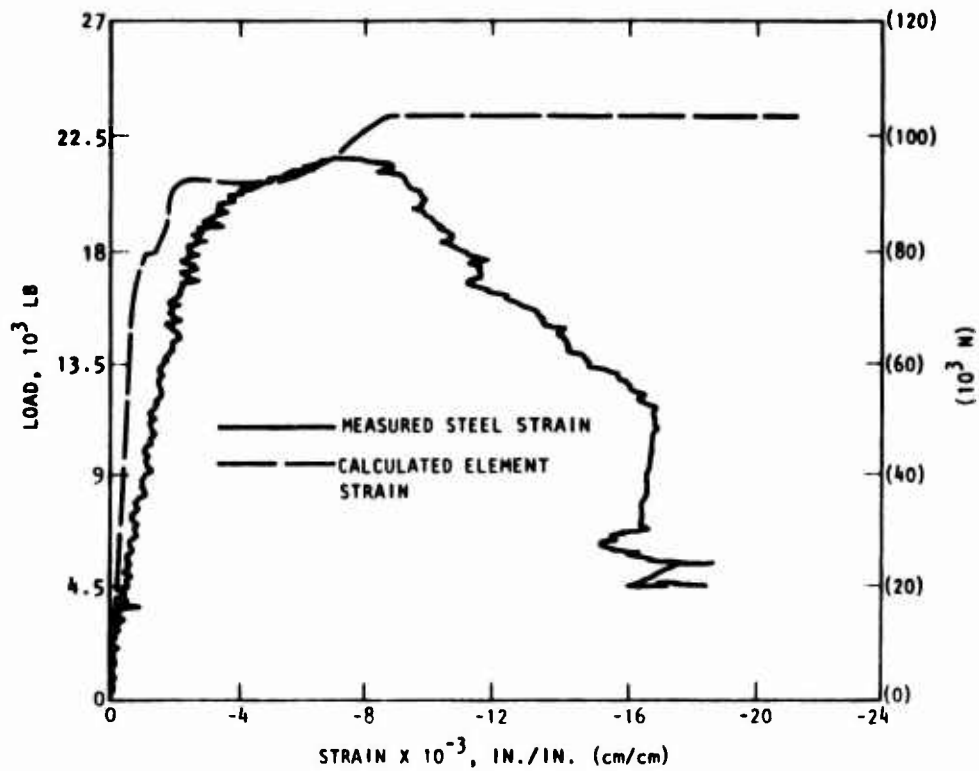
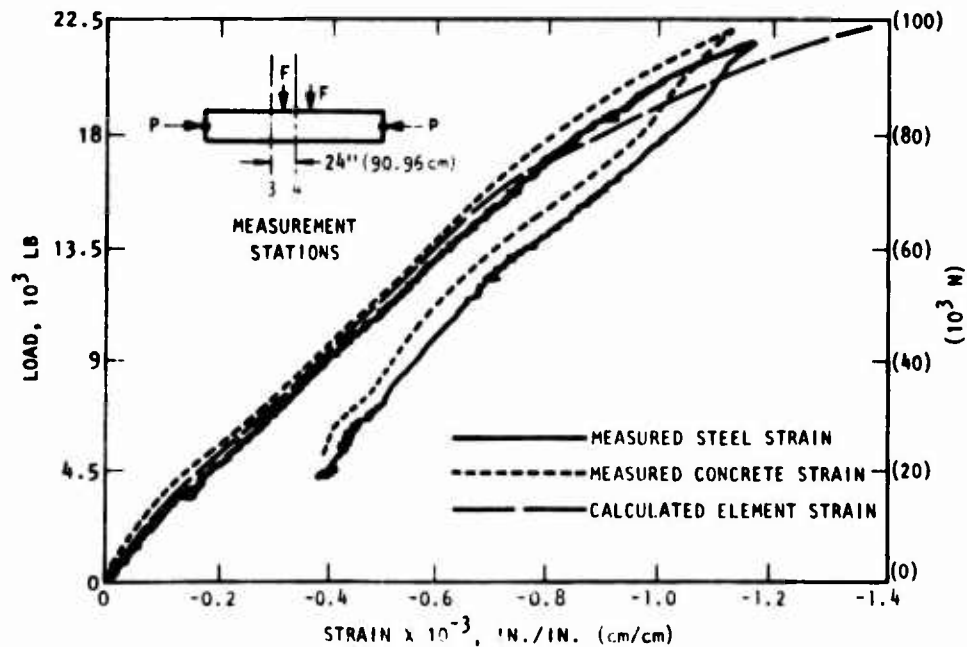


Figure 65. Measured and Calculated Vertical Load/Deflection Relationship at Middle of Beam Column 5-3-1, Case S-3 (Lane, Ref. 90)



(a) Strains calculated for Element 107 and strains measured in steel at bottom of the middle (Station 4) of the beam



(b) Strains calculated for Element 80 at top of beam (Station 3) and strains measured in both steel and concrete

Figure 66. Comparison of Measured and Calculated Strains, Beam 5-3-1, Case S-3 (Lane, Ref. 90)

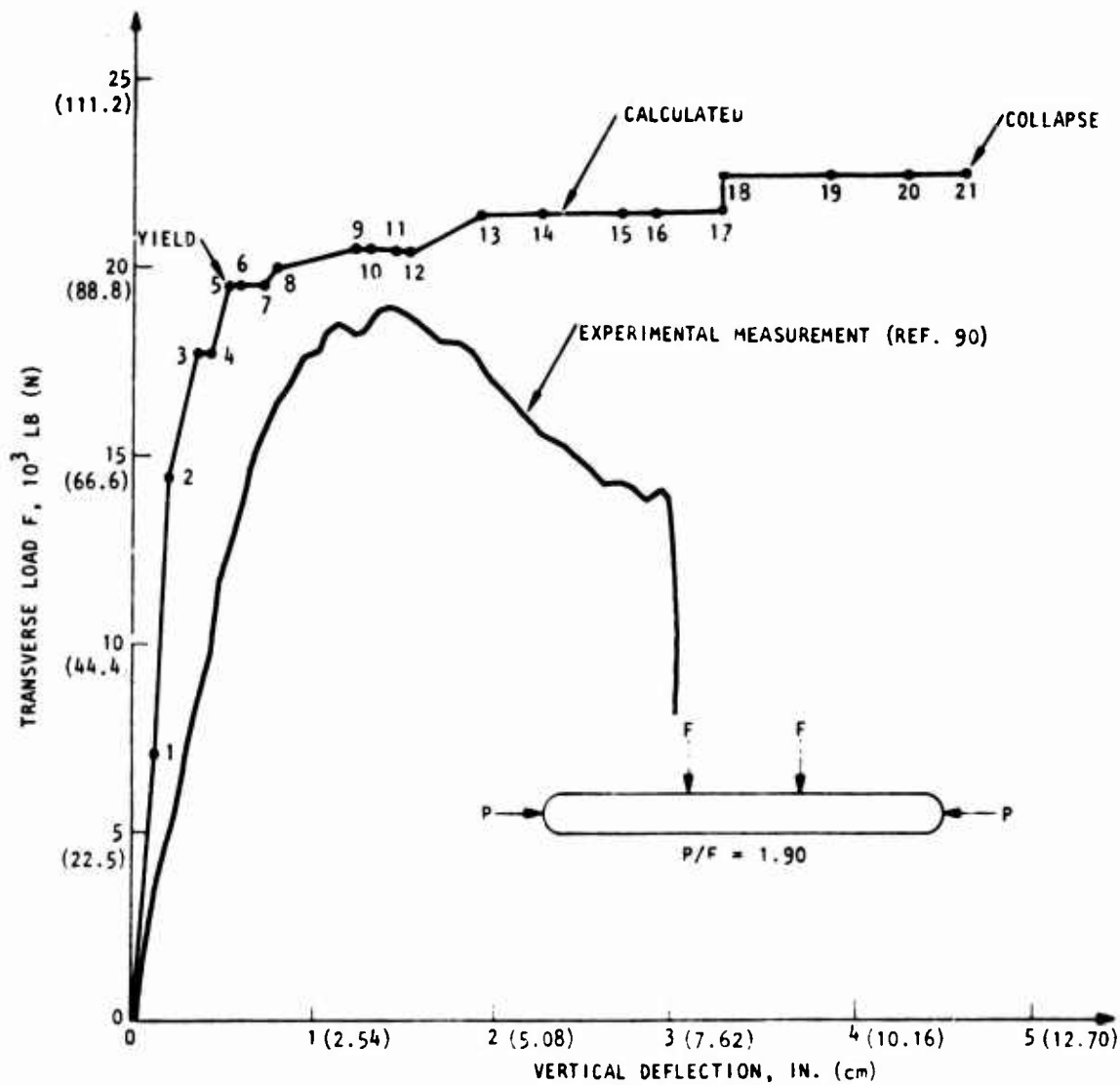
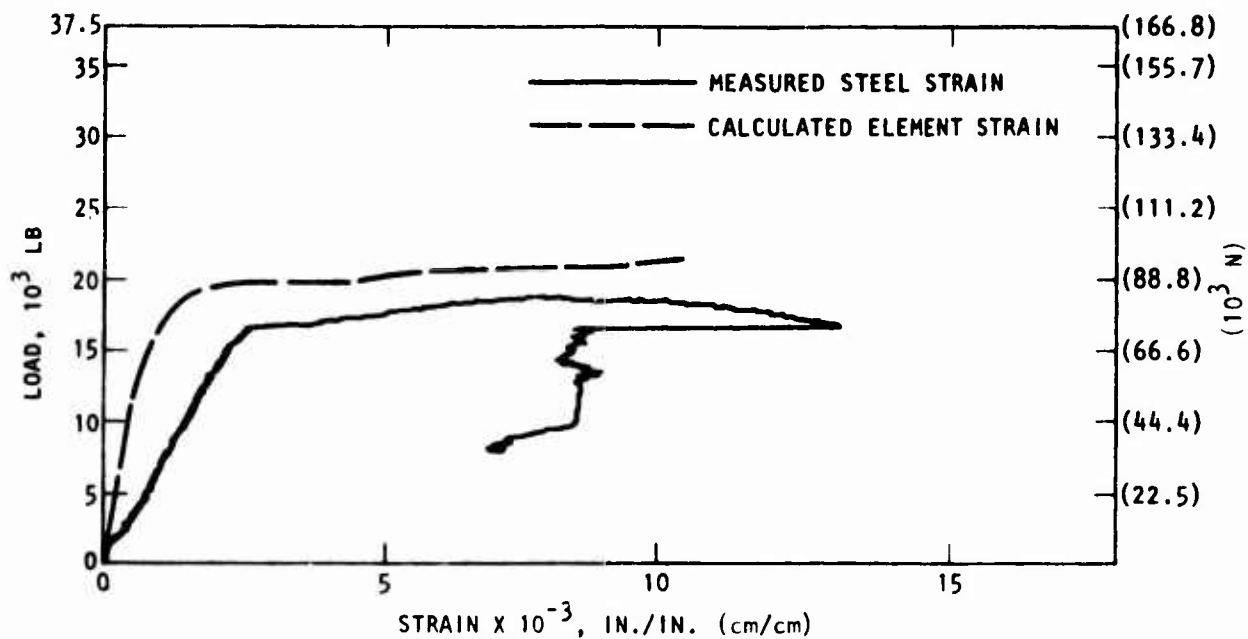
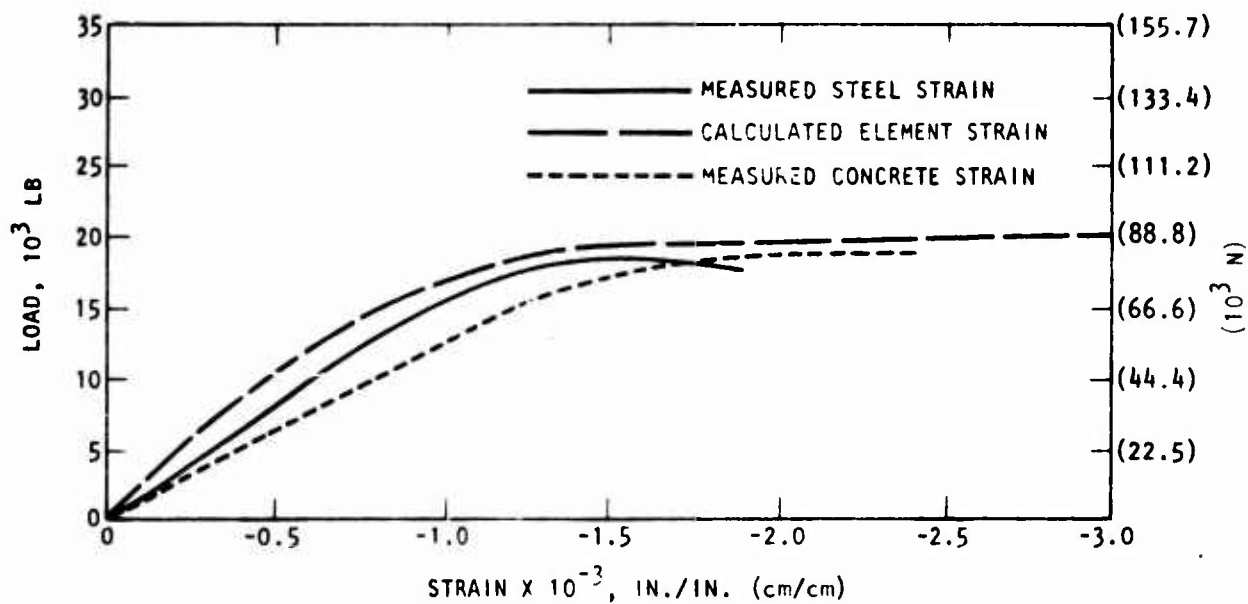


Figure 67. Measured and Calculated Vertical Load/Deflection Relationship for Beam Column 5-2-1, Case S-4 (Lane, Ref. 90)



(a) Strains calculated for Element 107 and strains measured in steel at bottom of middle of the beam (Station 4)



(b) Strains calculated in Element 113 at top of middle of the beam (Station 4)

Figure 68. Comparison of Measured and Calculated Strains, Beam 5-2-1, Case S-4 (Lane, Ref. 30)

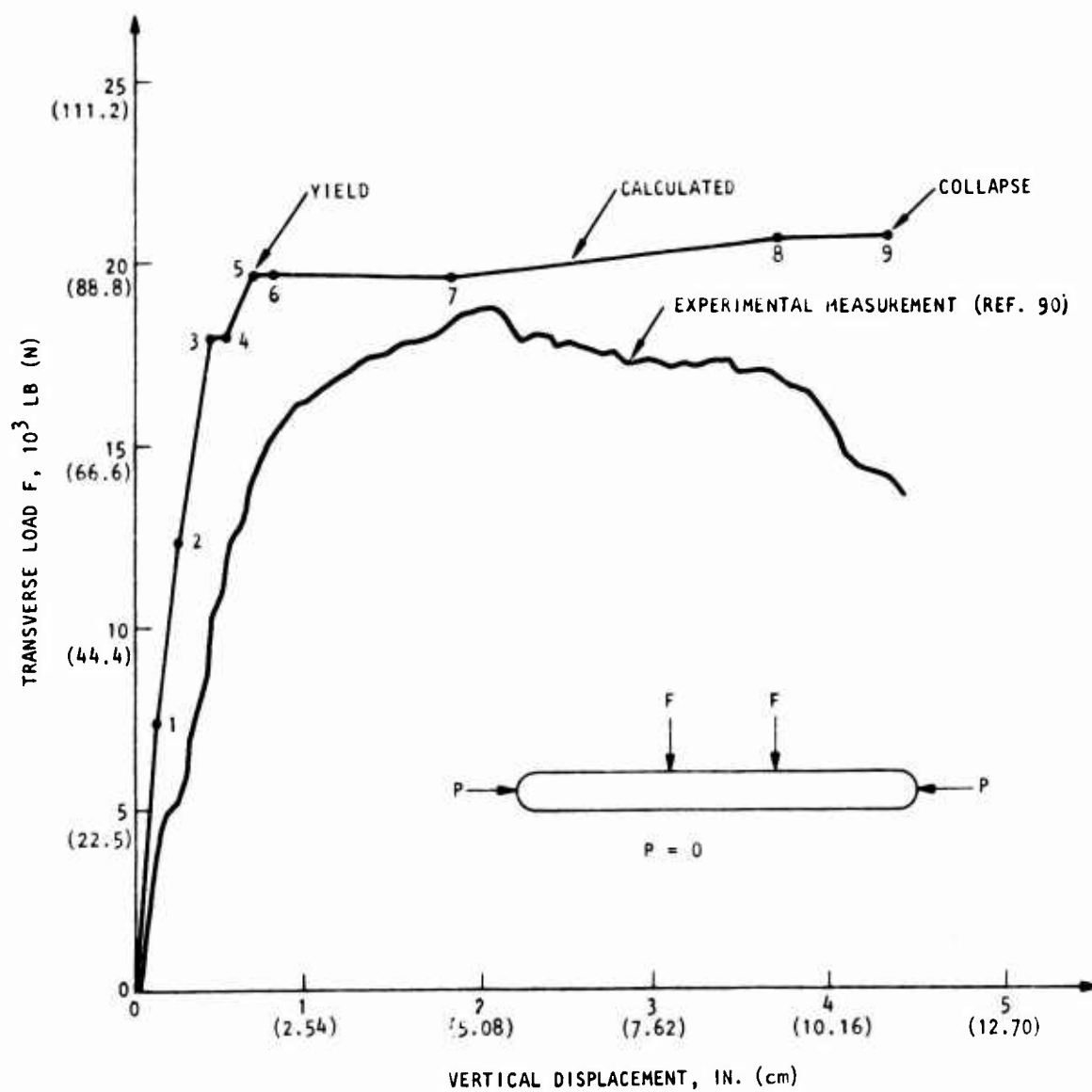


Figure 69. Measured and Calculated Vertical Load/Deflection Relationship at Middle of Beam Column 5-0-1, Case S-5 (Lane, Ref. 90).

the steel strain indicating that the analytical model overestimates the stiffness. The same trend was revealed in figures 70(a) and 70(b) for beam 5-0-1 of case S-5.

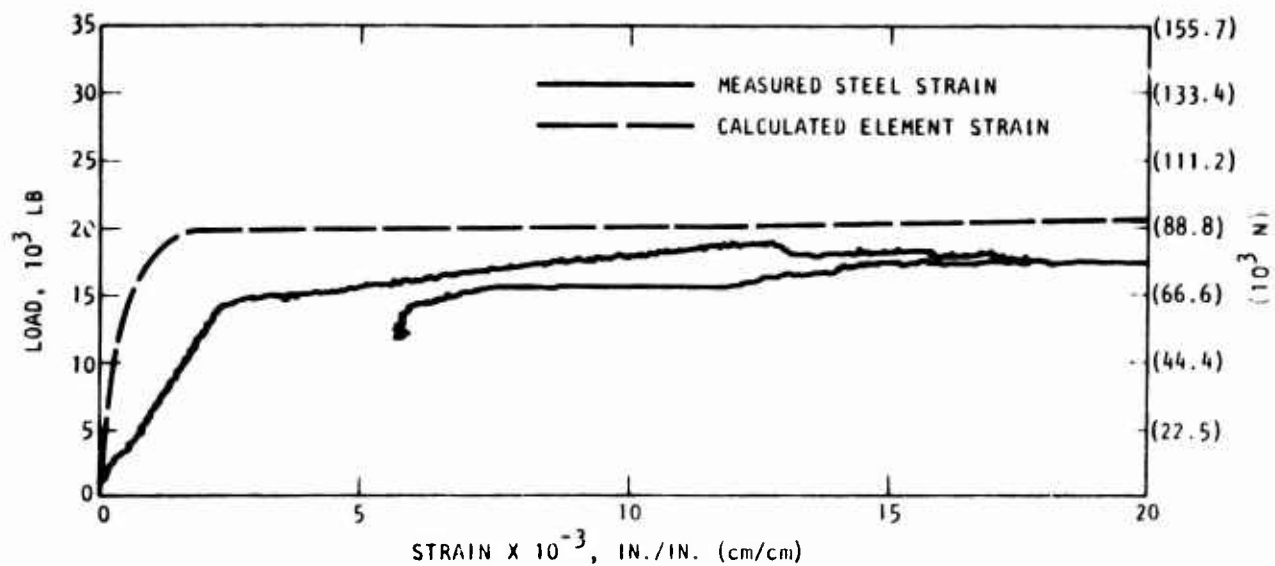
In each of the above discussed cases, the analysis slightly overestimated the collapse load. This appears to be partially due to applying too large a load increment.

5. CASE D-1 (DYNAMIC ANALYSIS OF BEAM 2D1.6-1 OF CRIST, REF. 89)

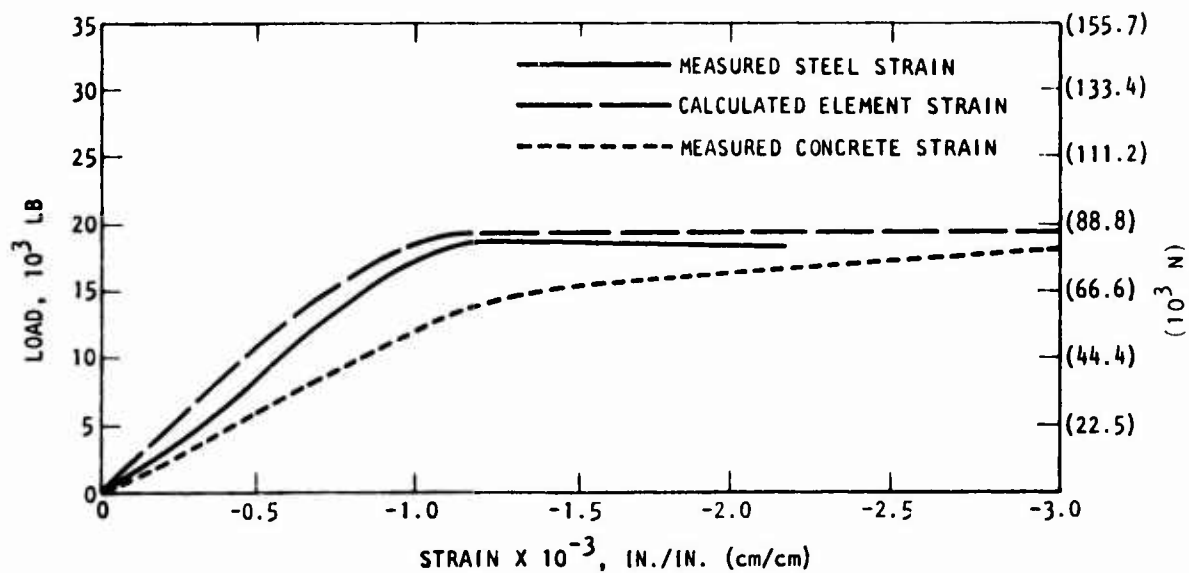
The dynamic analysis of beam 2D1.6-1 is performed using the finite element mesh shown in figure 56. Dynamic loading pulse L1, shown in figure 71, is applied to this model. There are two main differences between the dynamic analysis and the physical experiment. In the analysis the pressure load is assumed to be stationary, whereas in the physical experiment it is a traveling wave applied by a shock tube. This detail is omitted from the analysis for economy. The second difference is that the analytic model is assumed to rest on fixed supports, whereas the physical beam rests on flexible supports. This detail is omitted because it is irrelevant to the properties of the model being investigated but would affect comparisons.

The measured deflection/time history of the midpoint of the beam (Ref. 89) is compared with the computation in figure 72. The mathematical model underestimates the displacement of the physical beam, and has a fundamental period which is slightly longer. The time to the maximum measured centerline deflection of 0.32 in. (0.81 cm) was 8.5 msec. The maximum calculated centerline deflection of 0.19 in. (0.48 cm) occurred at 12 msec.

The above results are in general agreement with the previous static results which indicate that the analytical model is stiffer than the experimental model. However, since the test apparatus used (Ref. 89) allows the supports to move, which complicates the response of the test beam, the test results should be interpreted in the gross sense. The model beam undergoes a great deal of localized yielding, as is illustrated in figure 73, when measured and computed steel strains are plotted as a function of length along the beam. The comparison is made at two different times to allow for the possibility that the model and physical beams do not reach their maximum



(a) Strains calculated for Element 107 and strains measured in steel at bottom of the middle of the beam (Station 4)



(b) Strains calculated in Element 113 at top of middle of the beam (Station 4)

Figure 70. Comparison of Measured and Calculated Strains, Beam 5-0-1, Case S-5 (Lane, Ref. 90)

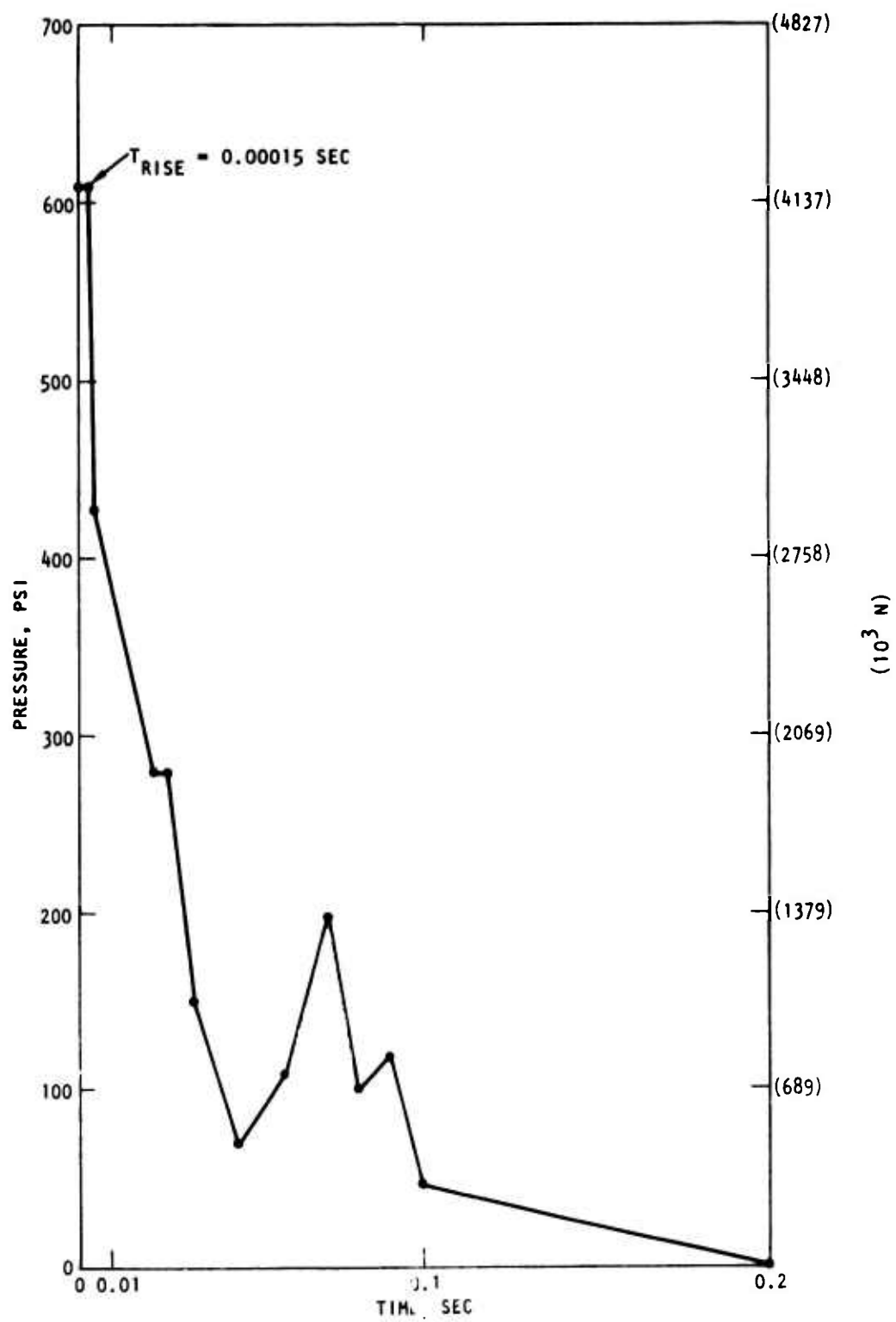


Figure 71. Pressure Loading for Beam 2D1.6-1, Case D-1, Loading L1 (Crist, Ref. 89)

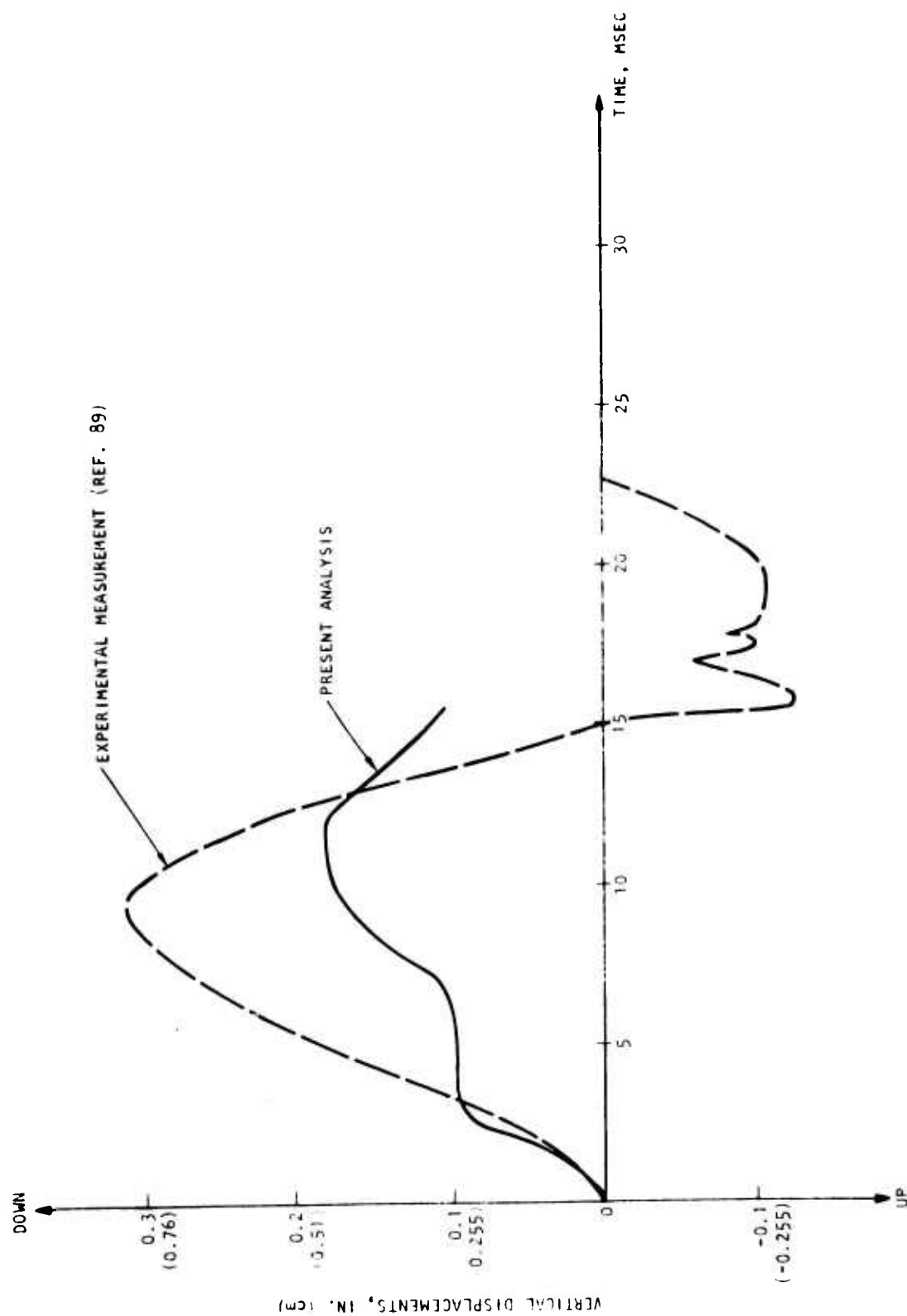
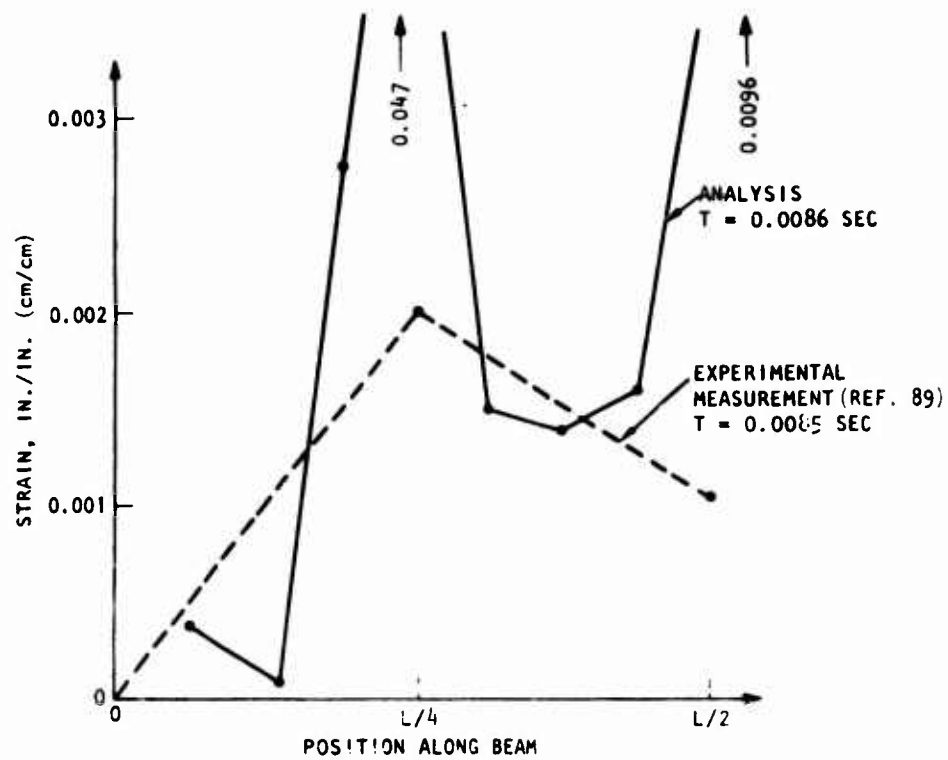
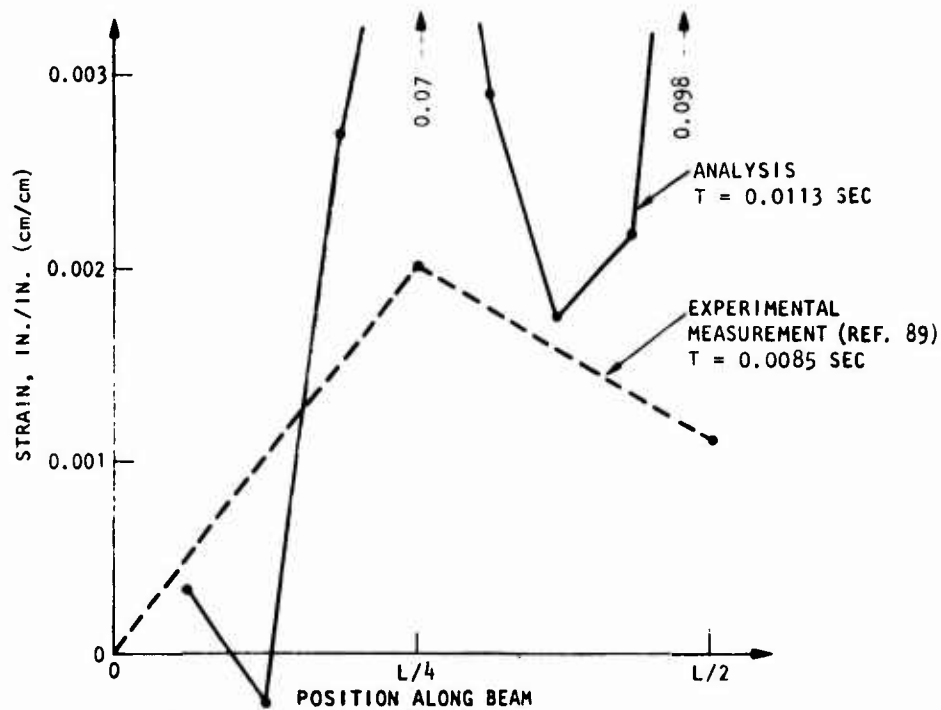


Figure 72. Comparison of Measured and Calculated Vertical Displacements at Middle of Beam 2D1.6-1, Case D-1



(a) $T = 0.0085$ sec for both physical and model beam



(b) $T = 0.0085$ sec for physical beam and
 $T = 0.01131$ sec for model beam

Figure 73. Comparison of Strains in Tension Steel in Analysis and Physical Experiment, Beam 2D1.6-1, Case D-1

responses at the same time. The comparison is favorable except that the analysis greatly exaggerates the strain concentration at several points. Figure 73(a) indicates that the strains calculated at 0.0086 sec correlate well with the measured strains in the middle of the range between $L/4$ and $L/2$ of the beam length. For the range of length between 0 and $L/4$, the calculated strains appear to fluctuate above and below the measured strains for approximately one-third of the length of the beam. After that point the calculated strains exceed the measured strains by a very large margin. The discrepancy between calculated and measured strains for the first one-third of the length of the beam may be due to the difference in end-support conditions. While the analytical model assumes rigid end supports, the experimental model rests on flexible supports.

The comparison of calculated strains at a later time of 0.0113 sec, with measured strains for $t = 0.0085$ sec, illustrated in figure 73(b), indicate a trend similar to that observed in figure 73(a). The strains obtained from the analysis are the average strains of the cracked and uncracked regions of the element. Since the strains in the cracked regions increase drastically with the increase in response, the average value of strain is expected to jump to a very high value also. Crist (Ref. 89) indicated that the experimental data were not complete because of gage damage, and there appeared to be radical deviation of steel strain in some cases. However, Crist indicated that this can be attributed to the proximity of the strain gages to the cracks. Therefore, conclusions should not be based on comparison of results at middle points or quarter points of the beam where both analytical and experimental results appear to be least reliable.

Comparison is made in figure 74 of measured and observed crack patterns. Although the calculations stop at about $T = 0.017$ sec, the computed cracks do not extend after about $T = 0.012$ sec. Hence, comparison with the final observed crack pattern is valid. The absence of cracks from elements 28, 29, 38, 45, 46, 47, 53, 54, 63, 70, and 71 indicates that the analytical model is stiffer than the experimental model. The overcracking in the compression zone that is observed in figure 74 requires additional investigation and has been discussed at the end of this section.

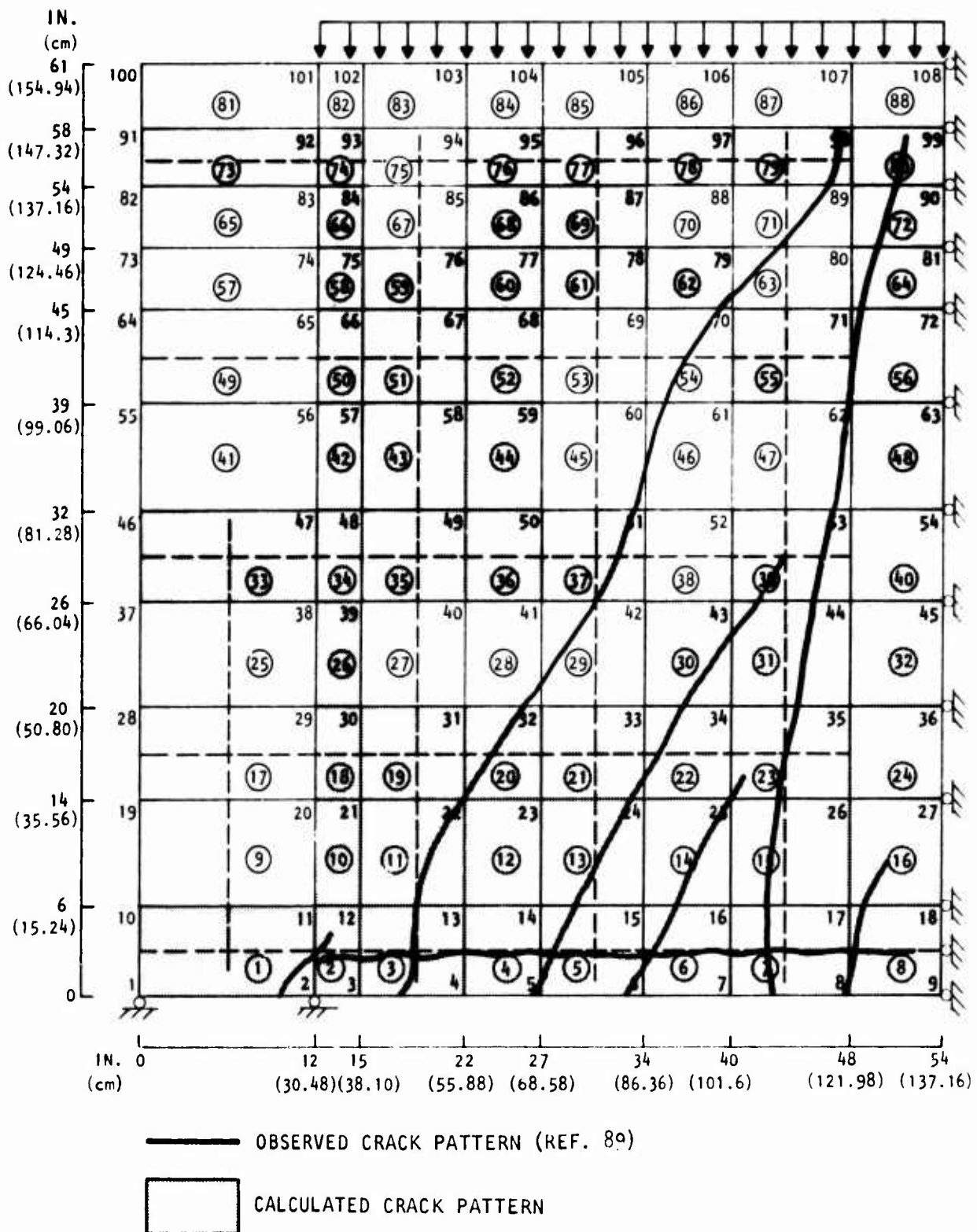


Figure 74. Comparison of Calculated and Observed Crack Patterns for Beam 2D1.6-1, Case D-1

6. CASE D-2 (DYNAMIC ANALYSIS OF BEAM COLUMN 5-3-1 OF LANE, REF. 90)

The dynamic analysis of a beam column was performed using the finite element mesh shown in figure 64. Dynamic loading, shown in figure 75, was applied simultaneously as axial and transverse forces. There is no physical experiment corresponding to this analysis.

The results of the analysis are presented in terms of velocity/ and displacement/time histories and in terms of stresses in the compression zone and along the tension steel. Figures 76 and 77 show the midspan displacement and velocity up to the time of collapse and numerical instability at $t = 0.025$ sec. Figure 78 shows the extreme fiber stress in the compression zone at midspan (Element 113). In the present case, onset of instability is signaled by a reduction in the compressive stress in Element 113, even though vertical load and rotation are increasing. Figure 79 shows the extreme fiber stress in the tension zone at midspan (Element 106). Cracking occurs at $t = 0.007$ sec. There is no longitudinal steel in this element, and hence the longitudinal stress drops abruptly to zero. Figure 80 shows longitudinal stress, normalized to stress at the yield point of steel in several elements containing tension steel. Near the center of the span, represented by Elements 90 and 107, the stress rises to the yield point. In other locations, represented by Element 58, the stress rises to a maximum value which is considerably less than the yield point and then fluctuates as a hinge forms in the center span of beam and redistribution takes place.

7. CASE D-3 (DYNAMIC ANALYSIS OF A REINFORCED CONCRETE RING)

The dynamic analysis of a reinforced concrete ring subjected to a traveling pressure pulse was performed using the finite element mesh shown in figure 81. The analysis simulates the behavior of an underground reinforced concrete silo when subjected to a pressure pulse traveling in the surrounding soil or rock.

Only half of the ring is modeled because of symmetry. The concrete is reinforced only in the circumferential direction, with 1 percent steel in the inner layer of elements and 0.37 percent in the outer layer of elements.

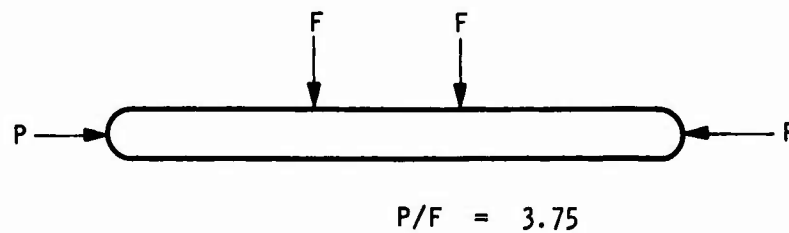
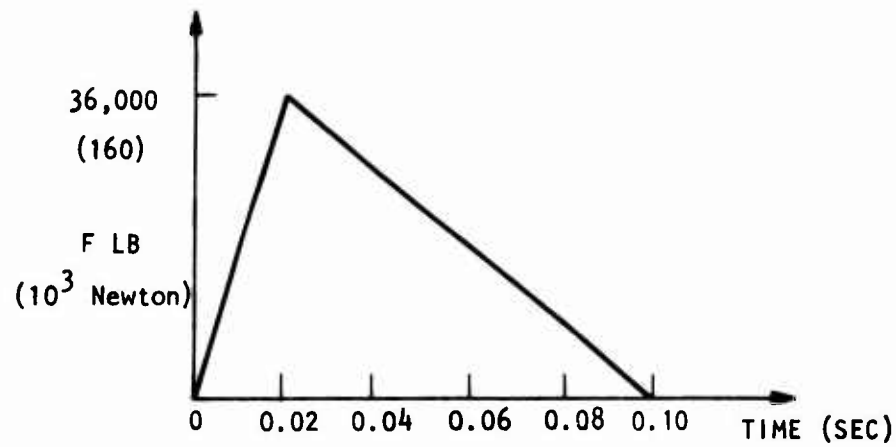
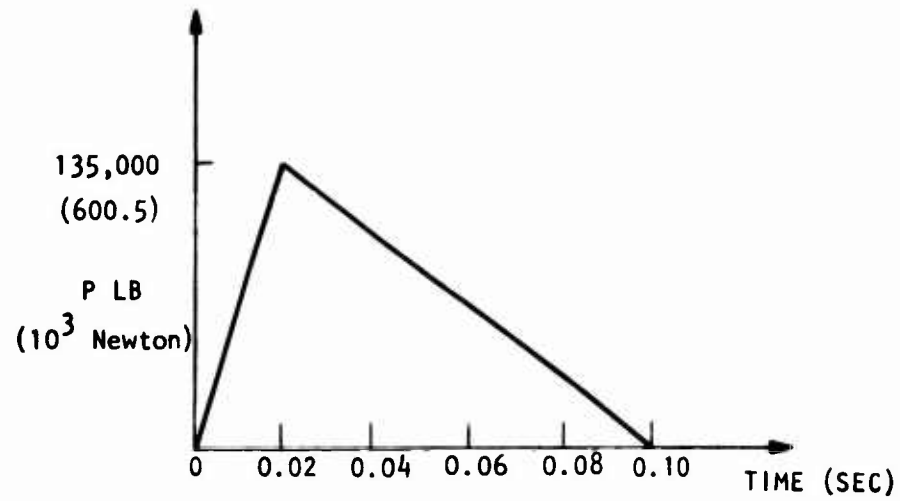


Figure 75. Axial and Transverse Loading for Beam Column 5-3-1, Case D-2, Having $P/F = 3.75$ (Lane, Ref. 90)

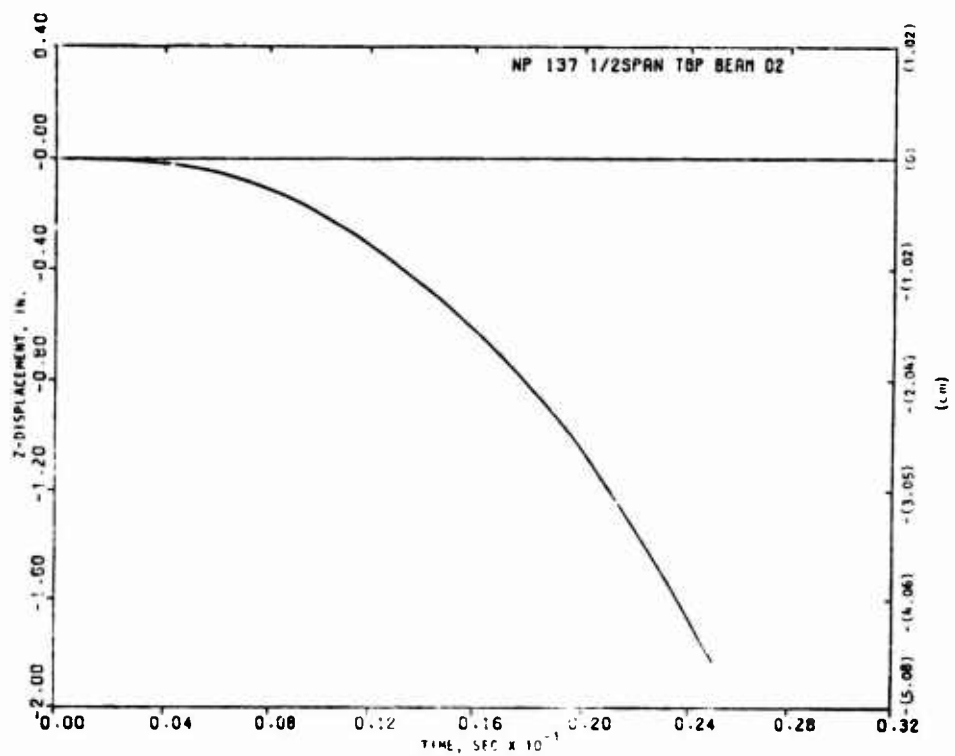


Figure 76. Calculated Midspan Displacement, Case D-2

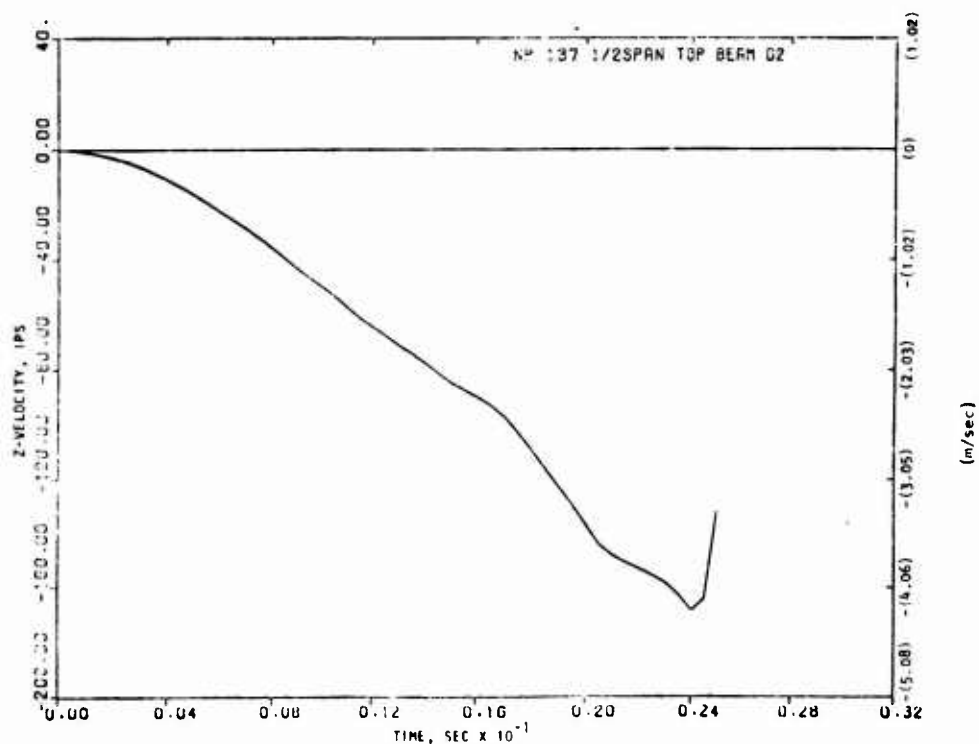


Figure 77. Calculated Midspan Velocity, Case D-2

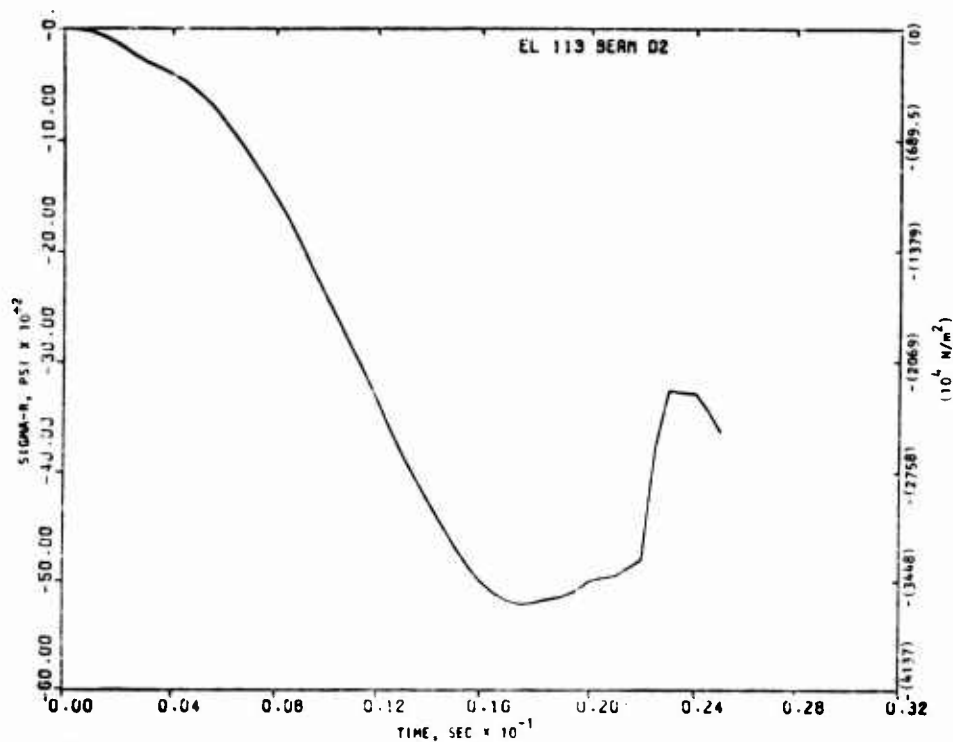


Figure 78. Extreme Fiber Stress in the Compression Zone at Midspan, Element 113, Case D-2

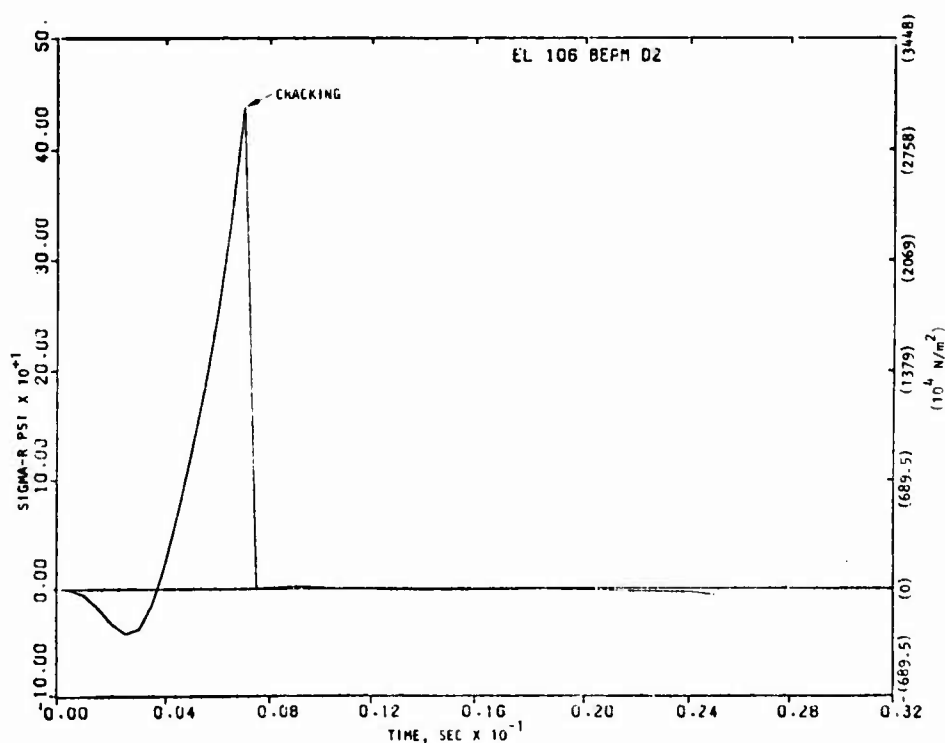


Figure 79. Extreme Fiber Stress in Tension Zone at Midspan, Element 106, Case D-2

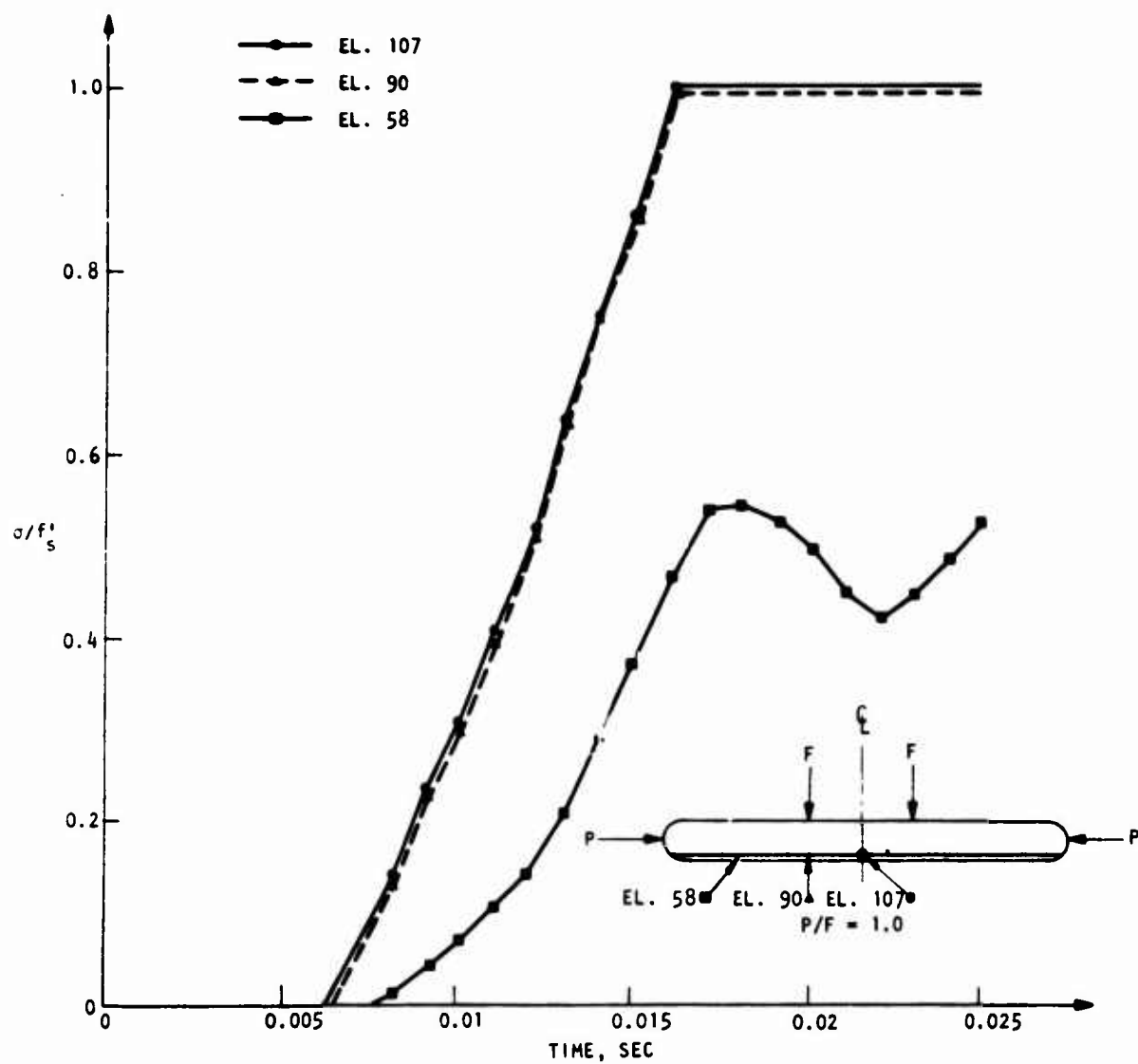


Figure 80. Steel Stress, Expressed as Fraction of Yield Stress, Versus Time Calculated for Beam Column, Case D-2

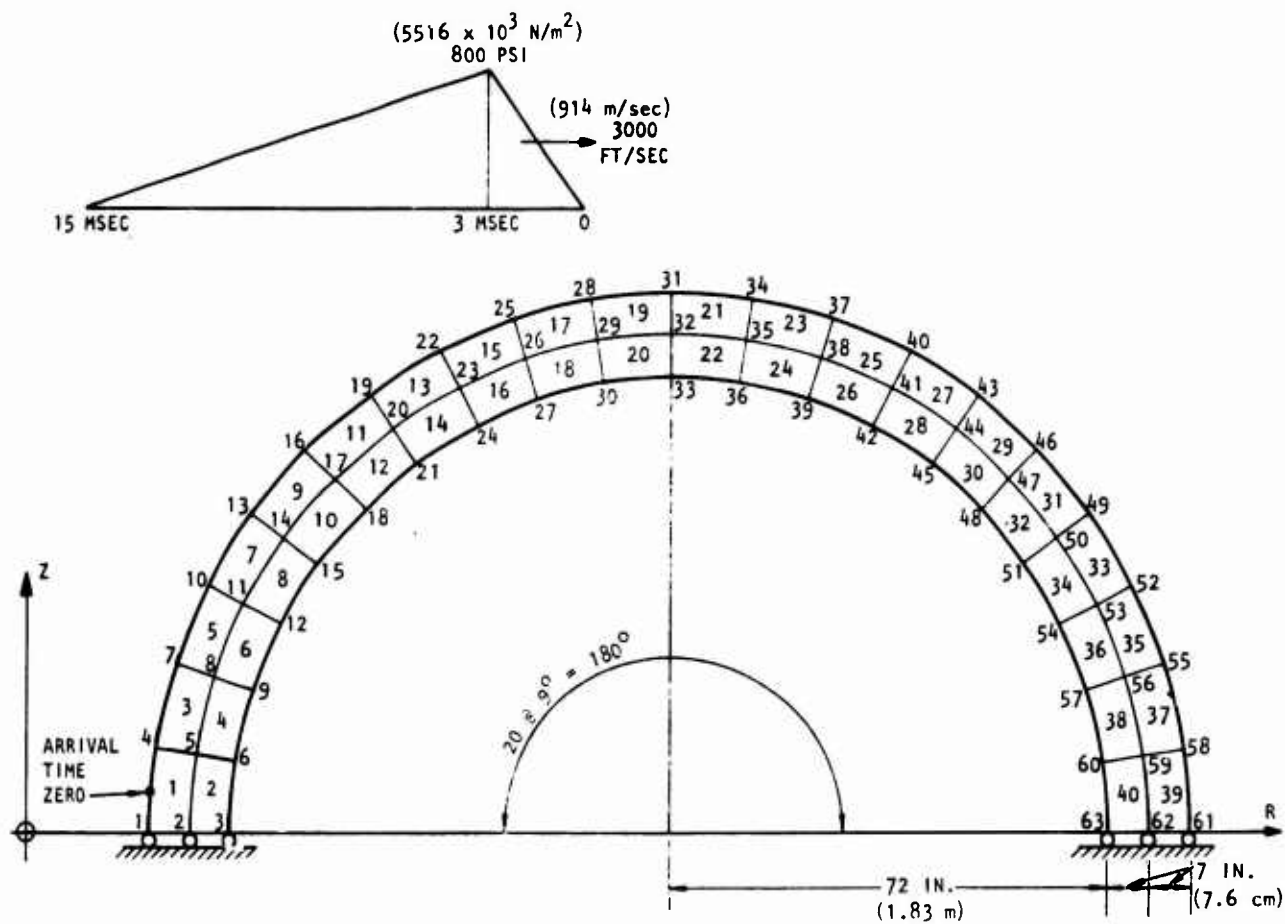


Figure 81. Finite Element Mesh for a Reinforced Concrete Ring and the Associated Traveling Pressure Pulse, Case D-3

The pressure pulse, which is triangular in shape as shown in figure 81, is assumed to travel at a constant velocity of 3000 ft/sec (914 m/sec) in the R-direction. The rise time is 0.003 sec with a peak pressure of 800 psi ($5516 \times 10^3 \text{ N/m}^2$). The pressure is applied normal to the outer surface of the ring. The arrival time for the midpoint of side 1-4 of Element 1 (Fig. 81) is taken as zero, and the arrival times for other loaded sides are determined on the basis of the 3000-ft/sec (914 m/sec) traveling speed and the differences between the R-coordinates of the midpoints of the loaded sides and that of the reference point with zero arrival time. Figure 82 shows the loading conditions at t sec.

A total of 240 integration time steps of 0.00005 sec each were taken for the inelastic analysis, resulting in 0.012 sec of response history. Figures 83 through 86 show the time histories of stress components (effectively, hoop stress) in four selected elements. There is, however, no experimental data for comparison.

The curves presented show that Element 1 cracks in tension at about 0.0108 sec while Element 40 cracks at about 0.0044 sec. In both cases, the stress level drops instantaneously to that corresponding to the 50,000 psi ($345 \times 10^6 \text{ N/m}^2$) yield point assumed for the reinforcing steel. This is because the steel percentages assigned to the outer and the inner element layers are both so low that the transfer of stress from concrete to steel accompanying the concrete cracking throws large enough load on the reinforcing steel to cause it to yield instantaneously. Notice that in the case of Element 40, the yielding of steel in tension is followed by a period of unloading and reloading of the element. The stiffness assigned to the steel during this period corresponds to the elastic modulus, while that assigned to the cracked concrete is taken as 5 percent of its initial tangent modulus--a more-or-less arbitrarily selected value.

Other elements which develop tension cracks and yielding of steel during the response are indicated in figure 87 together with the times at which the cracks first develop. The element which develops the largest compressive stress is Element 39. The time history of the stress σ_z in this element, which is almost directed in the circumferential direction, is given in figure 85.

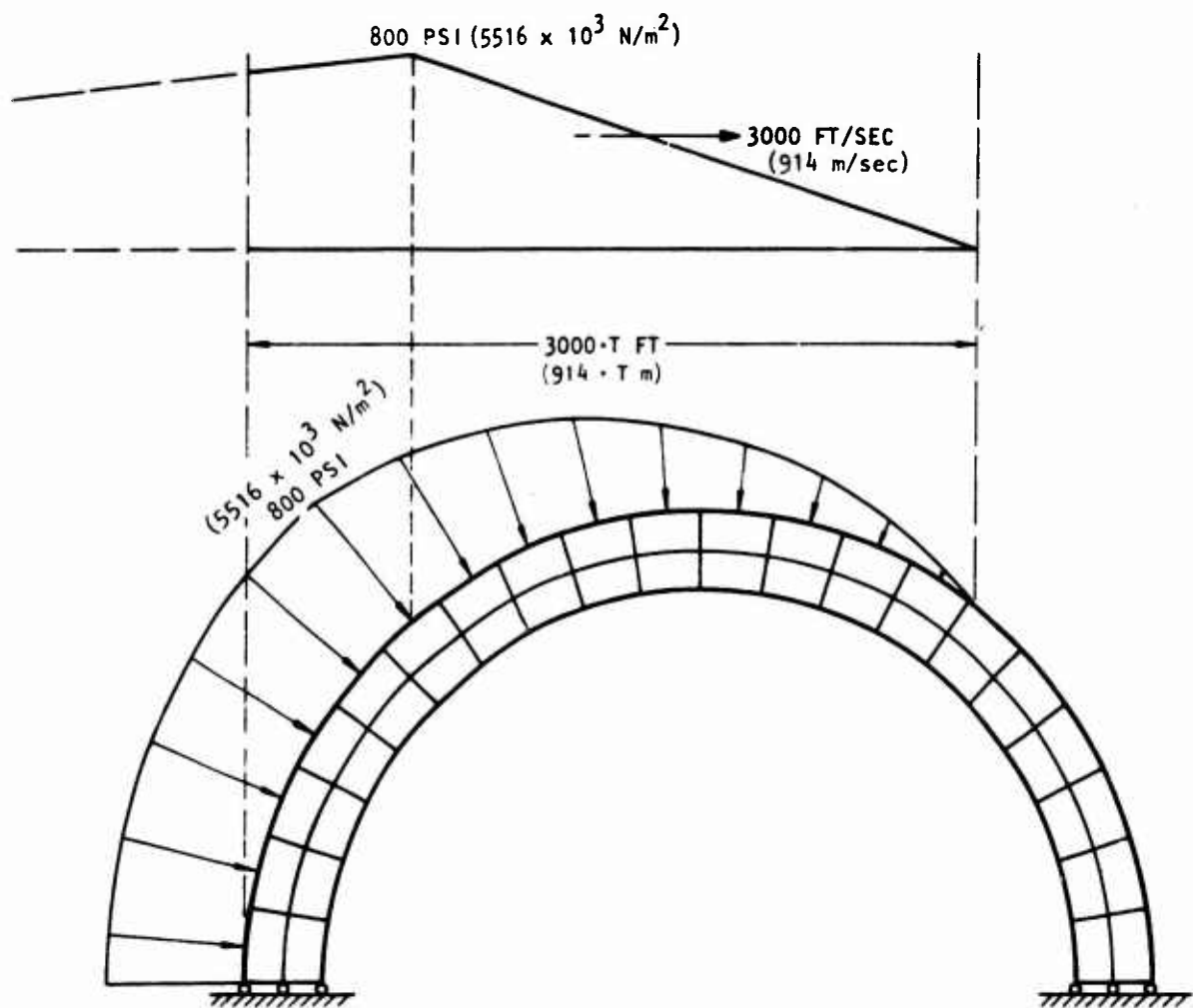


Figure 82. Loading Condition on the Ring at T Seconds, Case D-3

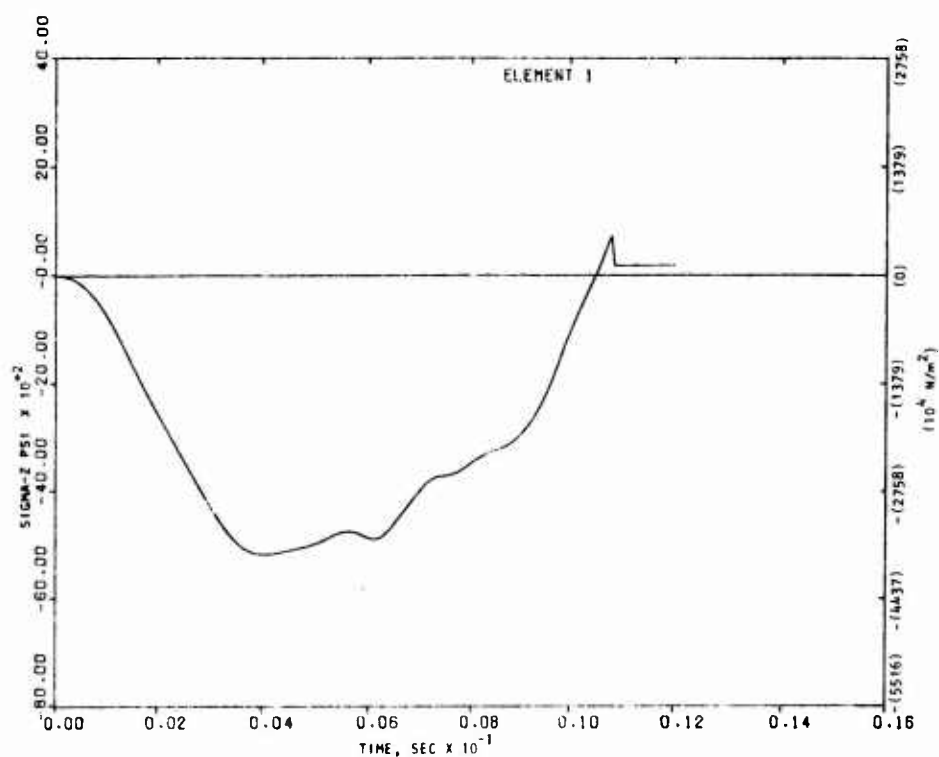


Figure 83. σ_z in Element 1 due to Traveling Pressure Pulse, Case D-3

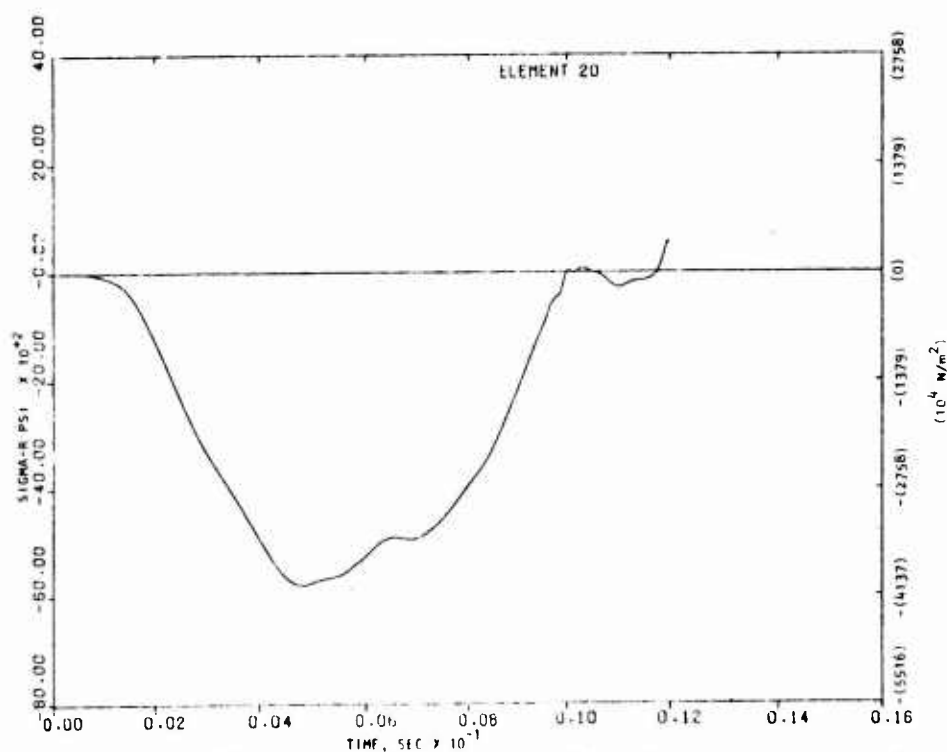


Figure 84. σ_R in Element 20 Due to Traveling Pressure Pulse, Case D-3

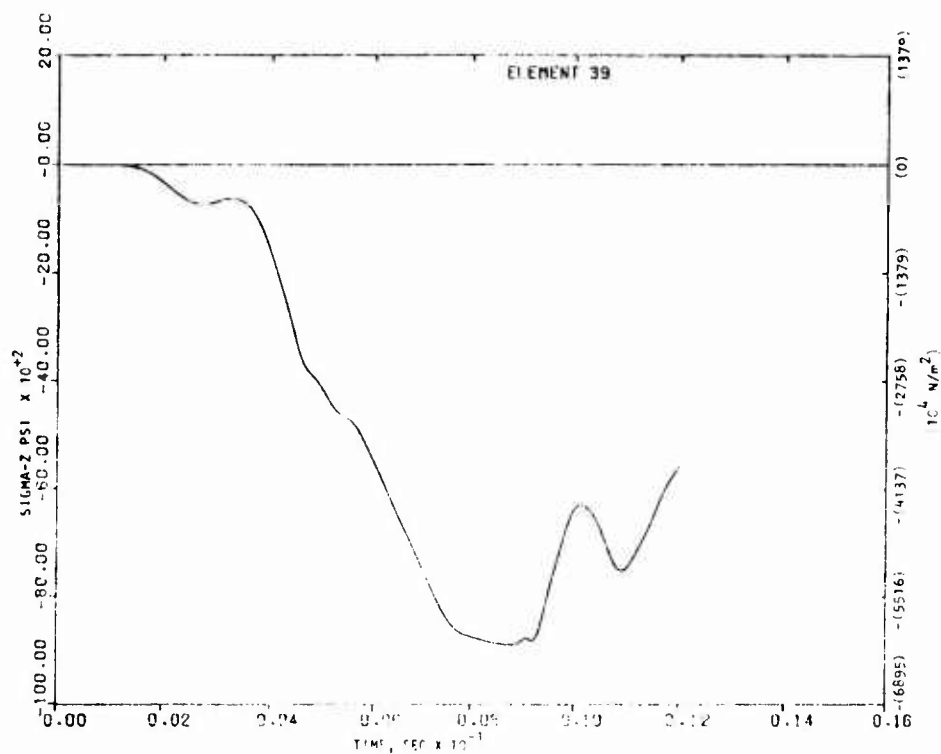


Figure 85. σ_z in Element 39 Due to Traveling Pressure Pulse, Case D-3

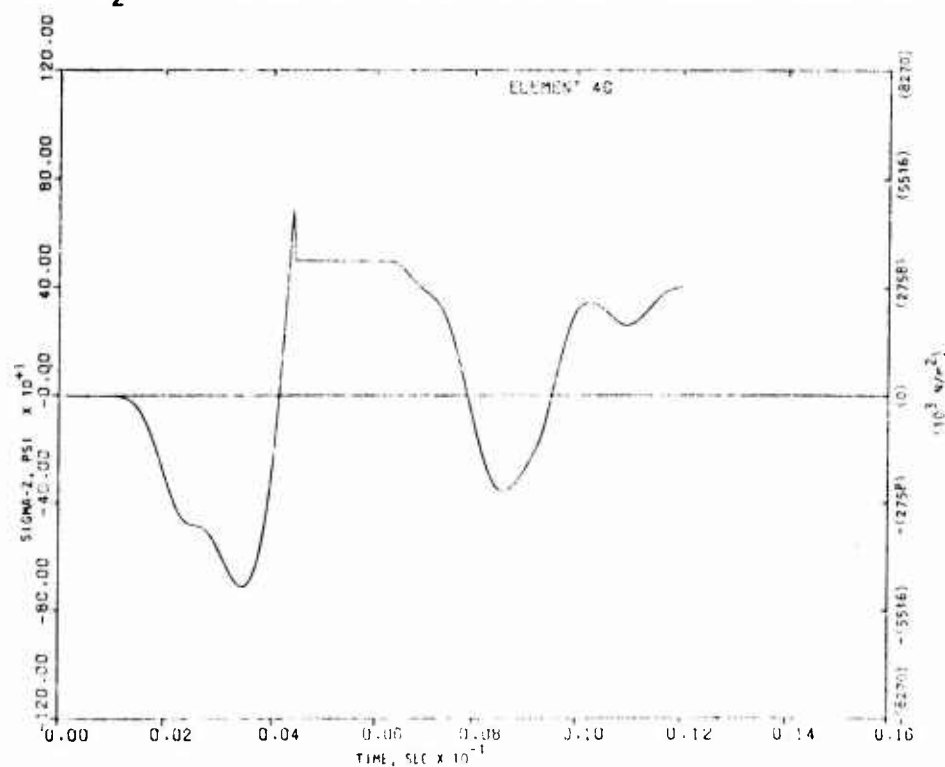


Figure 86. σ_z in Element 40 Due to Traveling Pressure Pulse, Case D-3

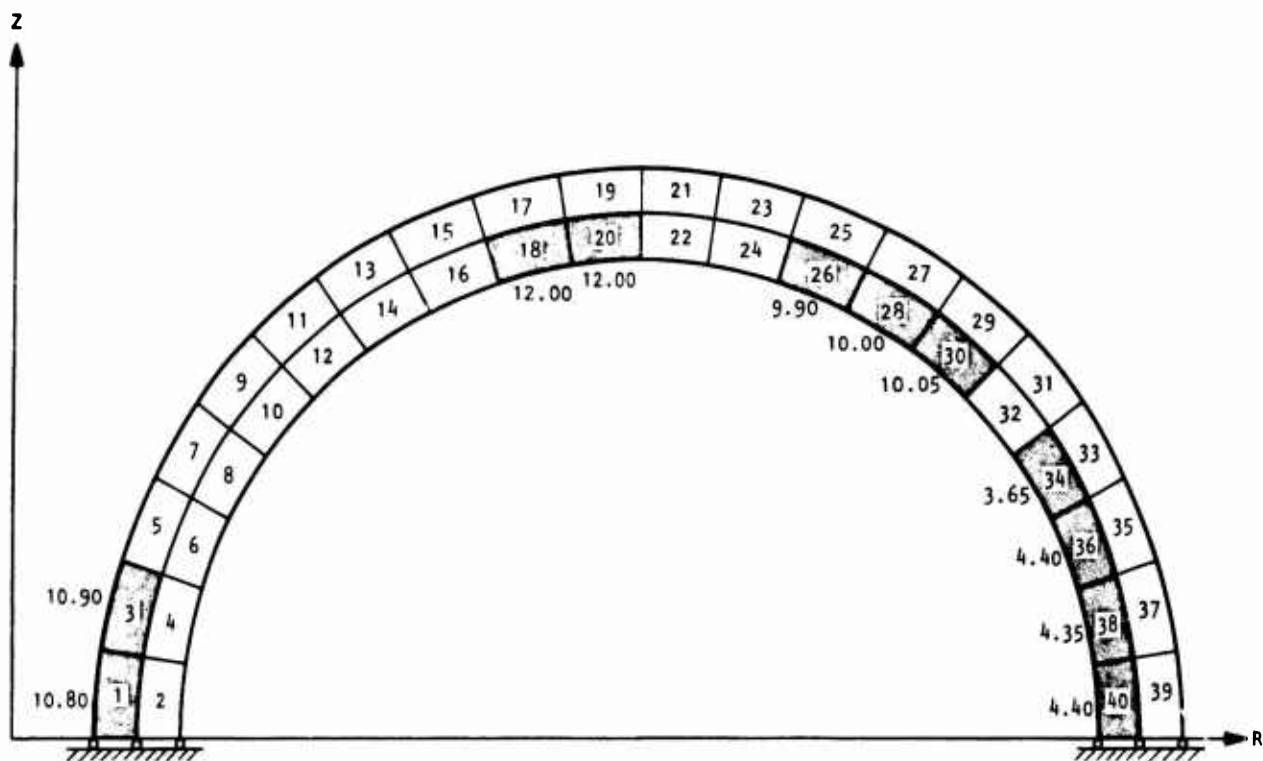


Figure 87. Elements Developing Tension Cracks and the Times in Milliseconds at Which the Cracks Develop, Case D-3

Figure 88 shows the deformed shape of the central axis of the ring at various times during the dynamic response. For convenience, the deformed shapes have been drawn with Nodal Point 62 considered stationary. It must be emphasized, however, that since the displacements are magnified 40 times in this figure, the deformed shapes are considerably distorted. Care must therefore be exercised in interpreting this figure. As an example, the figure suggests that some tension should develop in Element 33. The reason that no tension is indicated in the output may be due to the fact that Element 34 cracks in one direction very early in the history at 3.65 msec. Then at 5.05 msec, it cracks in a second direction perpendicular to the first. With Element 34 rendered almost useless structurally, only Element 33 remains in the locale to carry the load. Since a stress component calculated for an element is the average stress across that element, a single element is inadequate for determining bending stresses or the variation of stresses across an element. The absence of tensile stress in the output for Element 33, therefore, seems reasonable.

8. CASE D-4 (DYNAMIC ANALYSIS OF REINFORCED CONCRETE BEAM 3b2 OF FELDMAN, REF. 91)

a. Introduction

The dynamic behavior of a double-reinforced concrete beam under dynamic loads is investigated in this sample problem. The beam has a span length of 9 ft (2.7 m) and a cross section of 12 in. by 6 in. (30.4 x 15.2 cm), as shown in figure 89. The beam has two No. 9 bars as tension reinforcement, two No. 7 bars as compression reinforcement and No. 2 stirrups spaced at a constant interval of 4-1/4 in. (10.78 cm). The effective depth is 9.925 in. (50.5 cm). The ultimate strength of concrete (f'_c) as reported in reference 91 is 3.26 ksi (22×10^6 N/m²), the initial tangent modulus (E_c) is 3.74 ksi (26×10^6 N/m²), and the reinforcing steel has an average yield point of 45.8 ksi (316×10^6 N/m²).

The beam is simply supported and is subjected to two concentrated loads symmetrically placed at a distance of 18 in. (45.7 cm) from the midspan. The loads are derived from explosive sources, and each of them has a time variation as shown in figure 90. Since the load-time curve is very nearly triangular, it is idealized into what is shown in figure 90 for the actual numerical

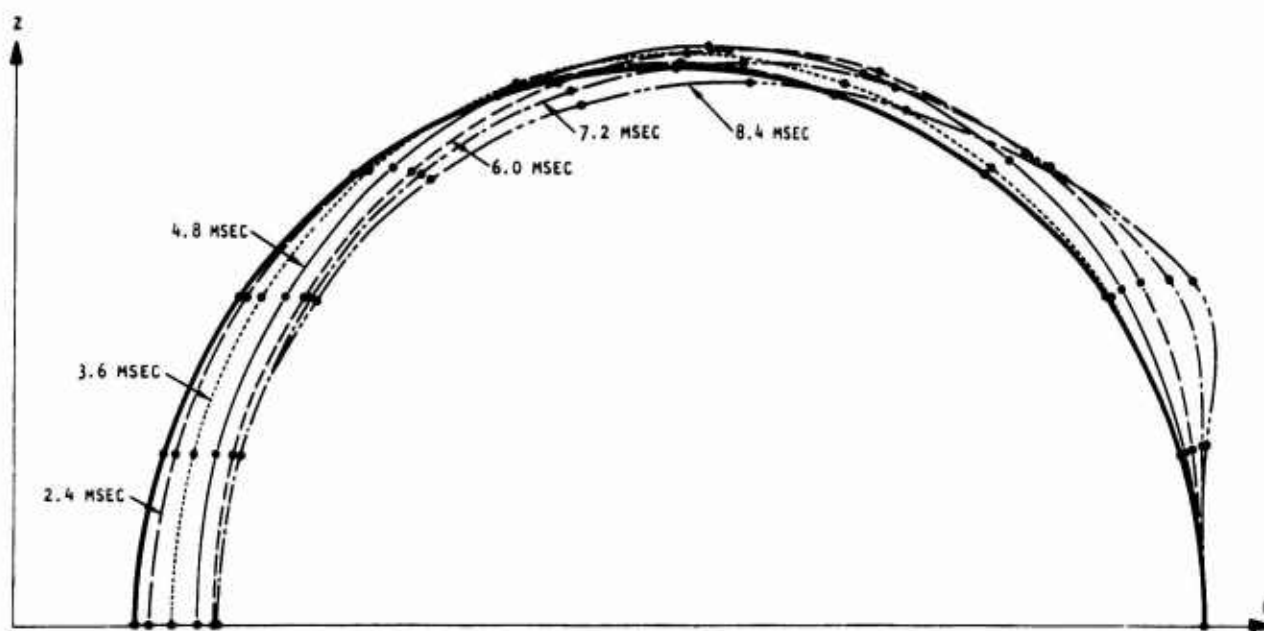


Figure 88. Deformed Shape of Ring at Various Times in Milliseconds (displacements magnified 40 times), Case D-3

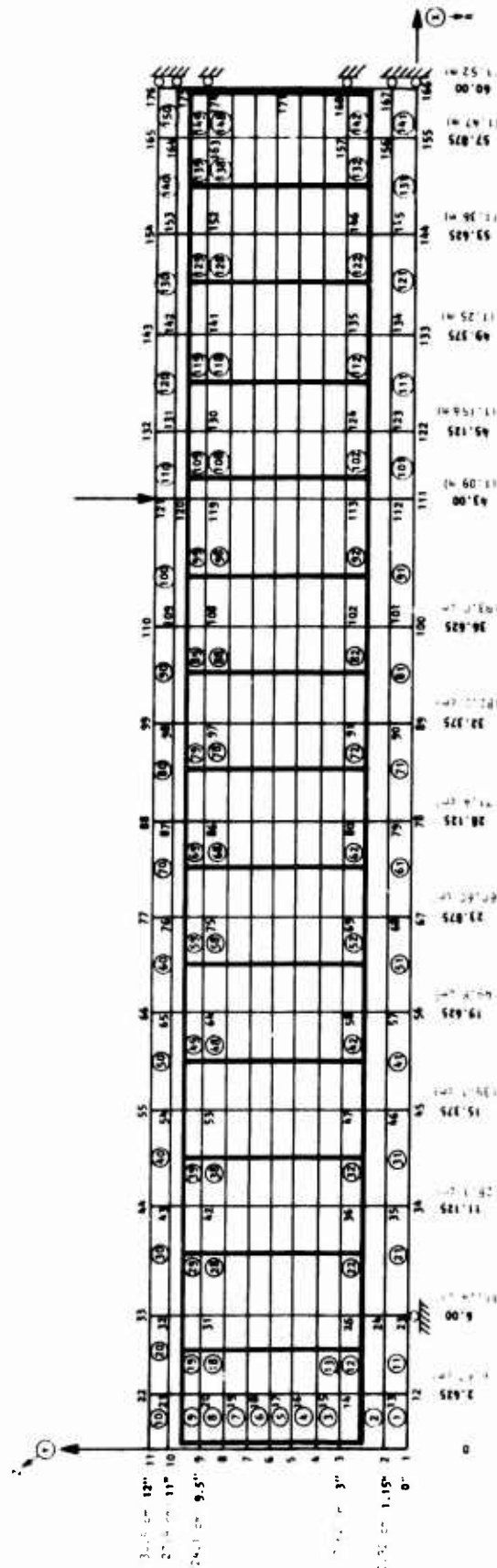
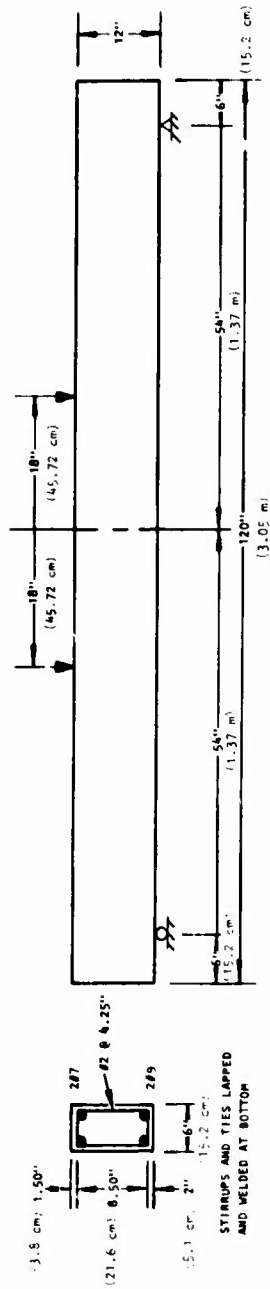


Figure 89. Finite Element Representation of Beam 3b2, Dynamic Case D-4

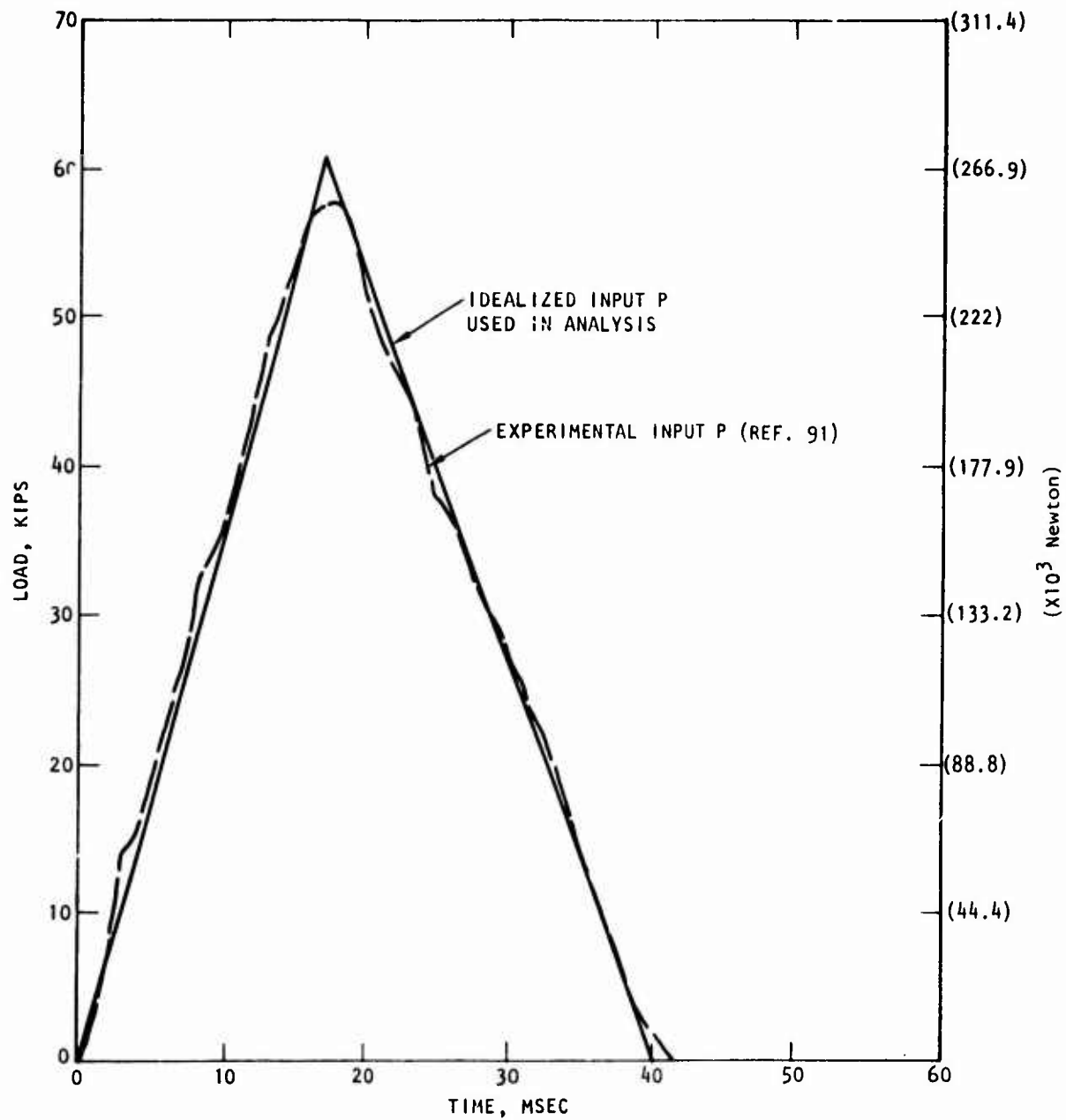


Figure 90. Load-Time History, Flow 1, Beam 3b2, Case D-4

calculation. In the actual test the beam is subjected to two additional force pulses (blows). However, only the first pulse is considered in the following analysis.

The finite element mesh used in the analysis is shown in figure 89. It is a 10 by 15 rectangular mesh with 176 nodal points and 150 elements. The right boundary corresponding to the midspan section is rollered to reflect the beam symmetry. Any element crossed by steel--whether tension steel, compression steel or a stirrup--is considered reinforced, with the steel percentage calculated as the steel area divided by the cross-sectional area of the element perpendicular to the steel reinforcement.

A total of 50 one-msec steps are taken, resulting in 50 msec of response history. The FEDRC code was used for the dynamic analysis, and the results are discussed in the following section.

b. Case D-4: Original Results and Modifications Required to Improve These Results

The dynamic analysis of Case D-4 was performed first using the original version of FEDRC code. Selected portions of the analysis are shown in figures 91(a), 92(a), 93(a), and 94. Figure 91(a) reveals that the beam started to crack at 4 msec. However, cracks were initiated in the second row in a heavily reinforced group of elements rather than at the lower tension fibers of the beam. An examination of the cracking criteria in the original version of the code indicated that the cracking of an element was established on the basis of overall element stress. Since the heavily reinforced elements have more stiffness, they developed higher stresses and therefore cracked first. To avoid this condition the cracking criteria were modified in such a manner that the onset of cracking would be decided by the concrete stress rather than the average element stress. The result of this modification is illustrated in figure 91(b). A similar comparison, at 5 msec, is shown in figures 91(c) and 91(d).

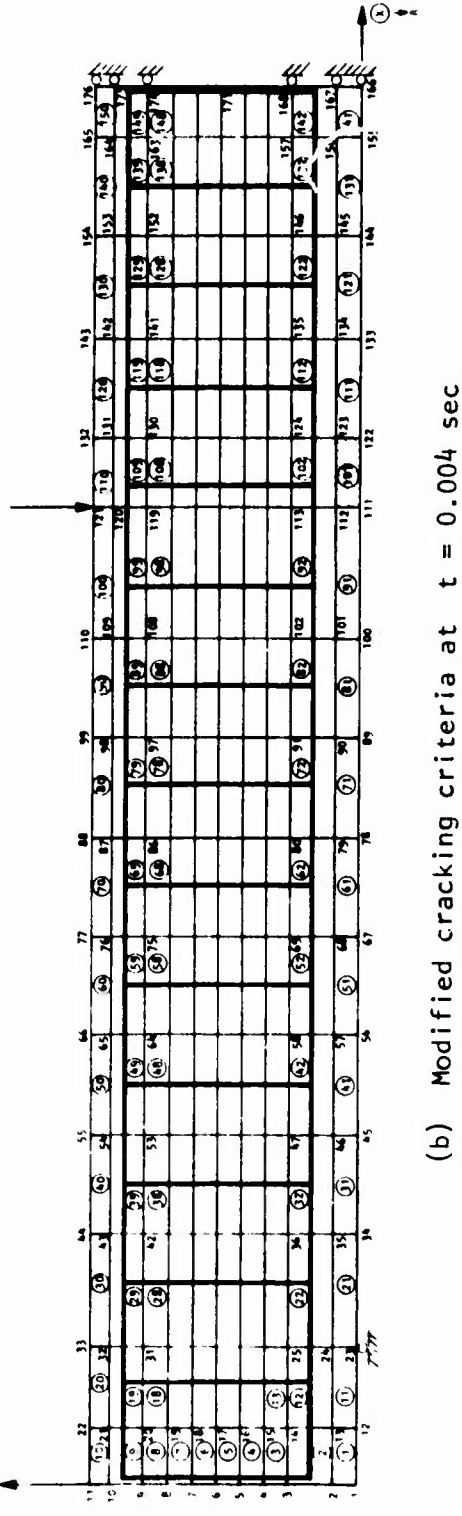
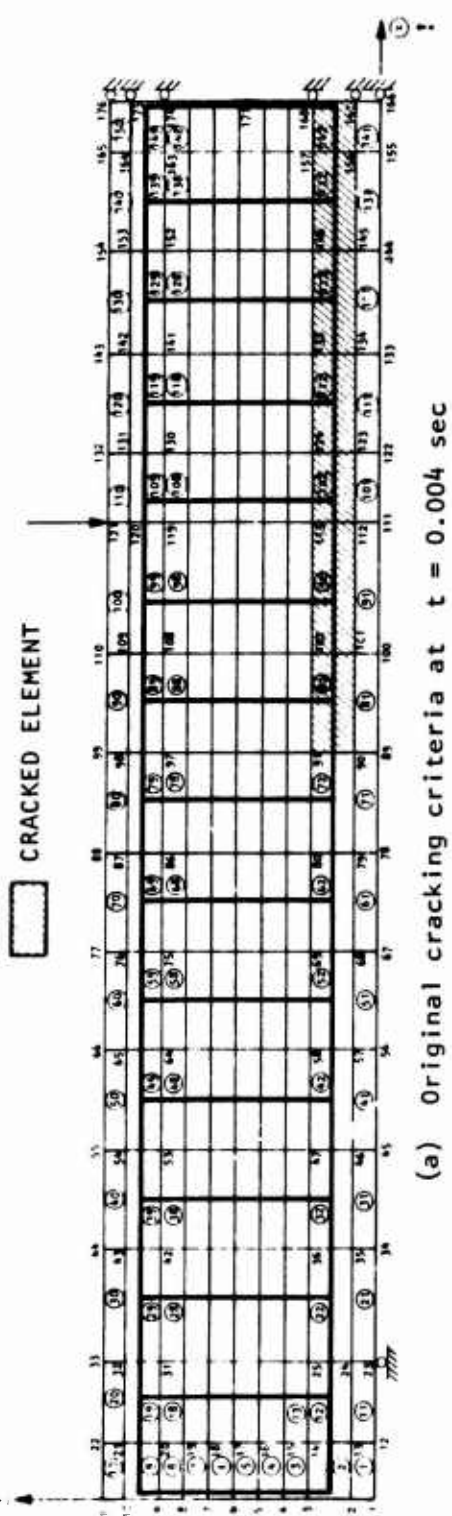
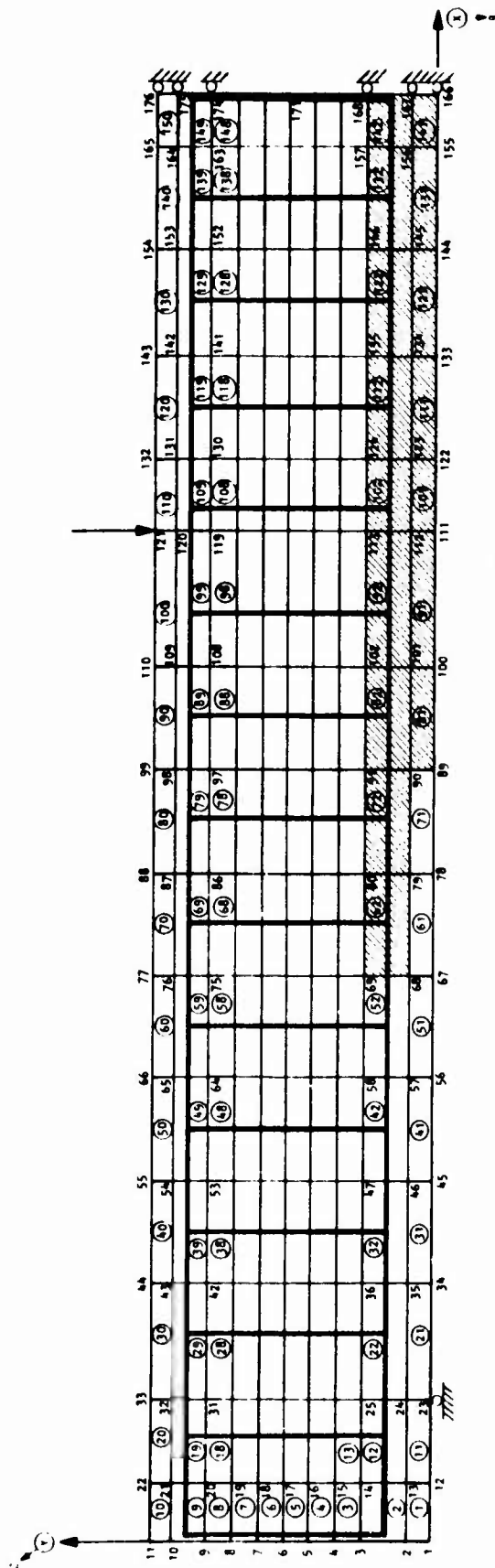
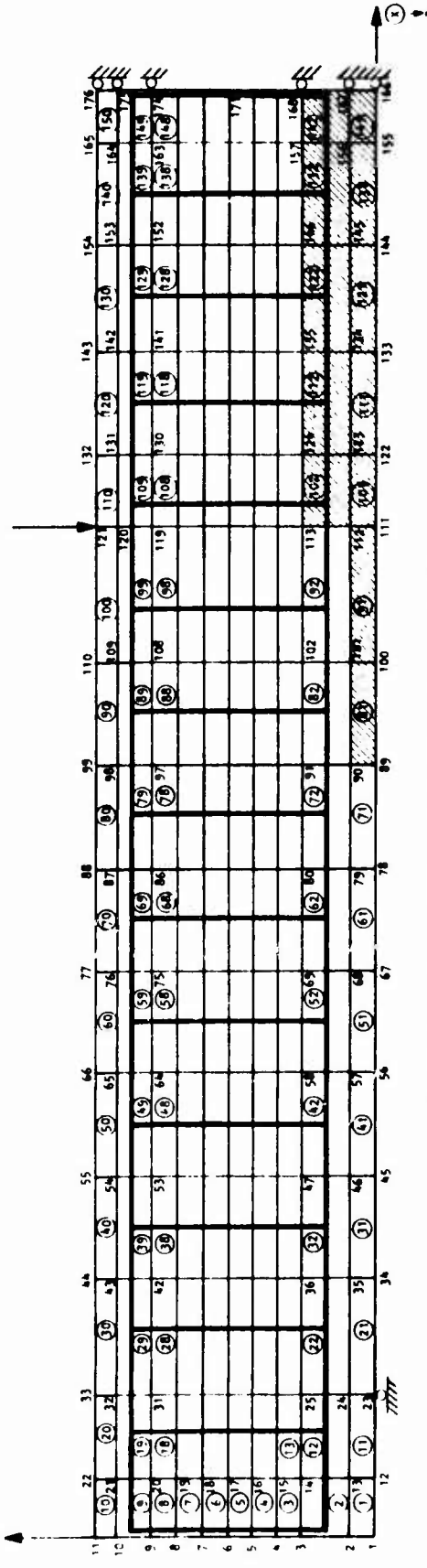


Figure 91. Comparison of Initial Crack Patterns Obtained from Original and Modified Cracking Criteria, Dynamic Case D-4



(c) Original cracking criteria at $t = 0.005$ sec



(d) Modified cracking criteria at $t = 0.005$ sec

Figure 91. Comparison of Initial Crack Patterns Obtained from Original and Modified Cracking Criteria, Dynamic Case D-4 (concluded)

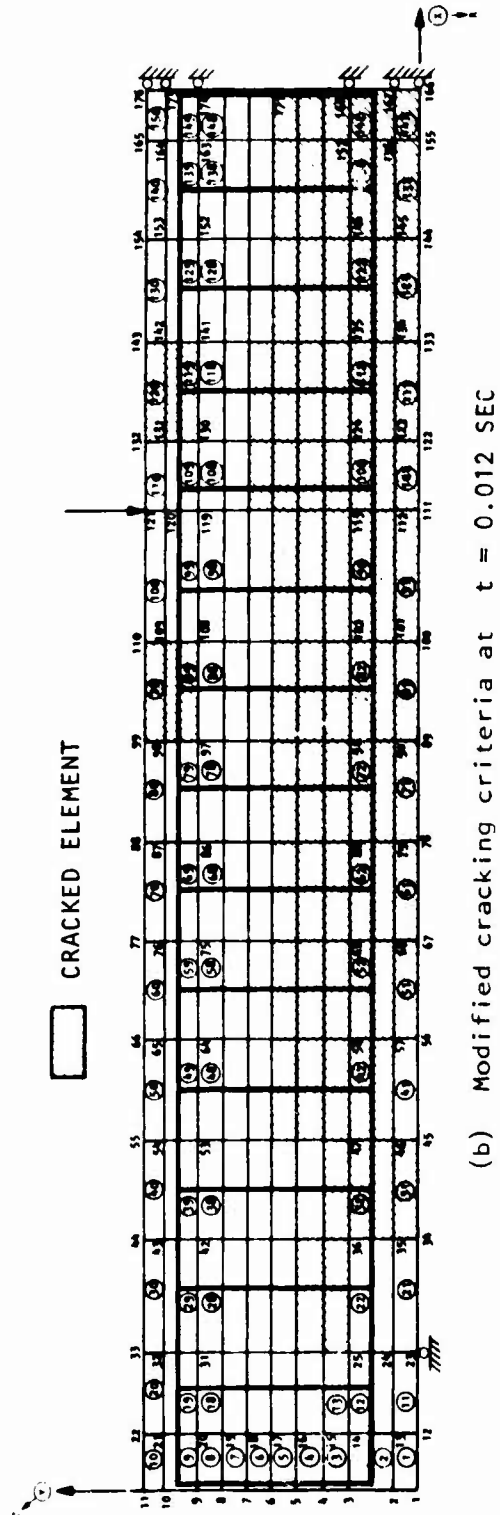
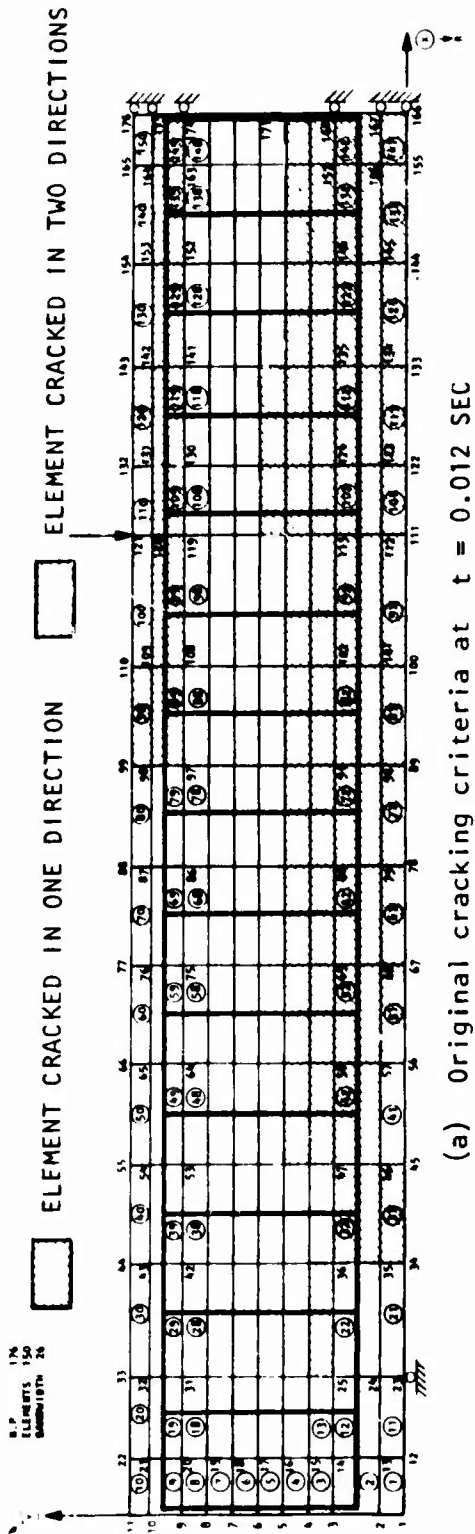


Figure 92. Comparison of Propagation of Cracks in Compression Zone Obtained by Original and Modified Cracking Criteria, Dynamic Case D-4

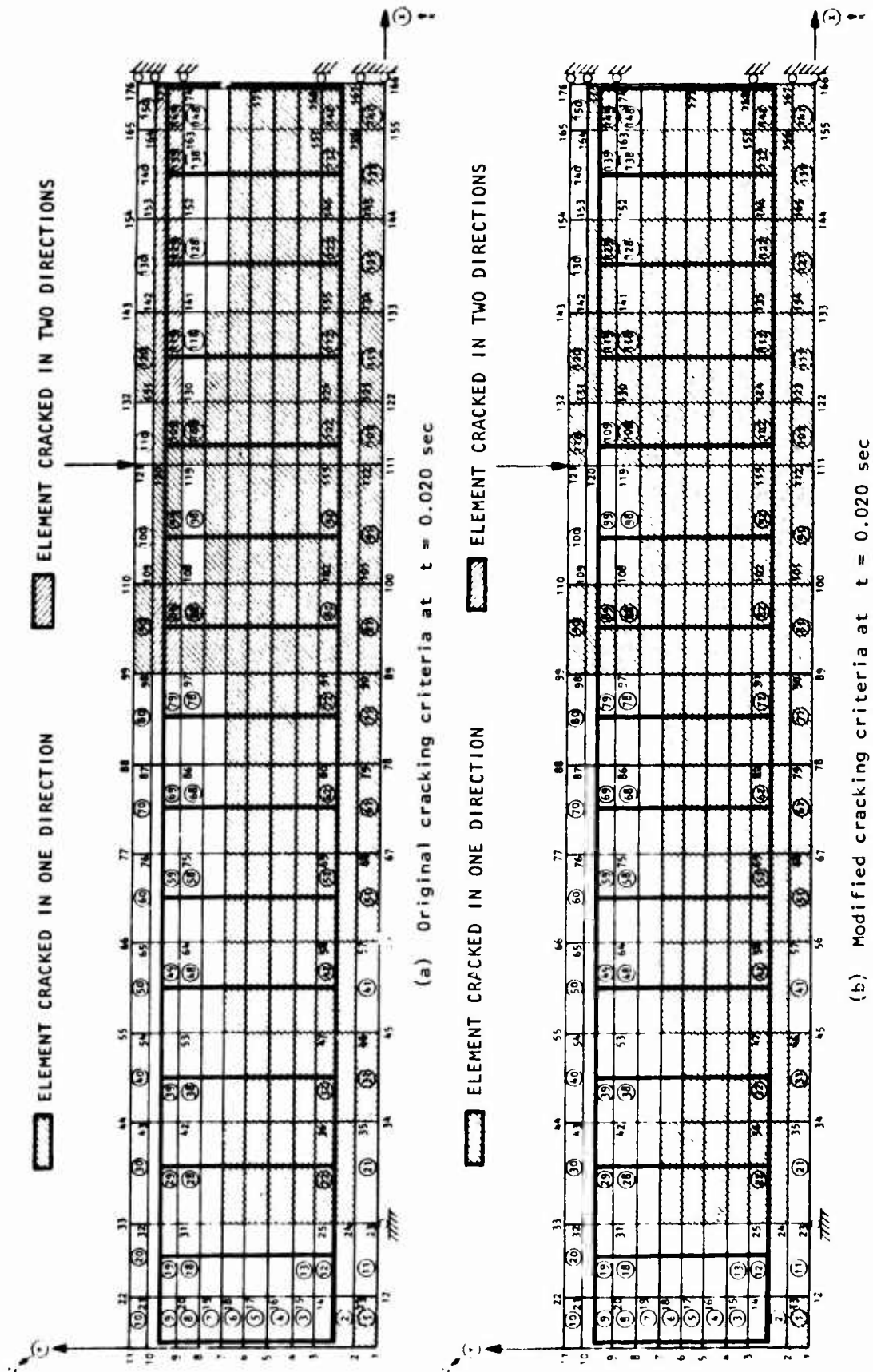


Figure 93. Comparison of Propagation of Cracks in Compression Zone Obtained by Original and Modified Cracking Criteria, Dynamic Case D-4

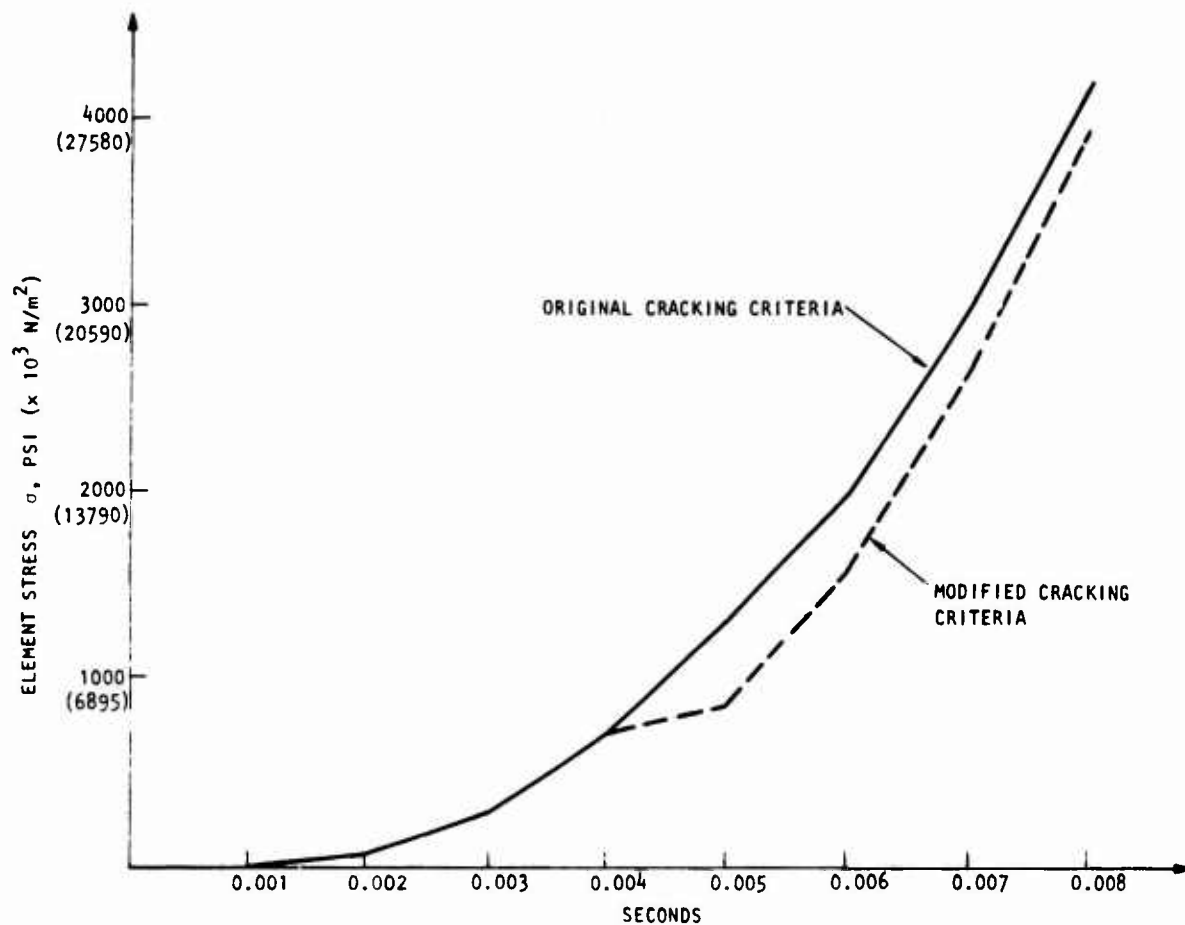


Figure 24. Comparison of Stresses in Tension-Reinforced Element 142 Obtained by the Original and Modified Cracking Criteria, Case D-4

Another discrepancy was observed in the propagation of cracks in the beam at 12 msec, as shown in figures 92(a) and 93(a). Double cracks were developed in the top compression zone, implying that tension stresses were developed in both directions.

An examination of the interaction formula (36a) for cracking (Fig. 15)

$$f'_t = f_t - \eta \sigma_2$$

where

f'_t = Tension cracking stress

f_t = Unconfined tensile stress of concrete (>0)

η = Sloping of cracking envelope (<0)

σ_2 = Minimum nonpositive stress normal to tension axis

indicates that if large compressive stress σ_2 is applied, f'_t will approach zero stress and cracks will be initiated at almost zero-tension stress (Fig. 15). Therefore, a slight numerical noise in the solution, which is almost unavoidable in any numerical solution, would trigger cracks in the compression zone.

To prevent the contaminating of crack patterns with unrealistic cracks, a 5-percent threshold was used as a numerical noise filter. As a result, only tension stresses that are above $0.05 f_t$ are allowed to produce tension cracks. The results of this modification are shown in figures 92(b) and 93(b).

Another feature that was added to the material package is the capability to simulate the sudden increase of strain at the onset of cracking. This is achieved by dropping the concrete stress level in the element immediately after cracks. The result of such modification is shown in figure 94.

The material package was further improved by including rebonding capability. With all the above improvements incorporated in the material package, a modified version of FEDRC became available. This version was first used to rerun the analysis of dynamic case D-4 and later to recalculate the static response of beam S-1. Both these calculations will be discussed in the following pages.

c. Case D-4: Results of Modified Dynamic Analysis

The results of case D-4 are shown in figures 95, 96, 97, 98, and 99. An examination of the displacement time-history plots (Fig. 95) reveals good agreement between calculated and measured displacements for the first 20 msec. This time almost coincides with the end of the loading part of the triangular pulse and the beginning of the unloading portion of this pulse (Fig. 90). However, for longer times, the calculated displacements are higher than the observed displacements. A maximum displacement of 2.40 in. (6.1 cm) was calculated at the middle of the beam, as compared to 1.25 in. (3.2 cm) obtained from experimental measurements (Ref. 91). Both velocity and acceleration time-history at the middle of the beam are shown in figure 96. The displacement time-history of nodal point 116, located 18 in. (45.6 cm) away from the center of the beam, is shown in figure 97. There is a general agreement between the shapes of the displacement time-histories obtained from analysis and experimental measurements as illustrated in figures 95 and 97.

The results recorded in figure 98 reveal good correlation between calculated and measured strains for a range that extends to 20 or 25 msec. This range again coincides with the ascending part of the input pulse given in figure 90. However, for the descending part of the loading (beyond 20 or 25 msec), calculated strains are larger than measured strains.

The stress time histories for selected elements are shown in figure 99. Figures 99(a), (e), and (h) indicate that the stress drops to a zero value immediately after cracks occur in unreinforced concrete elements. However, rebonding can be observed in figures 99(a), and (e), where a buildup of compression stress occurs beyond 45 msec.

An examination of figure 99(b) reveals that Element 52 cracked at approximately 7 msec, whereas steel bars yielded at approximately 15 msec. A similar trend was observed in figures 99(f) and 99(i). The stresses in elements at the top of the beam are illustrated in figures 99(d), (g), and (k). The rapid fluctuations of stresses from loading to unloading reflect the effect of waves and undamped oscillations on the response.

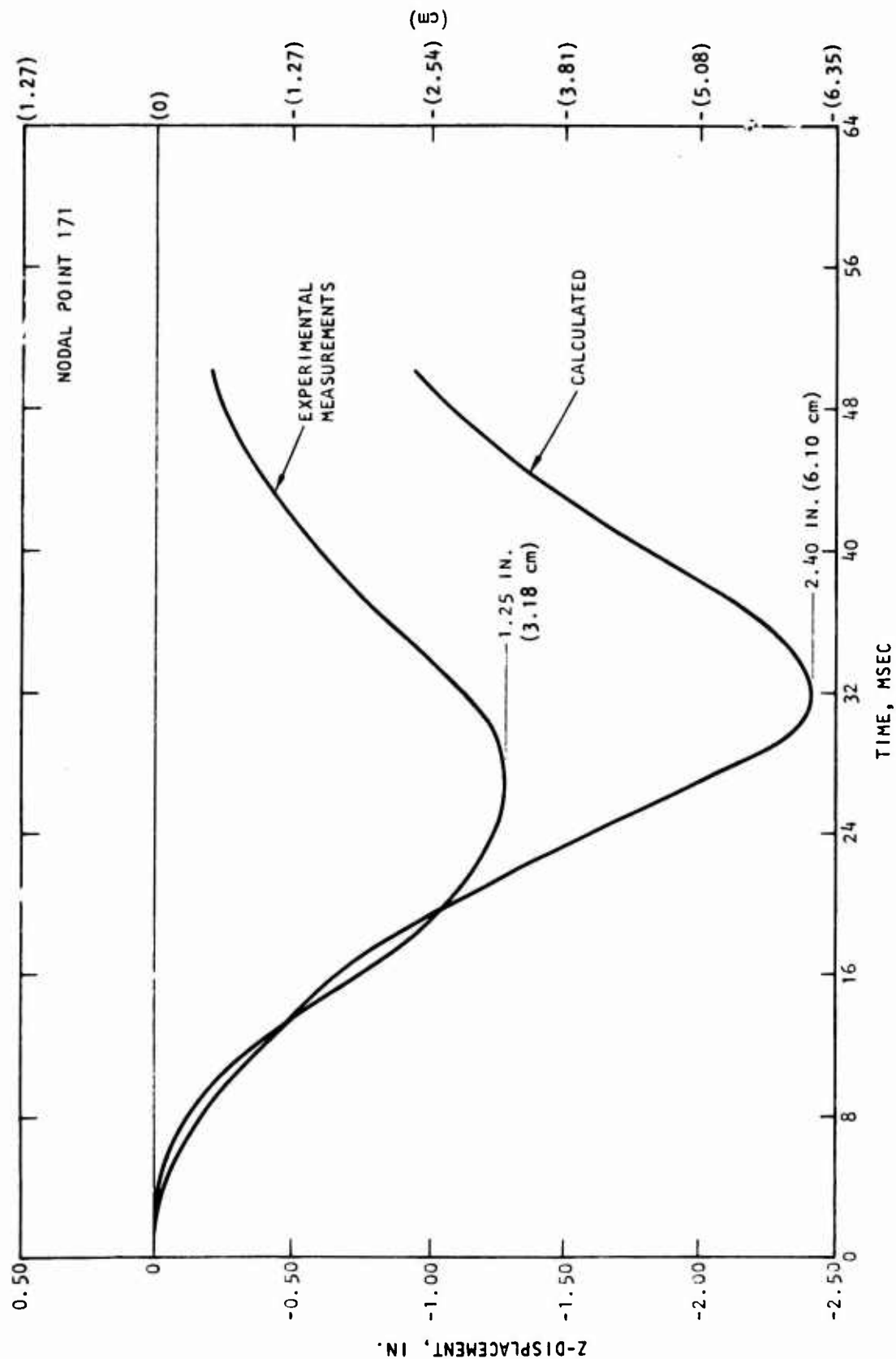
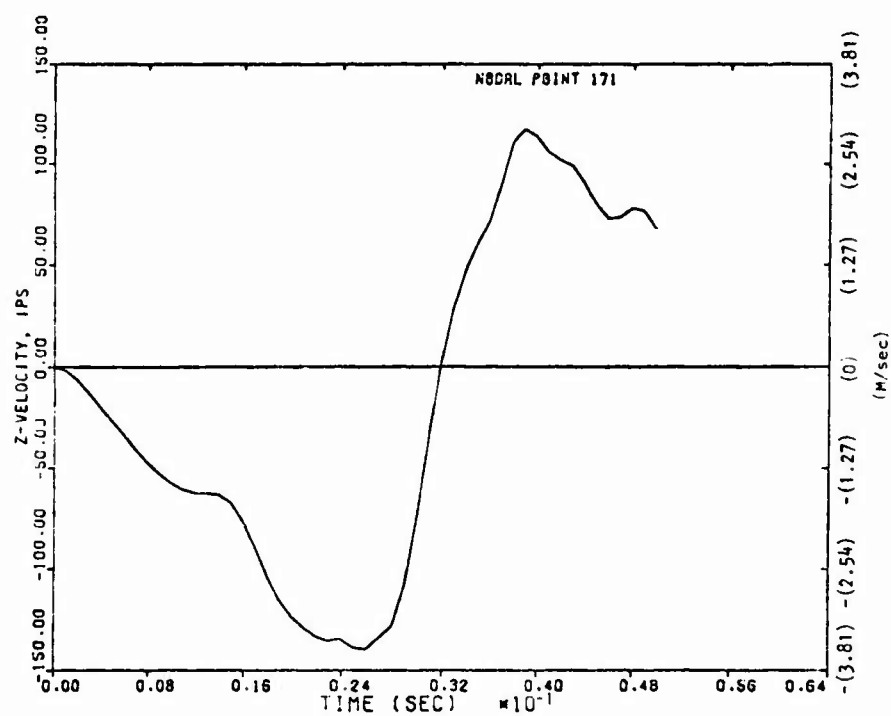
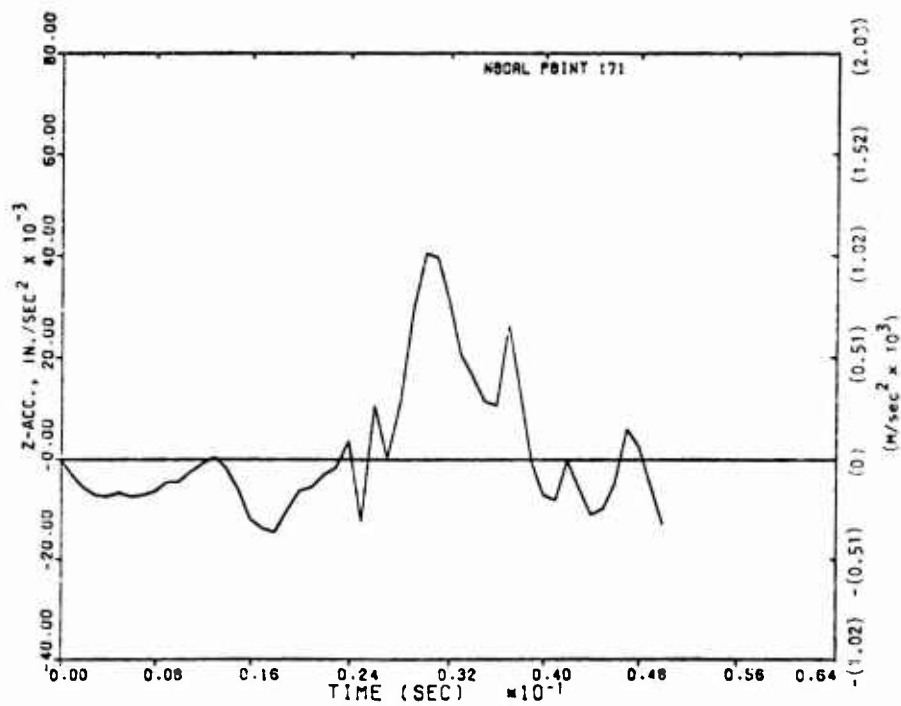


Figure 95. Vertical Displacement Time History at the Middle of the Beam Obtained from Experimental Measurements and Calculations, Case D-4



(a) Velocity time history



(b) Acceleration time history

Figure 96. Calculated Response at the Middle of the Beam, Nodal Point 171, Dynamic Case D-4

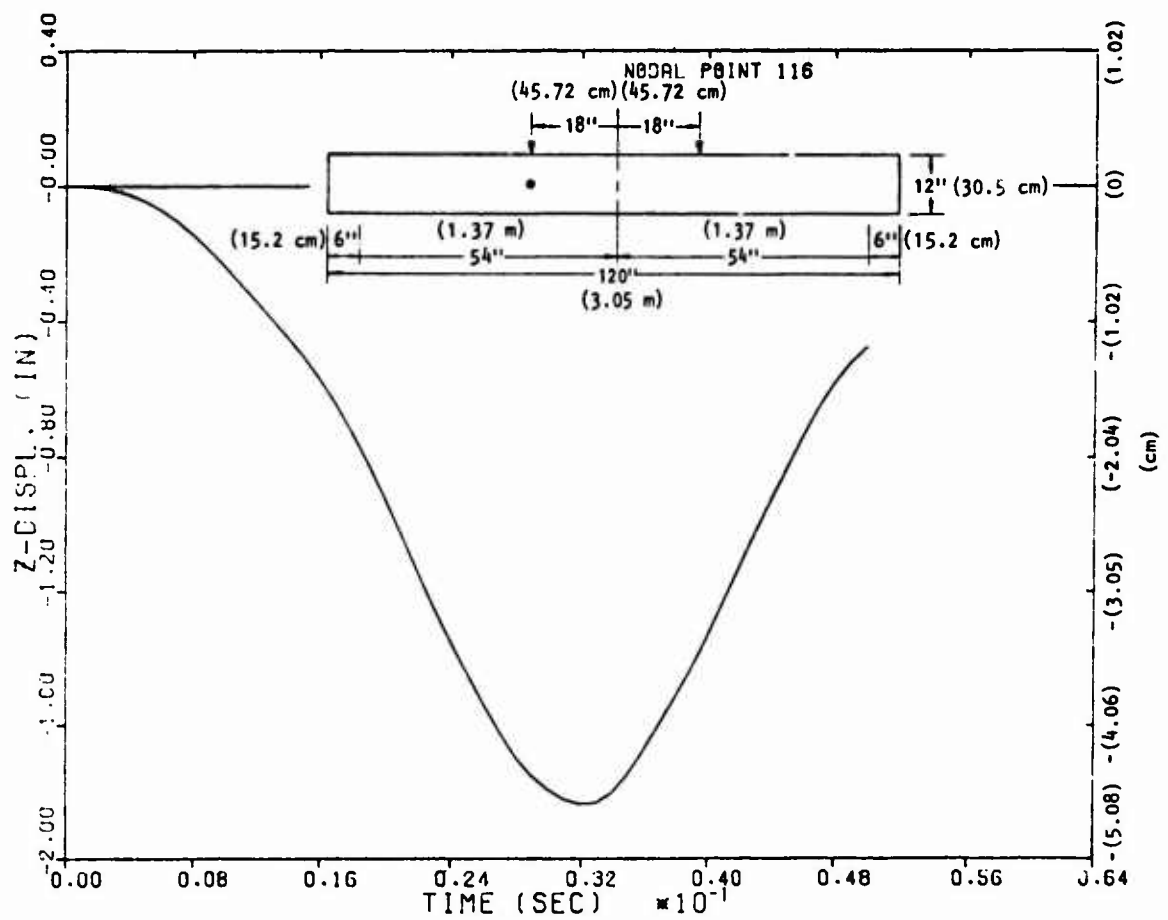


Figure 97. Calculated Vertical Displacement Time History at Nodal Point 116, Dynamic Case D-4

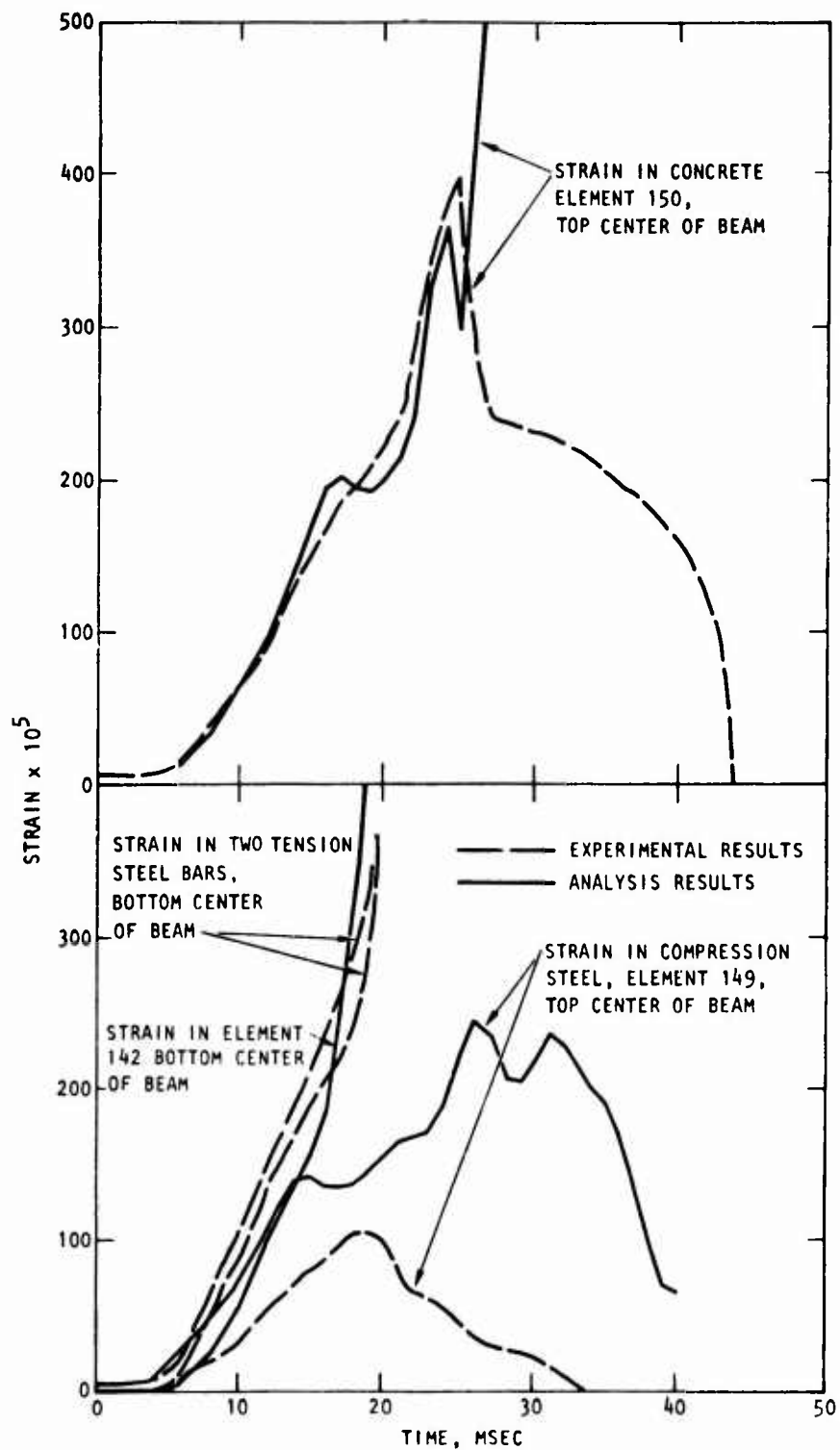
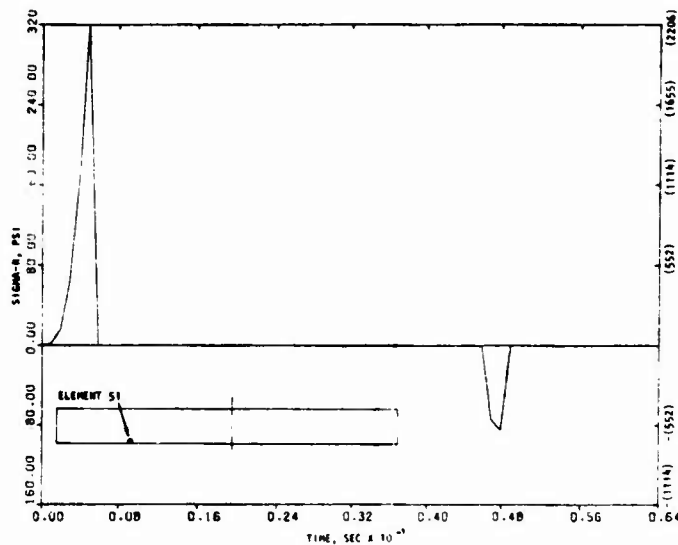
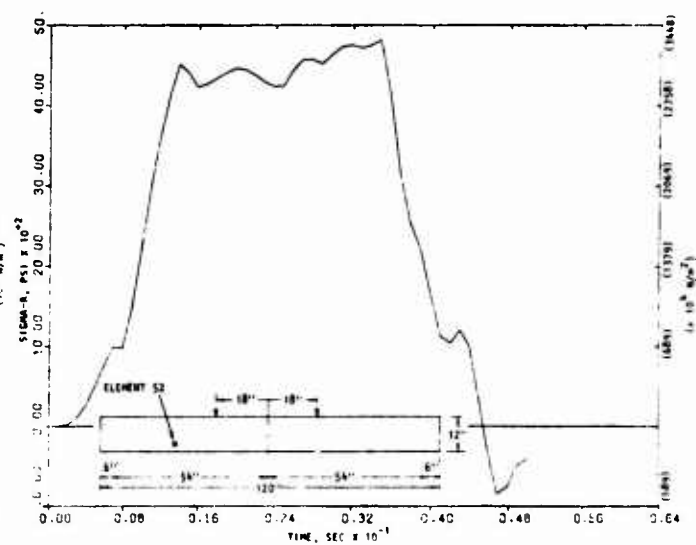


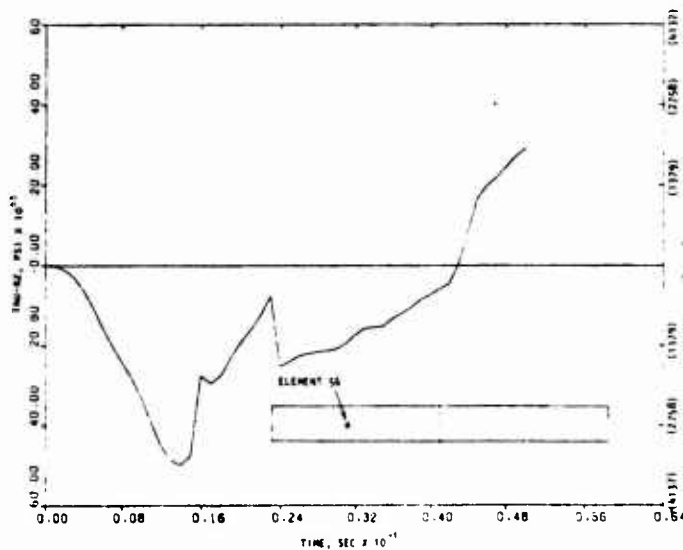
Figure 98. Comparison of Calculated and Observed Strain Histories, Blow 1, Beam 3b2, Case D-4



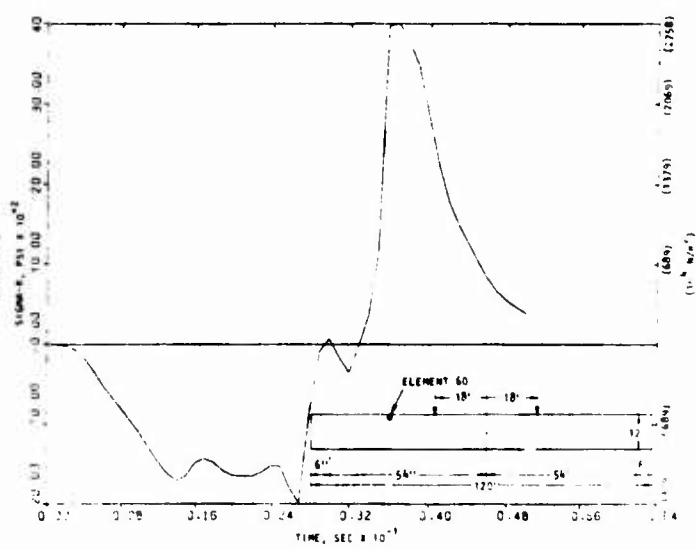
(a) Element 51



(b) Element 52 (tension reinforcement)

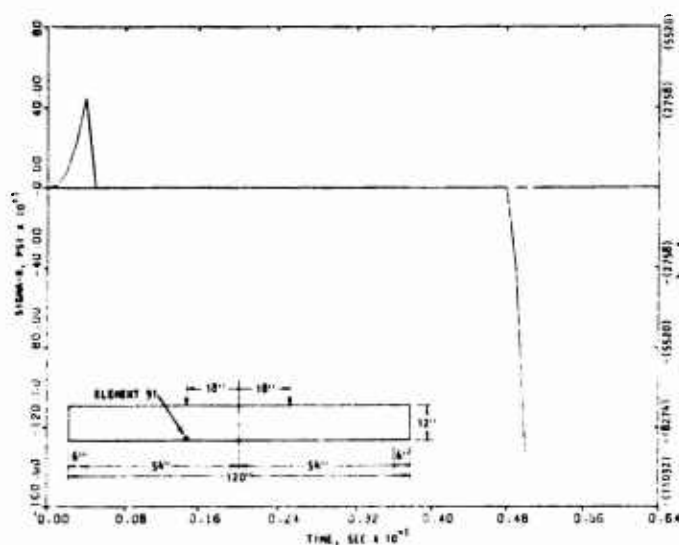


(c) Element 56

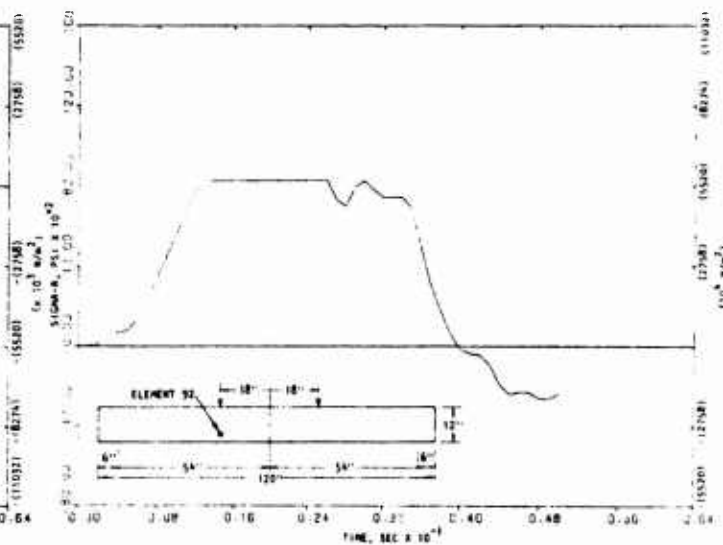


(d) Element 60

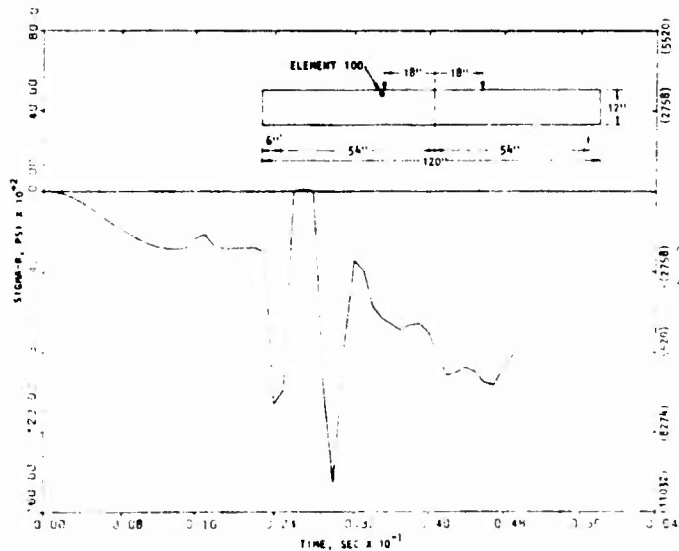
Figure 99. Dynamic Case D-4: Stresses in Beam 3b2



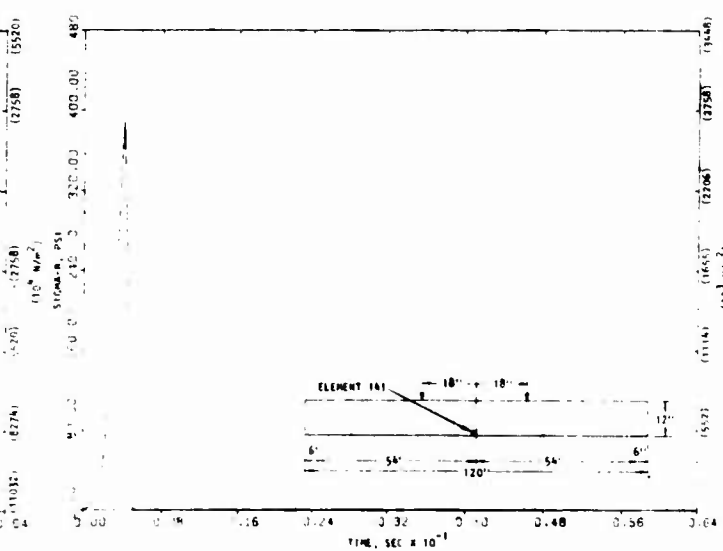
(e) Element 91



(f) Element 92

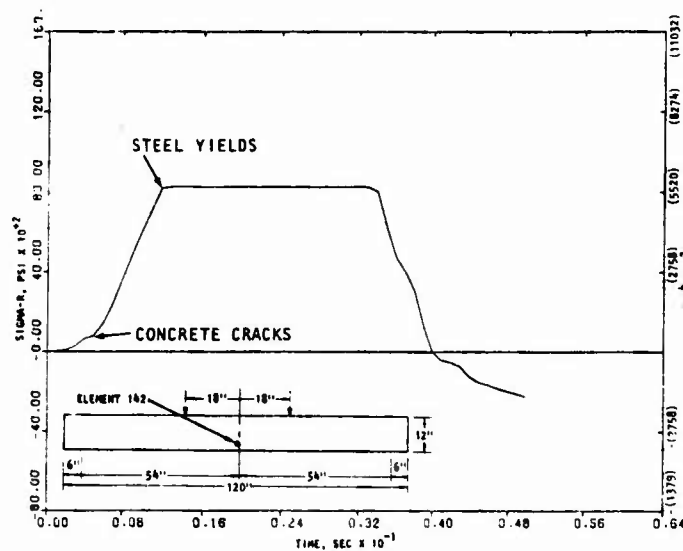


(g) Element 100

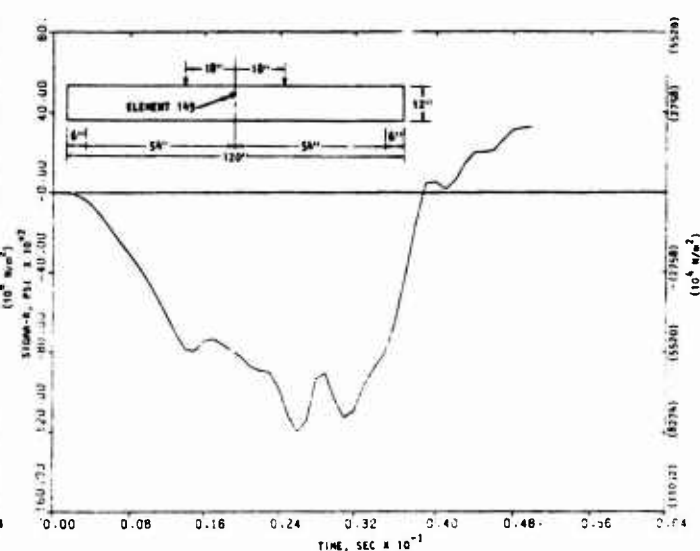


(h) Element 141

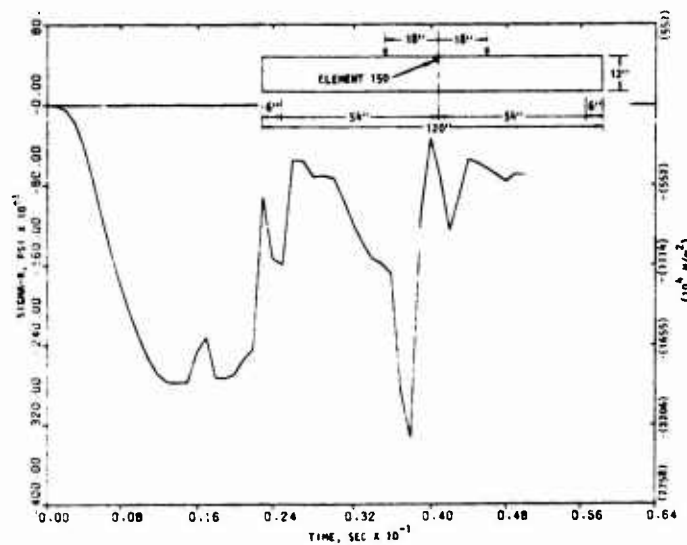
Figure 99. Dynamic Case D-4: Stresses in Beam 3b2 (continued)



(i) Element 142 (tension reinforcement)



(j) Element 149 (compression reinforcement)



(k) Element 150

Figure 99. Dynamic Case D-4: Stresses in Beam 3b2 (concluded)

It should be emphasized that the calculated strains are the average strains in the element. Higher strains would be expected in the cracked portion of the element while lower strains would be expected in the portion of the element where the bond is still intact. As a result the calculated strains would be higher than the strains measured at the same points on the beam where the bond is still intact. In addition reference 91 has indicated that strain damage occurred in some cases such as steel bars of element 142 (Fig. 98). It should also be observed that in the compression zone the concrete strain may exceed the steel strain, as clearly illustrated in the experimental work reported in section V (Figs. 52, 53, and 54). Therefore the average strain in element 149 near the top of the middle of the beam should be higher than the strain in the compression steel of the same element as revealed by figure 98.

The crack pattern calculated at the end of the 50 msec of the first blow is shown in figure 100. This figure indicates that cracks cover almost the entire beam.

d. Discussion of Results of Case D-4

From the preceding results it can be concluded that the modified analytical model can predict the response of beam 3b2 (case D-4) in the ascending portion of the loading, but overestimates the response in the unloading and free-vibration periods.

Since the dynamic behavior of a beam is more affected by damping in the unloading and free vibration portions of the response, a discussion of the damping mechanism is necessary.

The present material package accounts only for the hysteretic damping generated by the inelastic material behavior. Another type of damping that can be incorporated in a dynamic model is viscous damping. This type of damping is important at later stages of loading and would participate in damping of the free vibrations of a beam. Therefore, the absence of the viscous portion of the damping appears to contribute to the overestimation of the free vibration response of the beam.

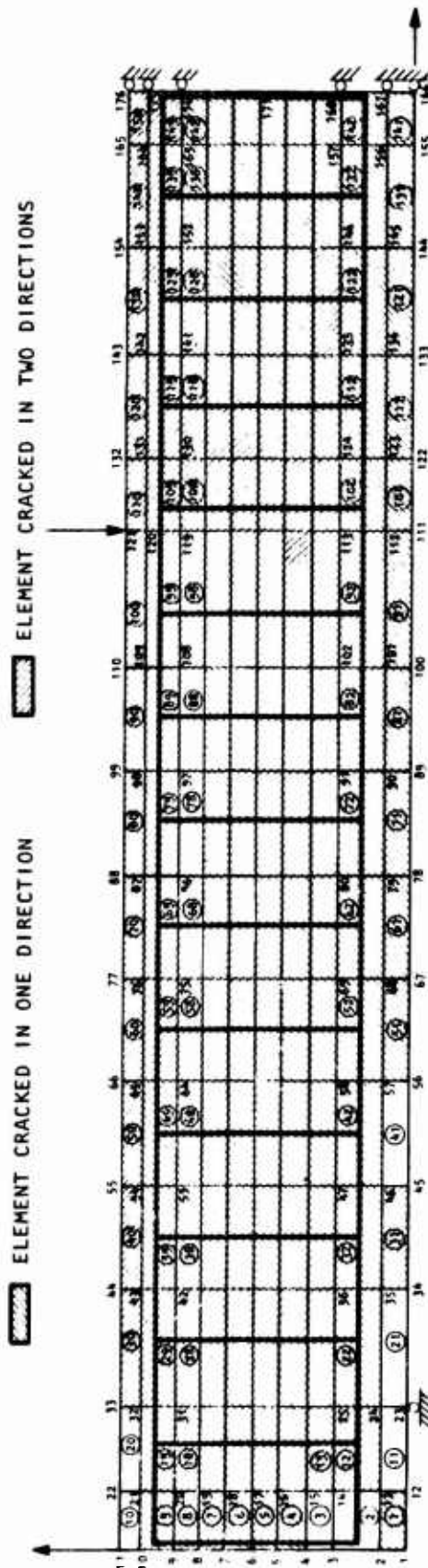


Figure 100. Dynamic Case D-4: Cracking Pattern at End of 50 msec

9. CASE S-1 (MODIFIED STATIC ANALYSIS OF REINFORCED CONCRETE BEAM 2S1.6-1 OF CRIST, REF. 89)

a. Introduction

The static response of beam S-1 (Fig. 56) was recalculated using the modified material package that was used earlier for the final dynamic analysis of case D-4. These results are also compared with both experimental measurements and original analysis results. Conclusions and recommendations are given at the end of this section.

b. Results of Modified Analysis

The results are shown in figure 101 and are compared to those obtained from both the original analysis and experiment (Ref. 89).

Figure 101 reveals that the modified analysis represents a considerable improvement over the original analysis. Collapse loads predicted by the modified analysis appear to correlate well with the experiment. Yielding occurs at approximately the same level of loading. Better correlation between calculated and measured displacements was also found in the postyielding range.

Figure 102 compares measured and calculated distribution of steel strain at the onset of yielding and at the collapse load. There is a general agreement between calculated and measured strains. However, since the analytical collapse load is taken as the load at which the solution begins to be unstable, the calculated strains must be interpreted within this approximation. The propagation of cracks in the deep beam is shown in figures 103 through 107. Figures 103 and 104 show the crack patterns calculated at 200 kips (890×10^3 N) loading by the original and modified versions respectively. The modified package appears to provide a better control of the cracks at this early stage of loading. However, figures 105, 106, and 107 indicate that the modified analytical model is affected by cracks over a wider area. Thus, it responds as a softer system, allowing better correlation between calculated and measured displacements.

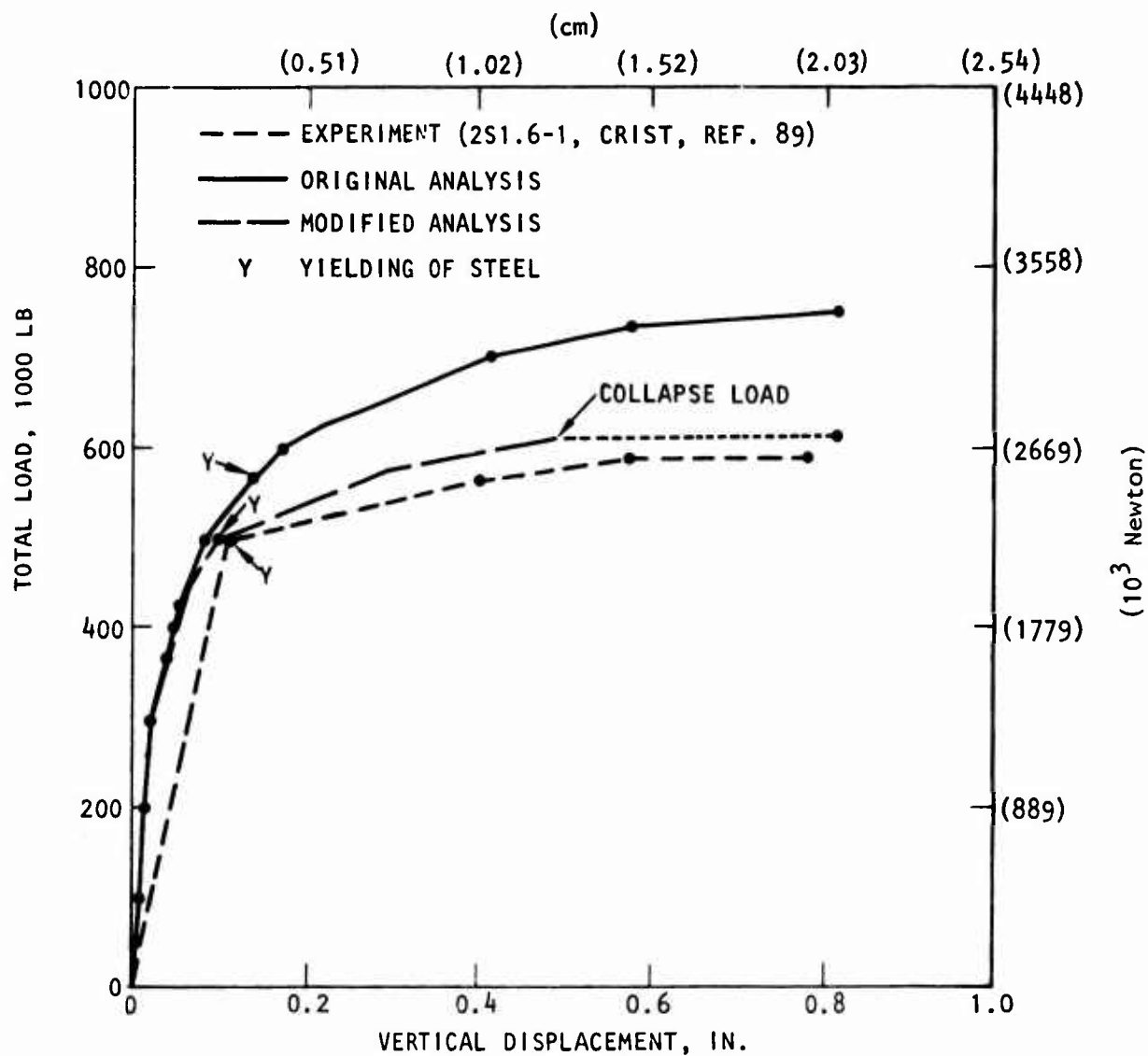


Figure 101. Comparison of Measured and Calculated (original and modified) Vertical Displacements at Midpoint of Beam 2S1.6-1, Case S-1

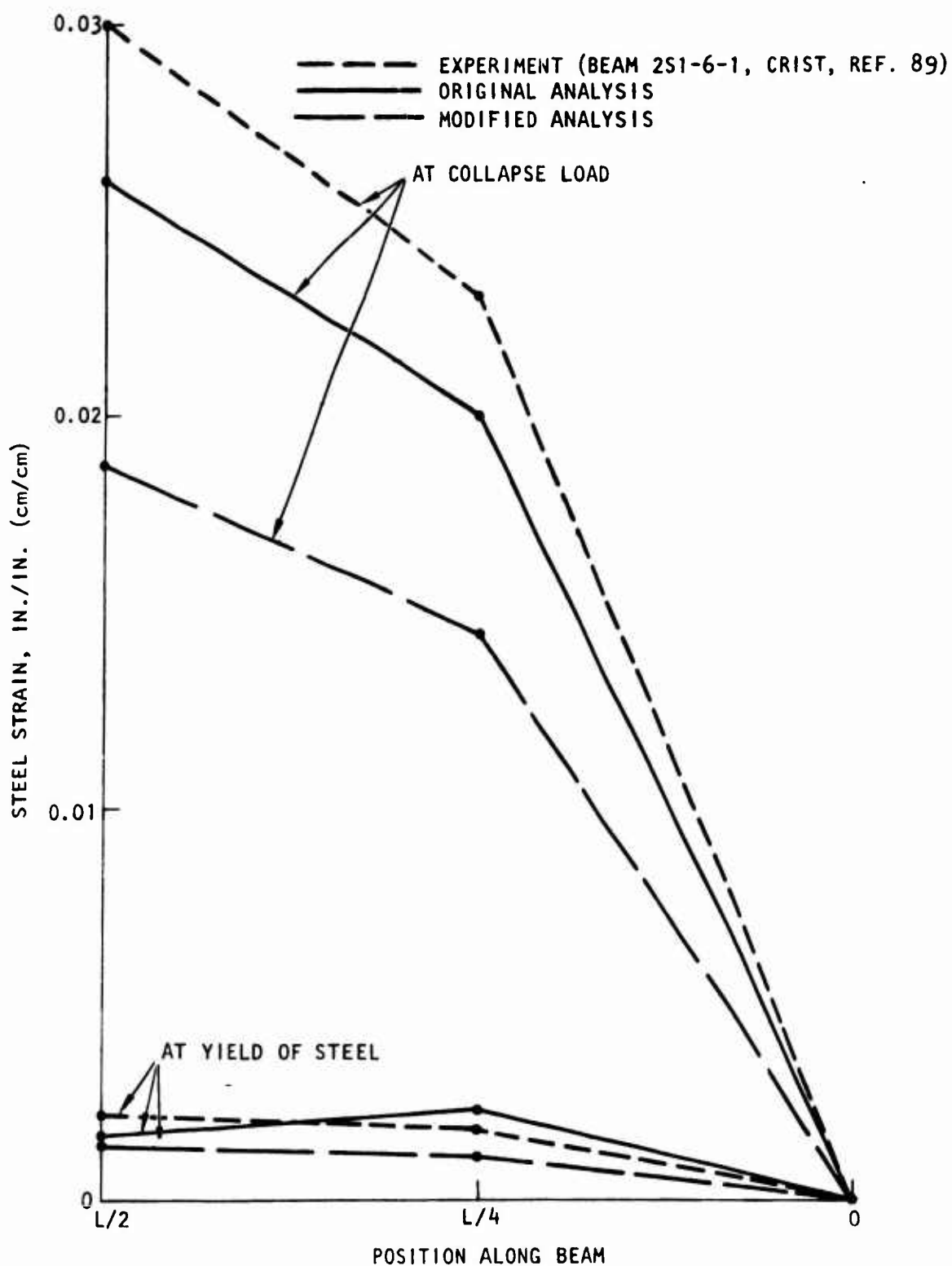


Figure 102. Comparison of Measured and Calculated (original and modified) Strains in Tension Steel, Beam 2S1.6-1, Case S-1

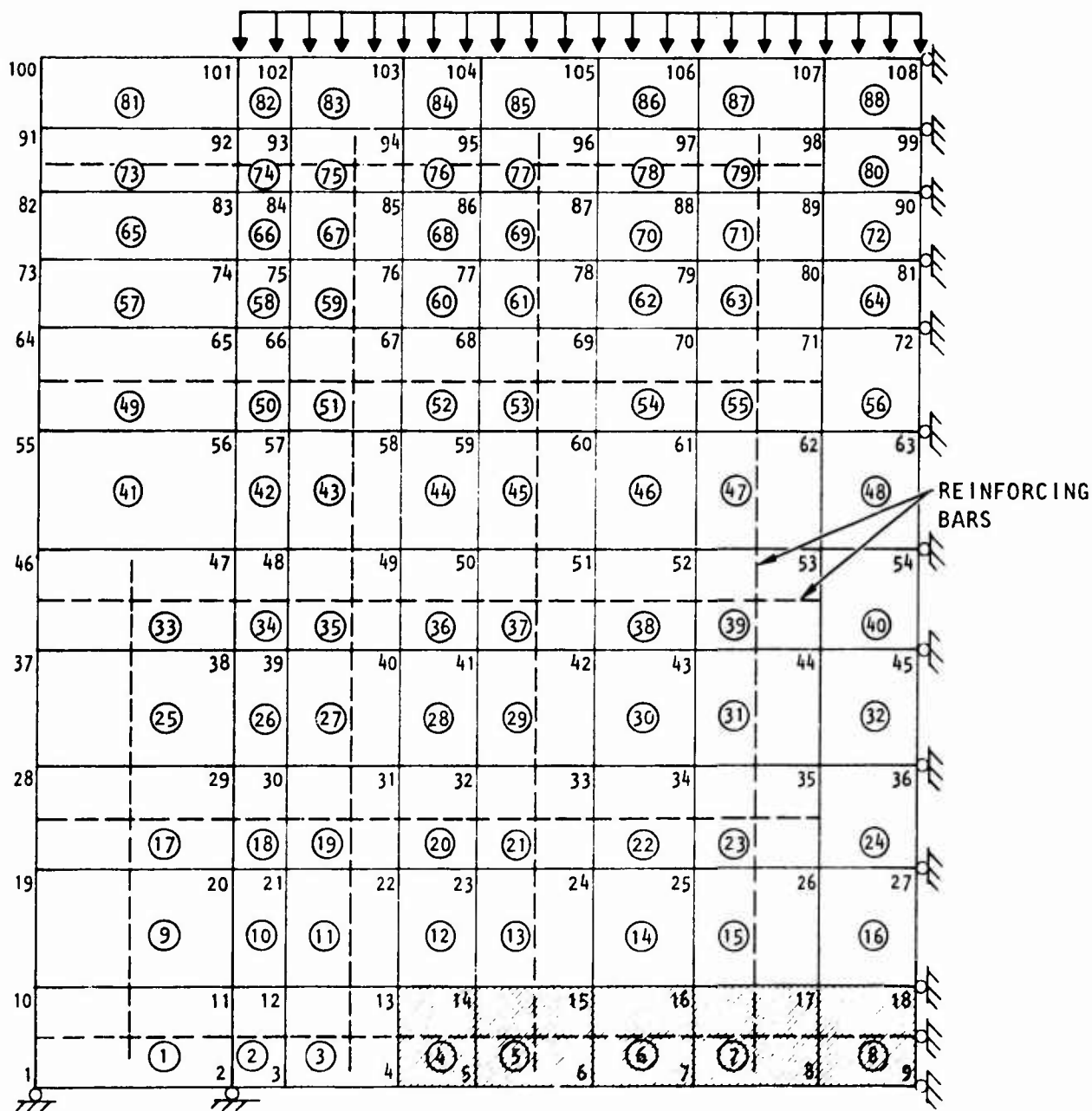


Figure 103. Crack Pattern Calculated by Original Version for Step 2 (200 kips), (890 x 10³ N), Case S-1

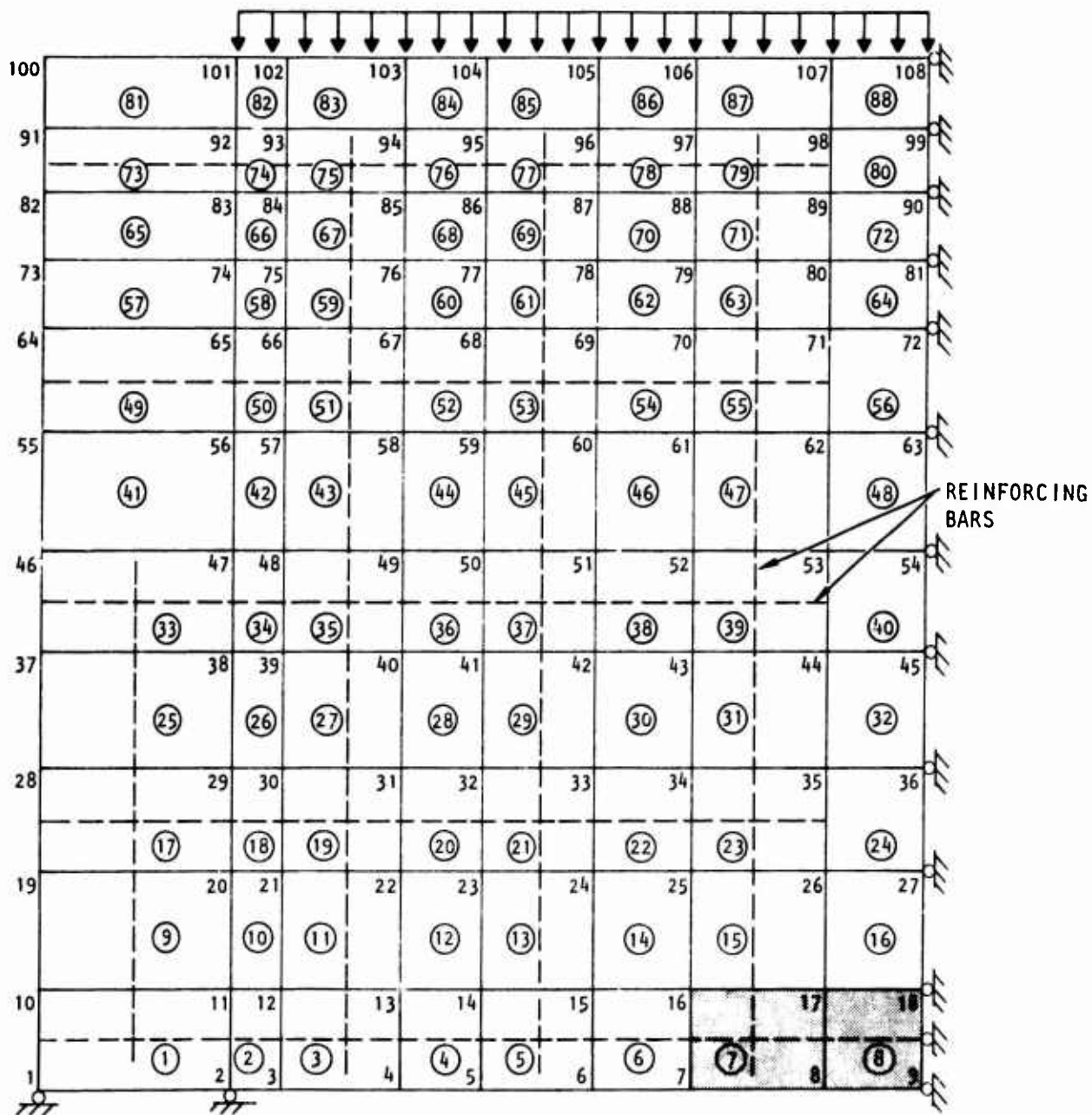


Figure 104. Crack Pattern Calculated by Modified Version for Step 2 (200 kips) (890×10^3 N), Case S-1

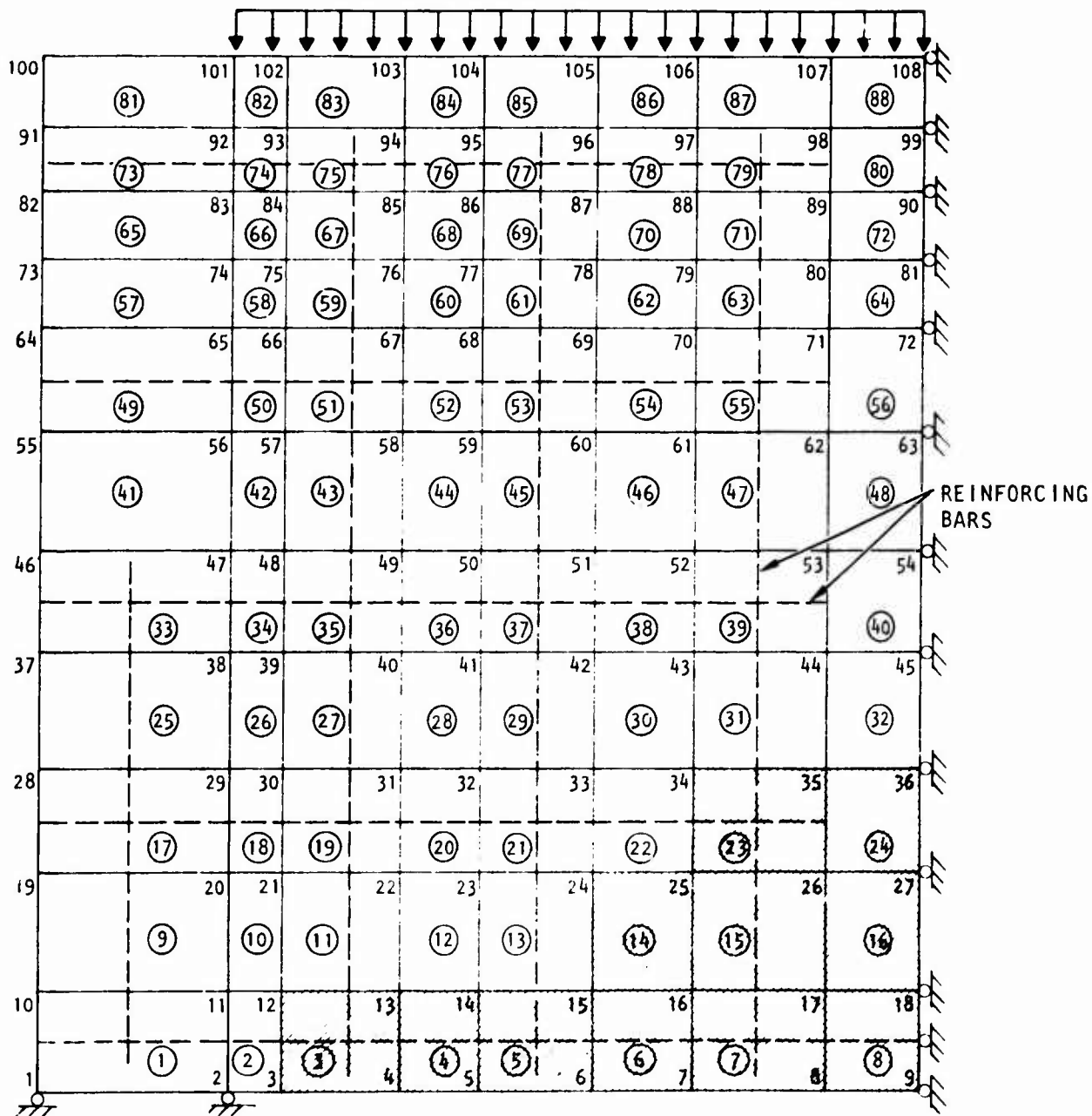


Figure 105. Crack Pattern Calculated by Original Version for Step 2 (300 kips) (1334×10^3 N), Case S-1

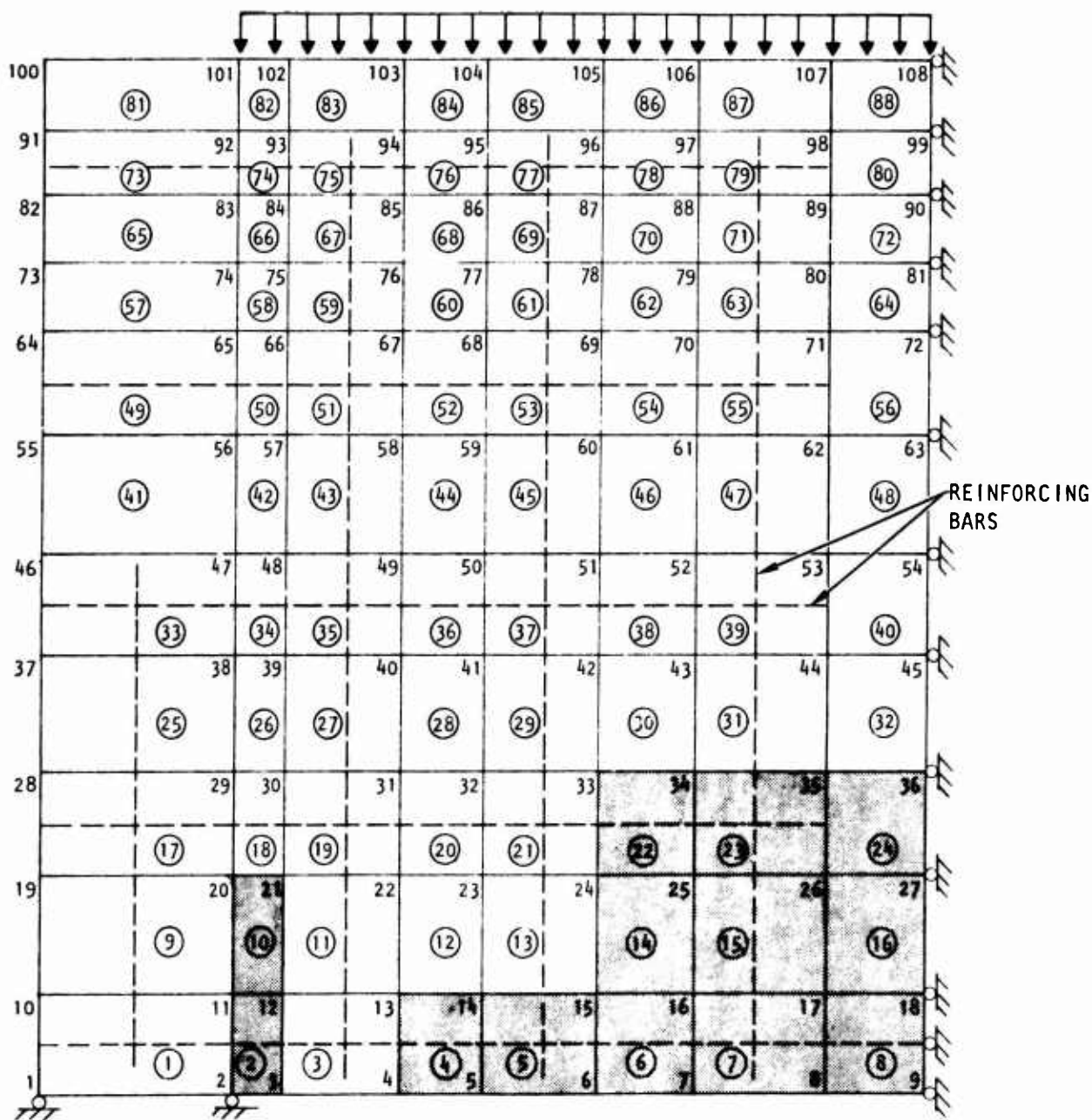


Figure 106. Crack Pattern Calculated by Modified Version for Step 2 (300 kips) (1334×10^3 N), Case S-1

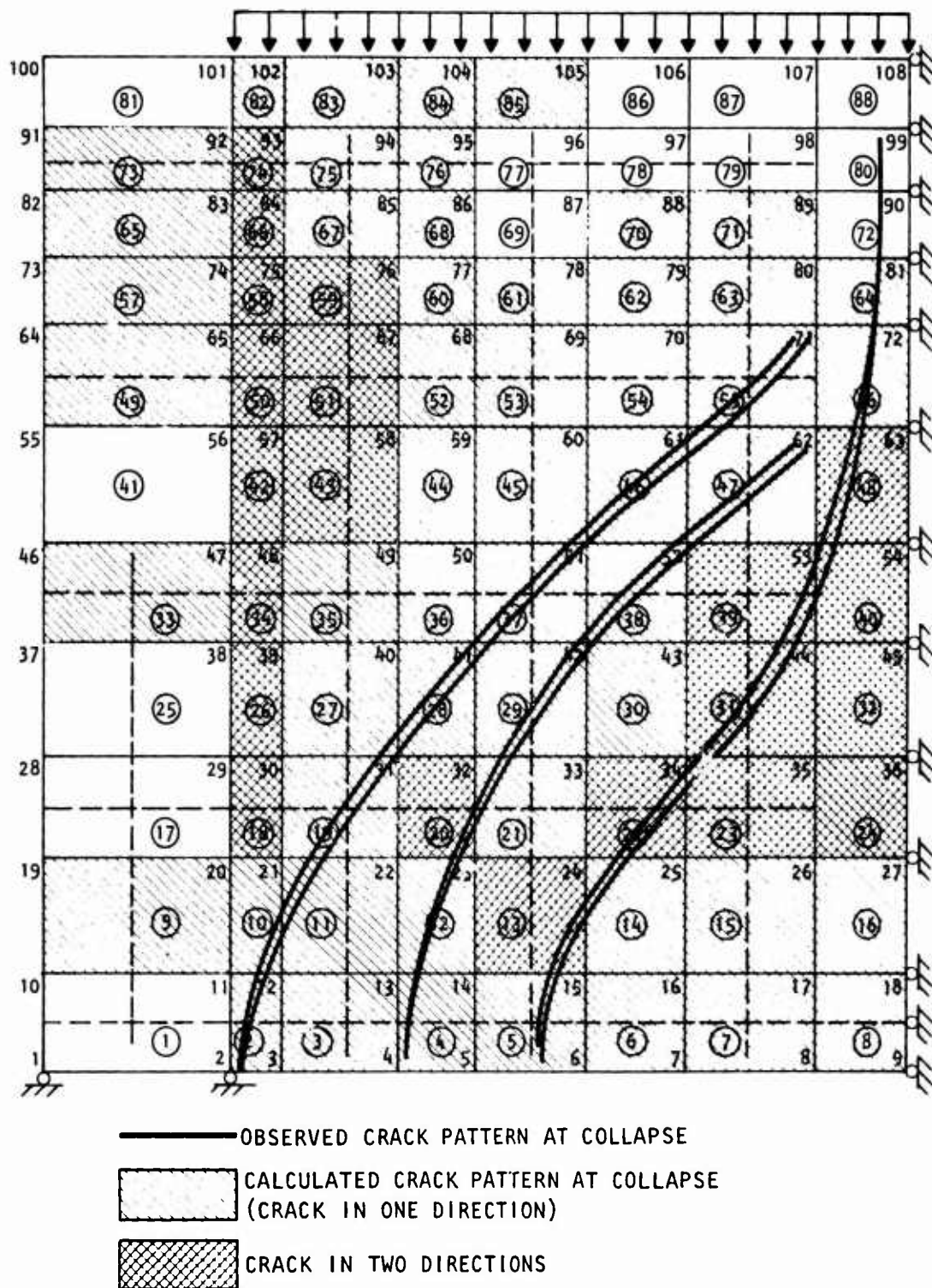


Figure 107. Comparison of Calculated (modified analysis) and Observed Crack Patterns at Collapse of Beam 2S1.6-1, Case S-1 (Crist, Ref. 89)

The results indicate that the modified model provides a better tool for predicting collapse load and deflection of static cases. However, overcracking was observed in the compression zone. Additional investigation of cracking under lateral confinement should be conducted, including comparative analysis for static and dynamic cases treated here. This will improve the prediction of crack formation and postcracking behavior.

SECTION VII

SUMMARY

The primary study objective was to incorporate recent advances in describing the behavior of plain and reinforced concrete in a computer program for analyzing plain or reinforced concrete structures. This objective was achieved in a three-step approach.

- a. Pertinent analytical and experimental data that became available after the development of the original Isenberg/Adham model were reviewed and evaluated (Secs. II and III).
- b. A material behavior package was prepared for incorporation in computer programs for analyzing typical reinforced concrete structures (Sec. IV and App. I).
- c. A matrix of cases was considered to validate the material behavior package by comparing the analytical results with the experimental data obtained from different sources (Sec. VI).

Secondary objectives were satisfied by a test program conducted to support the development of a model of bond slip incorporated in the material behavior package. A final objective was the provision of guidelines for performing analyses of typical reinforced concrete structures with the computer package developed in this study.

1. ANALYTICAL MODEL

The analytical model developed under this contract is essentially an improved version of the Isenberg/Adham model (Ref. 1). The improvements were based on pertinent studies reviewed at the beginning of this contract. Two types of plain concrete stress/strain models were incorporated in the present model: (a) variable modulus model and (b) plastic-capped model. The virgin stress/strain relations incorporated in the variable modulus model were based

on those suggested by Liu, Nilson, and Slate (Ref. 53). However, only the ascending branch of the stress/strain relations was activated in this study. The steel reinforcement is specified as a number of steel sets in any three arbitrary directions. The pertinent experimental data on bond slip were fitted by a bilinear relation. The dowel action was accounted for by a simplified relation based on the work of Dulacska (Ref. 65). The properties of steel and concrete were combined to represent composite behavior at various phases of loading. Rebonding capability was incorporated in the code and was based on adjusting the stiffness of the element to avoid numerical instability of the solution.

2. EXPERIMENTAL STUDIES

Experimental studies on bond slip were performed under this contract to add to the sparse data that were available, to verify analysis assumptions on bond slip and, in addition, to provide some guidance for element size selection. Four cylindrical specimens, each reinforced with a coaxial piece of rebar, were subjected to tension and compression. Strain gages were attached to the reinforcing bar and to the surface of the concrete.

Results of tension tests were compared to those of Ismail and Jirsa (Ref. 66) and Tanner (Ref. 67). The results were also compared to the analytical relationships of reference 1 and the bilinear relationship used in this study.

The two specimens tested for bond slip in tension had a total length less than the 30-bar diameter required to develop full bond. Their behavior was characterized by three stages: (a) gradual deterioration of bond caused by end effects, (b) sudden loss of bond when major cracks occur, and (c) some recovery due to aggregate interlock, followed by gradual deterioration of bond.

The two specimens tested in compression indicated that the effective stiffness of steel can be reduced by as much as 36 percent by application of the ultimate load in compression. The analytical model developed under this study assumes full contribution of steel in compression.

3. RESULTS OF DEMONSTRATION CASES

The material package developed for this contract was incorporated in FEDIA, a dynamic inelastic, two-dimensional continuum, finite element code developed by Agbabian Associates. The demonstration of the capabilities of this package included nine cases. Static cases were completed for two uniformly loaded, reinforced-concrete deep beams (cases S-1 and S-2) and for three reinforced-concrete beam columns (cases S-3, S-4, and S-5). Four dynamic cases were completed. Case D-1 was performed with the deep beam of static case S-1. Case D-2 was for the beam column configuration of case S-3. The third dynamic case, D-3, was designed to demonstrate the capability of the code to model the behavior of buried, reinforced concrete silos. Case D-4 represents the behavior of reinforced-concrete beams under impact loading. In addition, one static and one dynamic case were repeated using a modified version of the reinforced concrete model. The results of these demonstration cases were compared to available experimental data.

The results of the original analyses for the static cases appear to predict most of the cracked regions. However, the general results indicate that the analytical model is stiffer than the experimental model. The analysis appears also to overestimate the collapse load by about 20 percent. The modified material package behavior, which incorporated changes in the criteria for the onset of tensile cracking, showed marked improvement in predicting both initial cracking and the collapse load in the reanalysis of case S-1.

The results of case D-1, the dynamic analysis of the deep beam, indicate general agreement with test results, except that the analysis greatly exaggerates the strain concentration at the points corresponding to cracking and plastic hinge development, i.e., the points at one-fourth, and three-fourths of the length of the beam. It should be noted that the strains obtained from the analysis are the average strains of the cracked and uncracked regions of the element. Since the strains in the cracked regions increase drastically with the increase in response, the average value of strain is expected to jump to a very high value. Comparable deviation of steel strain

was observed in the experimental work of Crist (Ref. 89). The proximity of the strain gages to the cracks was suggested as one of the causes of this deviation. Therefore, both analytical and experimental results appear to be least reliable at the cracks. Peak dynamic deflection was underestimated by a factor of about two. Crack patterns in the tensile region were in reasonable agreement with test results, but excessive cracking was predicted for the compression zone. No direct experimental data were available for comparison with the results of dynamic cases D-2 and D-3. However, the general character of the results indicates that the analytical model is applicable to both the beam column and buried structure configurations treated.

Overestimation of collapse loads, already evident from the static cases, and observation of anomalous crack initiation during the early stages of calculating the dynamic response of case D-4, led to modification of the composite model of reinforced concrete behavior. The results of the modified dynamic analysis of case D-4, indicated favorable agreement with experimental data in the ascending portion of the loading. Strain in the tensile steel and compressive strain in the concrete were in excellent agreement during this stage. However, the analysis was found to overestimate the response in the unloading and free-vibration phases. This was attributed to the higher effective stiffness of finite elements with higher steel ratios, which attracted higher stresses.

The original test for cracking, which predicted crack initiation in reinforced elements on the basis of average element stress, was replaced by a criterion based on concrete stress above. This produced initial cracking at the beam edge, in agreement with experimental observation. Also, cracking in the compression region was found to be sensitive to the interaction of lateral stresses. Therefore, the criteria were modified to eliminate unrealistic cracks in the compression zone. Case D-4 was then reexamined using the improved reinforced concrete model.

The design of finite element meshes for the nine demonstration cases was guided by factors that included (a) uniform distribution of compression and tension steel, as well as stirrups in all elements, (b) elements wide enough to provide full bond at the start of the loading and to allow primary cracks to develop within the element, (c) elements small enough to allow for adequate description of stress and/or strain gradients, (d) integration time-step in a dynamic solution small enough to allow resolution of the dynamic input with sharp rises, and (e) budget limitations on size of problem and the number of integration time steps.

SECTION VIII

CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

From the previous discussion, the following conclusions can be summarized.

a. The experiments performed as part of this study presented additional data for the interpretation of the bond-slip behavior of reinforced concrete members. Concrete strength and load history appear to have a significant effect on bond-slip relations in tension. For bond-slip in compression, the present experimental results indicate that perfect bonding may not exist. Some reduction in the effective stiffness of steel in compression is expected.

b. The composite material model that was incorporated in a finite element computer program provided numerical solutions that were in general agreement with experimental results. In general the numerical solutions overestimated the stiffness and collapse loads. Agreement between analytical and experimental results was better for static than for dynamic cases.

c. In the two examples for which a more refined material model was used for comparison with earlier analyses of this study, considerable improvement was observed. The refined static analysis of case S-1 resulted in improved prediction of collapse load, while the refined dynamic analysis of case D-4 resulted in good correlation with measured displacements for the loading portion of the input. The refined model incorporates the following changes: (1) onset of cracking is described by the concrete stress rather than the average element stress, (2) initiation of cracking by numerical noise in regions of compressive confining stresses was avoided by introducing a "noise" filter, (3) sudden increase of strain at the onset of cracking was incorporated by dropping the concrete stress level in the element immediately after cracks are initiated, (4) rebonding capability was included.

d. The authors feel that without introducing additional damping mechanisms, the model is incapable of providing good correlation with experimental observations of dynamic behavior in the unloading and free-vibration portions of the response.

e. Onset of cracking and propagation of cracks depend to some extent on the distribution of finite elements and percent of steel in these elements.

f. Long elements in the tensile region allow better description of the bond at the beginning of loading, while shorter elements may underestimate the role of the concrete prior to cracking.

g. Smaller size elements allow better description of the response in regions of high stress or strain gradient.

h. Shorter integration time steps would allow more accurate description of the initial response to short rise-time dynamic input.

2. RECOMMENDATIONS

The results of the matrix of demonstration cases indicate that several areas need further study. It is recommended that improvements be made in the material package that would lead to better correlation between computed response and experimental observations. The recommended improvements and studies are summarized in the following:

a. Additional dynamic cases for which experimental measurements are available should be studied with the modified material package to determine the importance of damping mechanisms not presently accounted for.

b. The effects of finite element size and distribution and steel percent on the analysis results need more detailed examination.

c. The details of concrete strength under multiaxial loading, dowel action, bond slip, variation of Poisson's ratio, and rebonding need further experimental and analytical investigation.

d. The feasibility of using other elements in the composite model to describe (a) crack initiation and (b) failure under multiaxial stress conditions, as well as alternative forms of the basic constitutive relations that avoid limitations of the Liu, Nilson, and Slate formulation (Ref. 53) adopted here, should be investigated.

APPENDIX I

USER'S GUIDE AND DESCRIPTION OF THE COMPUTER PROGRAM MATPAC

1. INTRODUCTION

A material model package consisting of a set of subroutines has been developed for plain and reinforced concrete. Two types of models defining the constitutive properties of concrete have been incorporated in this package. These two types of models are

- Variable modulus model
- Capped plastic model

The routines are designed to be used with finite element or finite difference codes and can handle plane stress, plane strain, axisymmetric, and three-dimensional solids. Both models are capable of handling plain concrete. However, only the variable modulus model is equipped to handle cracked reinforced concrete.

The code calculates stress and strain at the end of the current step based on the current strain as stress increment. Incremental tangent stress/strain relations are also calculated.

The code is accessed by call to subroutine MATPAC with proper arguments whenever stress or tangential stress/strain relationships are necessary. The code operates on stress or strain increments.

The important options of the code are tabulated below.

- a. Variable modulus model or plastic-capped model controlled by variable ICOMPS
- b. Plane stress, plane strain, axisymmetric or three-dimensional state of stress controlled by variable NELTYP
- c. Steel orientation in global or prescribed directions controlled by variable NRB

- d. Stress increment input or strain increment input options controlled by the sign of variable NSPLIT
- e. Splitting of stress or strain increments for better accuracy controlled by the magnitude of variable NSPLIT

The controlling variables are defined in section 2, with suggested guidelines for choice of material properties. Guidelines for variable initialization and storage are also included.

A brief description of each subroutine and its functions is included in section 3. Section 4 contains the program flow charts. The listings of the routines are given in section 5.

Examples using the stress increment option and the strain increment option are shown in section 6. Section 7 contains the listing of the main program for testing the MATPAC program.

2. DEFINITION OF VARIABLES

The variables used as arguments of the material package (MATPAC) are described in detail below and in table VIII on page 221.

STRESS The six components of stress are stored in this location. In rectangular coordinate system the stress components are stored in the following order: $[\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}]$. In plane stress and plane strain problems the last two components are not used. In cylindrical coordinate system (r, z, θ) the stress components are arranged as follows: $[\sigma_r, \sigma_z, \sigma_\theta, \tau_{rz}, \tau_{z\theta}, \tau_{\theta r}]$. For axisymmetric problems the last two components are not used.

STRAIN The six strain components are stored in this location. The strain components are arranged in the same order as STRESS.

STRINC The incremental stress ($NSPLIT > 0$) or the incremental strain (if $NSPLIT < 0$) components are stored in this location. The components are arranged in the same order as STRESS.

EPSEL For the plastic-capped model the elastic strain components are stored in this location. For the variable modulus model strain components are stored in this location to indicate loading or unloading paths.

COEFF Fifteen constants defining material property for reinforced concrete for the variable modulus model are stored in this location. The arrangement of the properties is as follows:

- COEFF(1) = E_c , initial modulus of elasticity for concrete
- COEFF(2) = E_s , modulus of elasticity for steel
- COEFF(3) = $|f'_c|$, ultimate compressive strength of concrete
- COEFF(4) = f_y , yield strength of steel
- COEFF(5) = ν , Poisson's ratio for concrete
- COEFF(6) = f'_t , cracking strength of unconfined concrete in tension
- COEFF(7) = n , a constant reflecting change in cracking stress due to confinement ($\sigma_{\text{crack}} = f'_t - n^*\sigma$, σ = nonpositive confinement) = f'_t/f'_c = negative quantity
- COEFF(8) = n , dowel coefficient of local compression for shear (usually of the order 4.0) see equation 62
- COEFF(9) = G_s , shear modulus for steel
- COEFF(10) = Fraction of E_c during rebonding after crack (recommended value ≤ 0.5)
- COEFF(11 through 15) = Currently not used

BLKMOD Bulk modulus for concrete (used only for plastic-capped model)

SHRMOD Shear modulus for concrete (used only for plastic-capped model)

YCOF Twenty-five material constants defining the plastic-capped model are stored in this location. The values suggested for these material

parameters for a concrete with an ultimate unconfined compressive strength of f'_c (in psi, or English units) are as follows:

$$\begin{aligned}
 YC\emptyset F(1) &= \left| \frac{f'_c}{7} \right| \\
 YC\emptyset F(2) &= 12.2 \\
 YC\emptyset F(3) &= 11.0 \\
 YC\emptyset F(4) &= 40000.0 \\
 YC\emptyset F(5) &= 800.0 \\
 YC\emptyset F(6) &= 700.0 \\
 YC\emptyset F(7) &= 2.0 \\
 YC\emptyset F(8) &= 1.0 \times 10^{-6} \\
 YC\emptyset F(9) &= 100.0 \\
 YC\emptyset F(10) &= 120.0 \\
 YC\emptyset F(11) &= 1.0 \times 10^{-4} \\
 YC\emptyset F(12) &= 2.0 \\
 YC\emptyset F(13) &= 7.0 \times 10^{-6} \\
 YC\emptyset F(14) &= -2.4 \times 10^{-4} \times \left| \frac{f'_c}{7000} \right| \\
 YC\emptyset F(15) &= 4000.0 \\
 YC\emptyset F(16) &= 8.0 \times 10^{15} \\
 YC\emptyset F(17) &= -7.02 \times 10^{-12} \times \left| \frac{f'_c}{7000} \right| \\
 YC\emptyset F(18) &= 1.35 \times 10^{13} \\
 YC\emptyset F(19) &= 6.0 \times 10^4 \\
 YC\emptyset F(20) &= 0.0005 \\
 YC\emptyset F(21) &\left. \begin{array}{l} \\ \\ \end{array} \right\} \\
 YC\emptyset F(22) &= 0 \text{ (not currently used)} \\
 YC\emptyset F(23) & \\
 YC\emptyset F(24) &= 0 \\
 YC\emptyset F(25) &= 5.0 \times 10^7
 \end{aligned}$$

DC A 3 x 3 direction cosine table indicating principal orthotropic or cracked directions (for variable modulus model only)

DCS A 3 x 3 direction cosine table indicating steel orientation if different from global coordinate directions (for variable modulus model only).

1st column direction cosine of 1st steel direction

2nd column direction cosine of 2nd steel direction

3rd column direction cosine of 3rd steel direction

The steel directions need not be mutually orthonormal.

AST Steel ratios $\left(\frac{\text{Steel area}}{\text{Total cross-sectional area}} \right)$

AST(1) Steel ratio in direction 1

AST(2) Steel ratio in direction 2

AST(3) Steel ratio in direction 3

(for variable modulus model only)

NRB A flag. If $NRB \leq 0$, the steel directions are assumed to be in the global coordinate directions. If $NRB > 0$, steel is oriented in specified directions indicated by DCS (for variable modulus only).

ZK1,ZK2 Cap parameters to be stored for the plastic-capped model.

SI2P Second plastic deviatoric strain invariant to be stored for the plastic-capped model.

ICOMPS A flag.

ICOMPS = 1 for variable modulus model

ICOMPS = 2 for plastic-capped model

NELTYP Type of problem

NELTYP = +3 for plane stress problem

= -3 for plane strain problem

= 4 for axisymmetric problem

> 4 for general 3-D problem

C A 6 x 6 incremental stress/strain transformation matrix.

$$- d\sigma = C de$$

For plane stress, plane strain, and axisymmetric problems the first 4 x 4 block of the C-matrix is used.

ICRACK A counter designating number of cracked planes for variable-modulus model.

ICRACK = 0 (uncracked concrete)
 = 1 (1 plane cracked)
 = 2 (2 \perp planes cracked)
 = 3 (3 \perp planes cracked)

NSPLIT A flag designating stress increment or strain increment input. The flag also designates the number of splits made to the stress or strain increment (for variable modulus model only).

If NSPLIT < 0, strain increment input and
 |NSPLIT| = Number of splits

If NSPLIT > 0, stress increment input and
 |NSPLIT| = Number of splits

The following quantities must be initialized to zero. The material package updates these quantities.

STRESS, STRAIN, EPSL, DC^{*}, ZK1^{**}, ZK2^{**}, SI2P^{**}, ICRACK^{*}

The following quantities must be stored for each element (or points where stress is calculated).

STRESS, STRAIN, EPSL, DC^{*}, ZK1^{**}, SI2P^{**}, ICRACK^{*}, AST^{*}, DCS^{*}, NRB^{*}

^{*}Applicable for variable modulus model only.

^{**}Applicable for plastic-capped model only.

Table VIII. Definition of Arguments of Program MATPAC

Argument	Size	Definition
STRESS	6	Stress components. The routine updates the stress.
STRAIN	6	Strain components. The routine updates the strains.
STRINC	6	Strain increments or stress increments based on NSPLIT
EPSEL	6	Elastic strains.** The routine updates the elastic strains.
CØEFF*	15	Coefficients defining material property (see next page).
BULKMOD**	1	Bulk modulus of concrete.
SHRMØD**	1	Shear modulus of concrete.
YCØF**	25	Yield coefficients defining capped yielding of concrete
DC*	3 x 3	Direction cosines of cracked directions. (If NRB > 0)
DCS*	3 x 3	Direction cosines for steel directions.
AST*	3	Steel ratios as fraction of cross-sectional area
NRB*	1	A flag. NRB ≤ 0 steel in global coordinates (DCS not necessary). NRB > 0 steel in direction DCS.
ZK1**	1	Constant in first yield surface.
ZK2**	1	Constant in second yield surface.
SI2P**	1	Second plastic deviatoric strain invariant.
ICØMPS	1	A flag. ICØMPS = 2, for capped plastic model. ICØMPS = 1, for variable modulus model.
NELTYP	1	Type of problem. NELTYP = 3 for plane stress = -3 for plane strain = 4 for axisymmetric > 4 for 3-dimensional stress
C	6 x 6	Tangent stiffness relating stress/strain increments ($d\sigma = C d\epsilon$).
ICRACK*		A counter designating number of cracked planes.
NSPLIT	1	A flag. NSPLIT < 0, strain increment input option NSPLIT > 0, stress increment input option The stress or strain increments are subdivided by NSPLIT . For better accuracy NSPLIT may be increased.

*Applicable for variable modulus model only.

**Applicable for plastic capped model only.

The following quantities must be stored for each different material.

COEFF*, BLKMOD**, SHRMOD**, YCOF**

The following flags must be appropriately set.

ICOMPS to exercise the variable modulus model or the plastic-capped
 model

NSPLIT to exercise the stress increment input or the strain increment
 input options

3. SUBROUTINES AND THEIR FUNCTIONS

The material property package contains 22 subroutines. The functions of each of these routines are described below. The routines appear in alphabetical sequence.

(1) Subroutine CADJST

CADJST adjusts the C-matrix defining incremental stress/strain relationship for plane-stress problems. CADJST is called from subroutine ELFUNN.

(2) Subroutine CHANGE

CHANGE interchanges orthotropic coordinate axes such that axis normal to cracked planes always precede axis normal to uncracked planes. CHANGE is called from subroutine CRACKS.

(3) Subroutine COMPOS

COMPOS calculates composite modulus of steel and concrete in directions normal to cracks according to equation 79. COMPOS is called from subroutine VARMOD.

*Applicable for variable modulus model only.

**Applicable for plastic-capped model only.

(4) Subroutine COMPUT

COMPUT computes incremental stress by premultiplying incremental strains by the C-matrix. It then updates the stress vector. COMPUT is called from subroutine ELFUNN.

(5) Subroutine CRACKS

CRACKS checks for crack-initiation in principal orthotropic directions. CRACKS calls subroutine CHANGE to reorder coordinate system. CRACKS is called from subroutine ELFUNN.

(6) Subroutine CTRANF

CTRANF transforms C-matrix from orthotropic to global coordinate directions. Transformation procedures are described in section IV. CTRANF is called from ELFUNN.

(7) Subroutines DOWEL1 and DOWEL2

DOWEL1 and DOWEL2 calculate shear stiffness due to dowel action. These routines are called from subroutine VARMOD.

(8) Subroutine ELFUNN

ELFUNN is a major subroutine of the package. ELFUNN calls PRNSTR to evaluate principal stresses and directions, calls TRANSF to evaluate coordinate transformation matrices, calls MATPLY to convert stress and strain from one coordinate system to another, calls MAXMIN to evaluate principal stress directions in a plane, calls CRACKS to determine cracked planes if any, calls STLTRN to transform steel ratios from steel directions to principal orthotropic directions, calls VARMOD to evaluate the C-matrix in principal orthotropic directions, calls CTRANF to convert C-matrix from principal orthotropic direction to global direction, calls COMPUT to compute incremental stress from incremental strains, and calls PLASTK for the capped plastic model computations. ELFUNN is called from MATPAC.

(9) Subroutine FORMC

FORMC formulates the C-matrix (Eq. 71) in the principal orthotropic directions. FORMC is called from subroutine VARMOD.

(10) Subroutine JACOBI

JACOBI is a standard subroutine to find eigenvalues and eigenvectors. It is used here to find the principal stresses and their directions. JACOBI is called from subroutine PRNSTR.

(11) Subroutine LODUN

LODUN determines loading or unloading/reloading in directions normal to cracked planes. LODUN is called from subroutine VARMOD.

(12) Subroutine MATPAC

MATPAC controls the material package program. It bifurcates the program for the stress and strain increment options. It calls ELFUNN to obtain stress from strain increment and SOLVEX to convert stress increment to corresponding strain increments.

(13) Subroutine MATPLY

MATPLY is a matrix multiplication routine used to transform stress and strain from one coordinate system to another. MATPLY is called from subroutine ELFUNN.

(14) Subroutine MAXMIN

MAXMIN calculates the principal stress directions in a plane. MAXMIN is called from subroutine ELFUNN.

(15) Subroutine MODULI

MODULI calculates the tangent modulus of uncracked concrete in compression according to equation 29. MODULI is called from subroutine VARMOD.

(16) Subroutine PLASTK

PLASTK calculates stress increments from strain increments based on the plastic capped model. PLASTK is called from subroutine ELFUNN.

(17) Subroutine PRNSTR

PRNSTR evaluates principal stresses and directions. PRNSTR calls JACOBI to solve the general eigenvalue problem. PRNSTR is called from ELFUNN.

(18) Subroutine REBOND

REBOND checks for rebonding after a crack is initiated. If rebonding occurs, the concrete modulus is set to a user-specified fraction of the initial concrete modulus. REBOND is called from VARMOD.

(19) Subroutine SOLVEX

SOLVEX is an equation-solver routine using Gaussian reduction techniques. SOLVEX converts stress increments to corresponding strain increments. SOLVEX is called from MATPAC.

(20) Subroutine STLTRN

STLTRN transforms steel ratios from original steel directions to the principal directions of orthotropy. Transformation principles are discussed in section IV. STLTRN is called from ELFUNN.

(21) Subroutine TRANSF

TRANSF evaluates transformation matrices for stress and strain in order to change from one coordinate system to another. Transformation principles are discussed in section IV. TRANSF is called from subroutine ELFUNN.

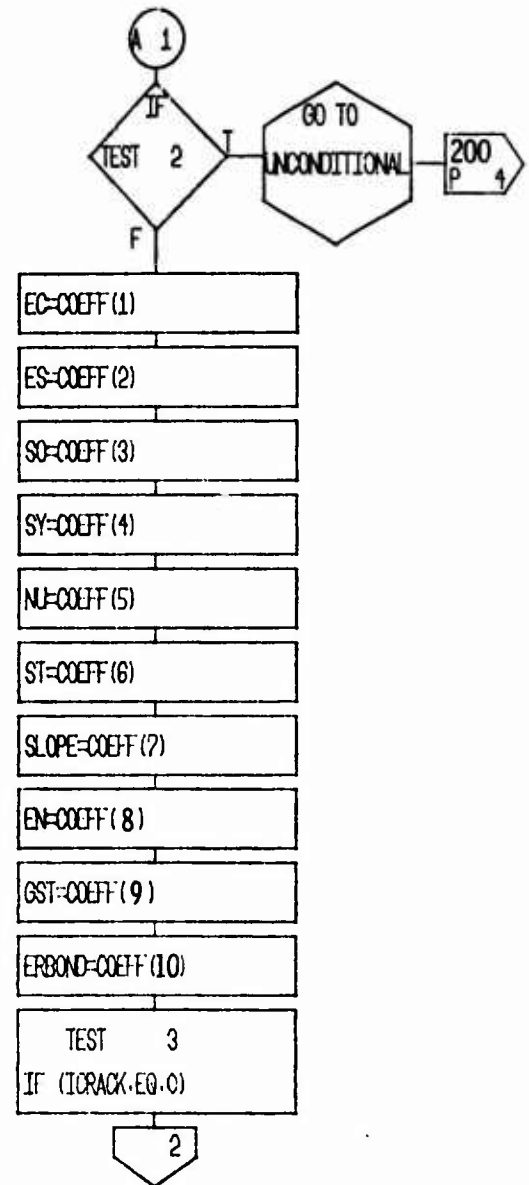
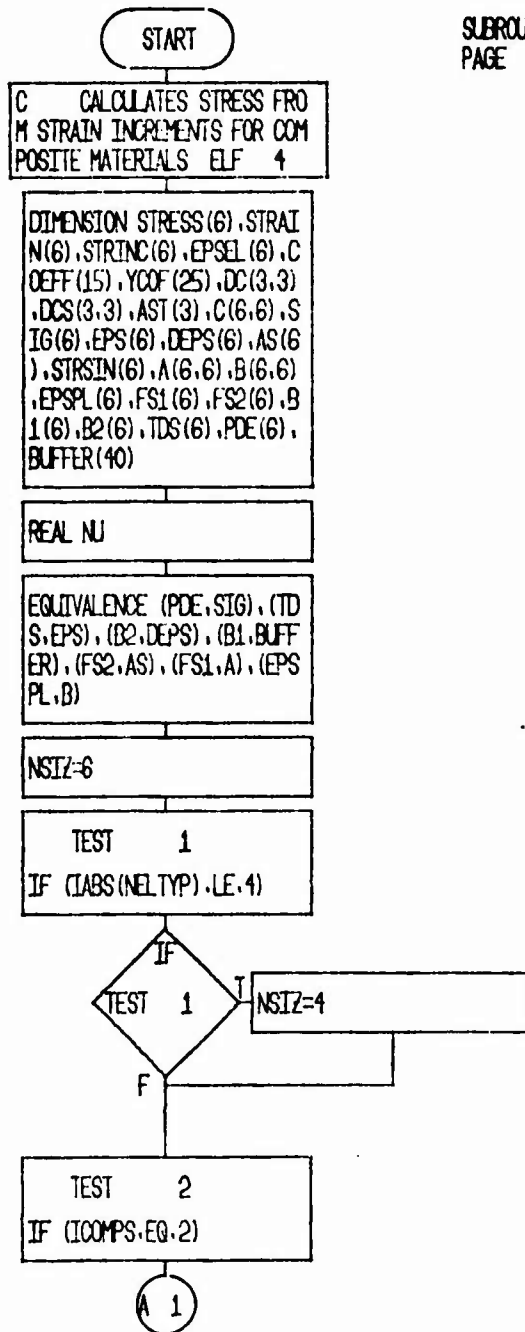
(22) Subroutine VARMOD

VARMOD computes the tangent stress-strain relationship for the variable modulus model.

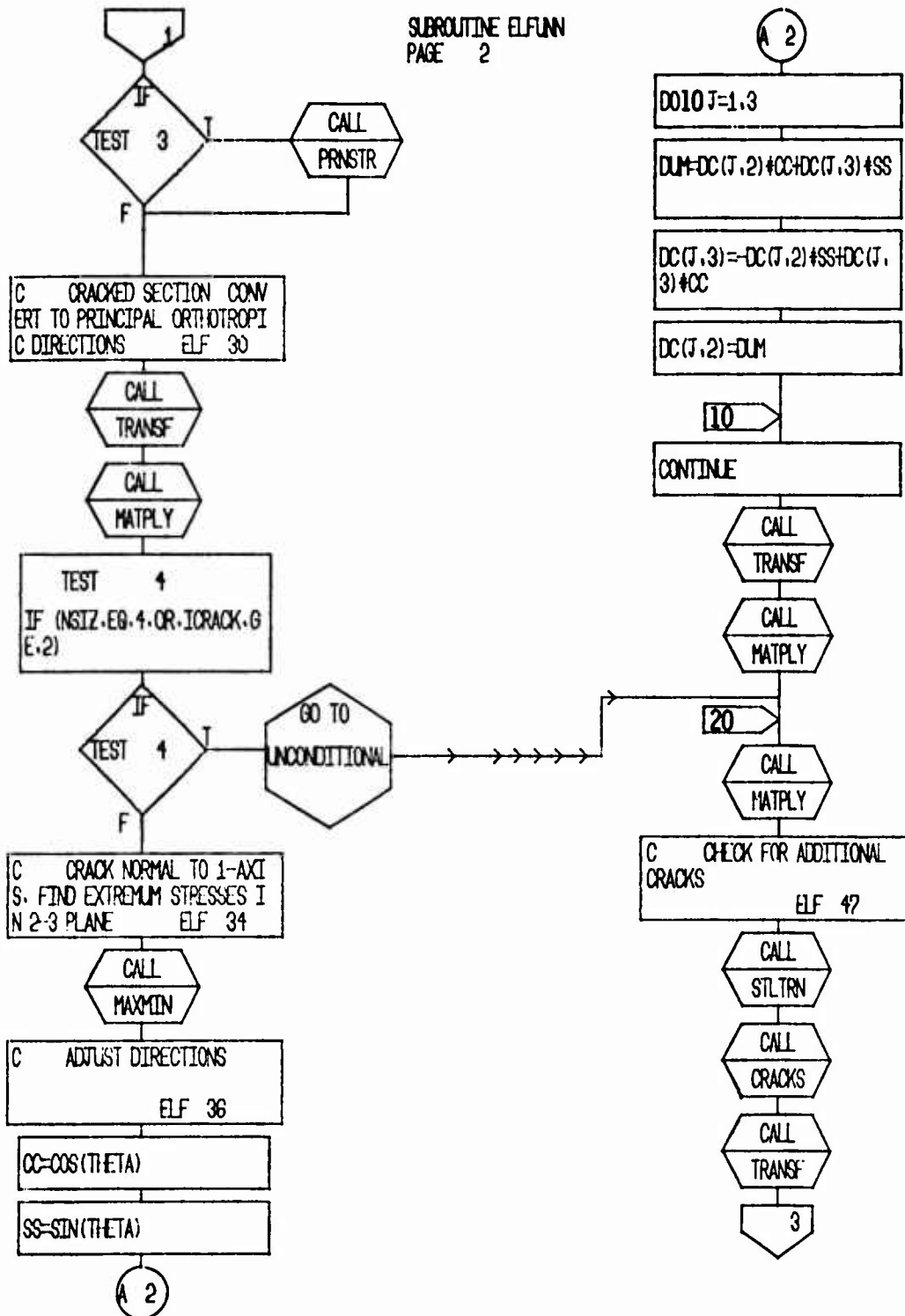
VARMOD calls LODUN to check loading/unloading in cracked directions; calls COMPOS to obtain composite modulus in directions normal to cracked planes, calls MODUL1 for tangent moduli in directions normal to uncracked planes, calls DOWEL1 and DOWEL2 for shear stiffness due to dowel action, and finally calls FORMC to assemble the C-matrix defining incremental stress-strain relationship. VARMOD is called from ELFUNN.

4. FLOW CHARTS FOR MATPAC PROGRAM

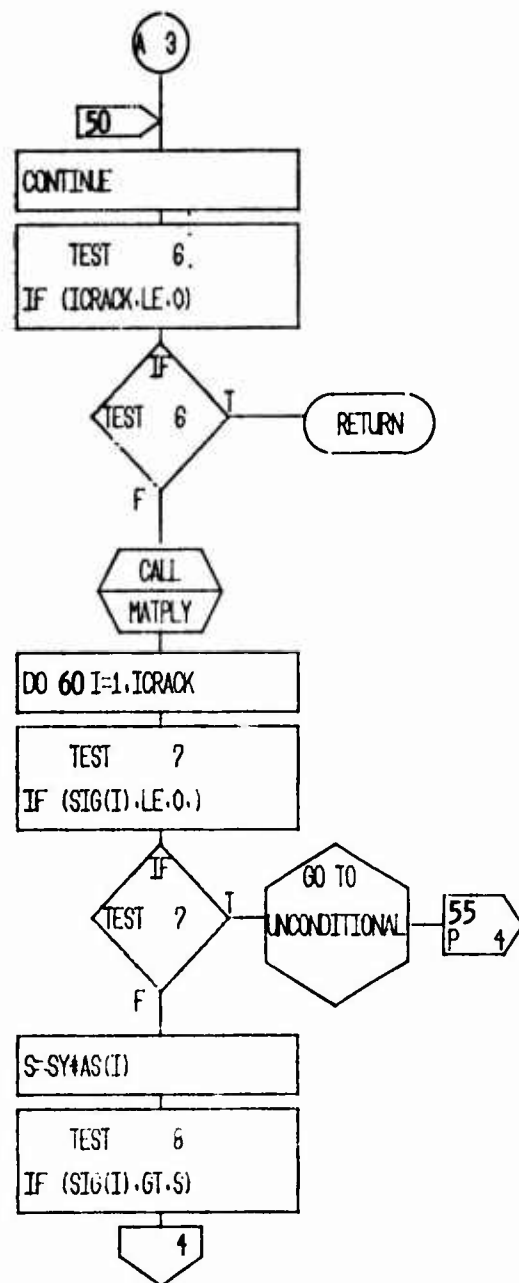
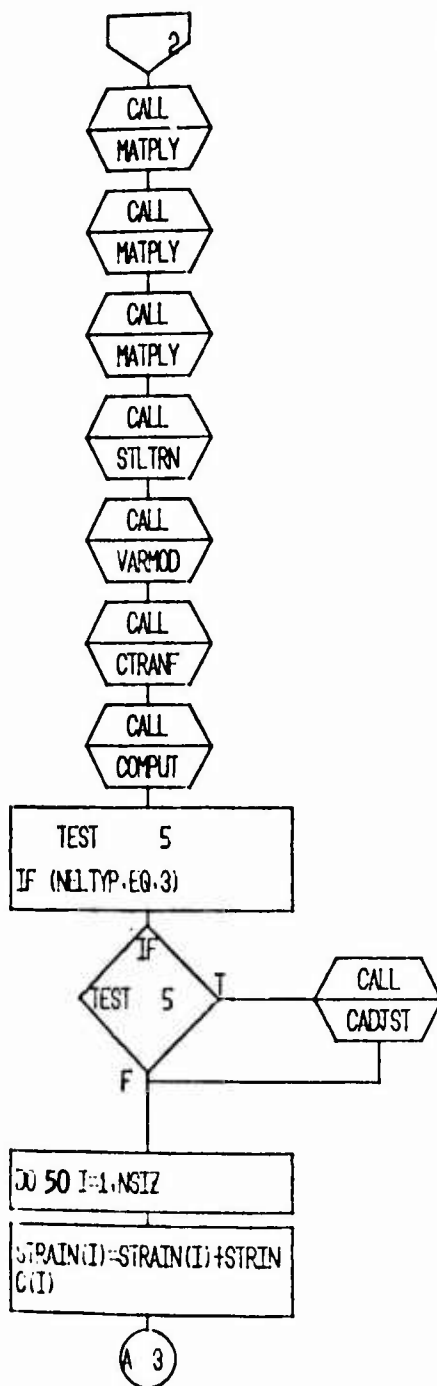
SUBROUTINE ELFUN
PAGE 1



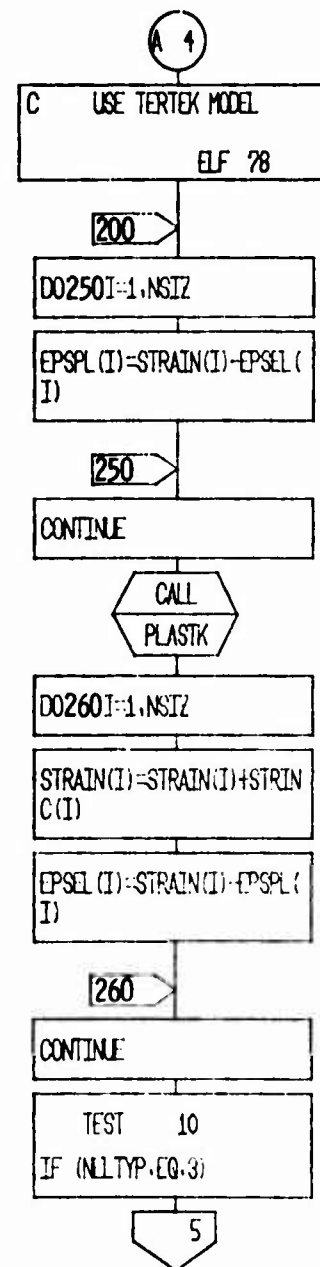
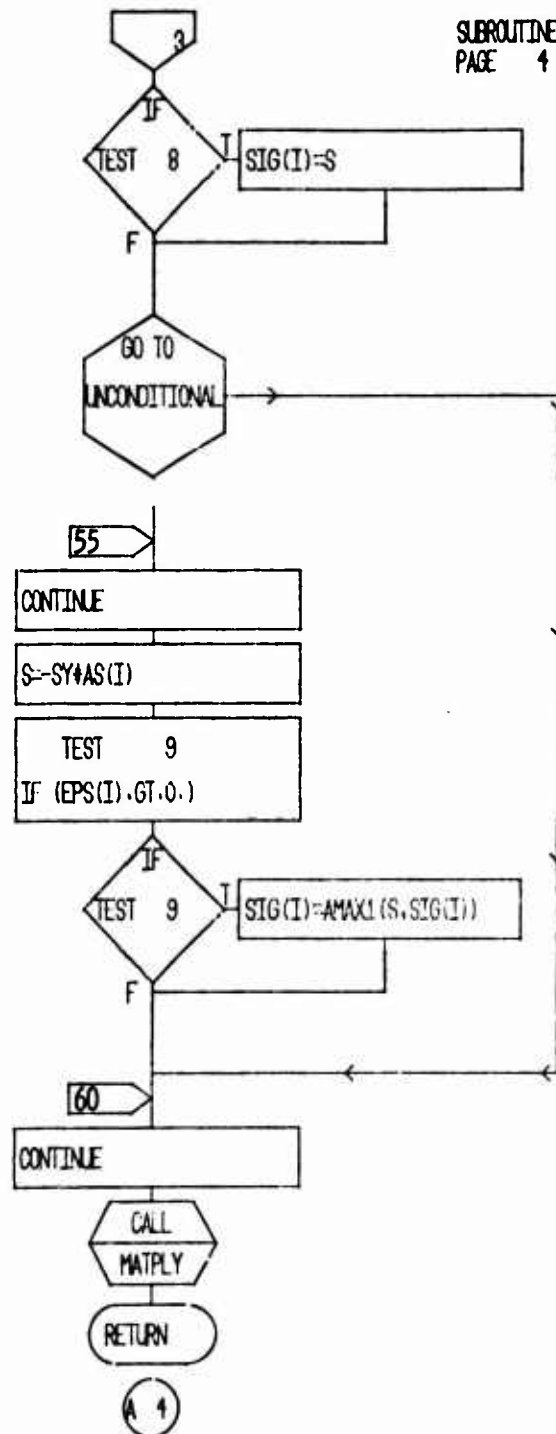
SUBROUTINE ELFUNN
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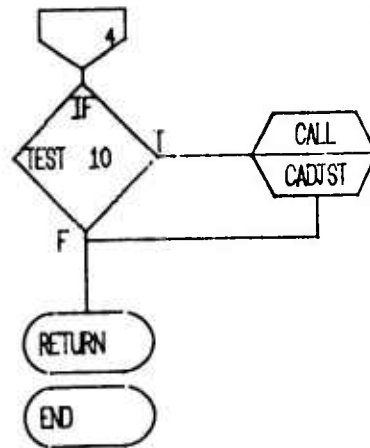
SUBROUTINE ELFUNN
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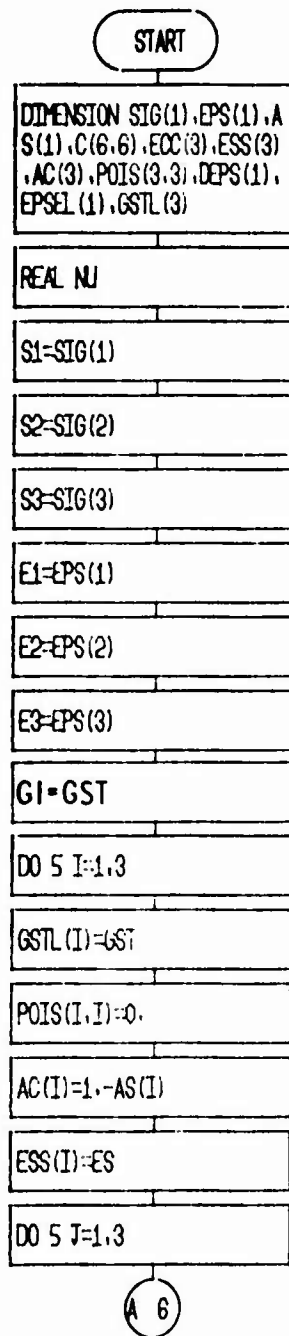


SUBROUTINE ELFUN
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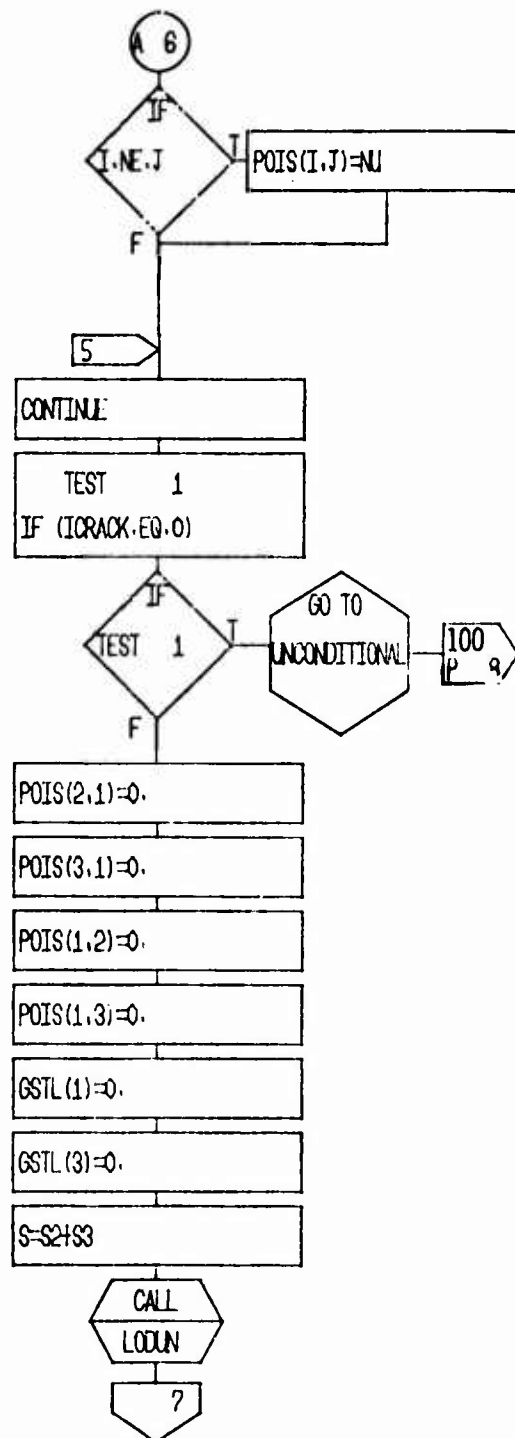


SUBROUTINE ELFUNN
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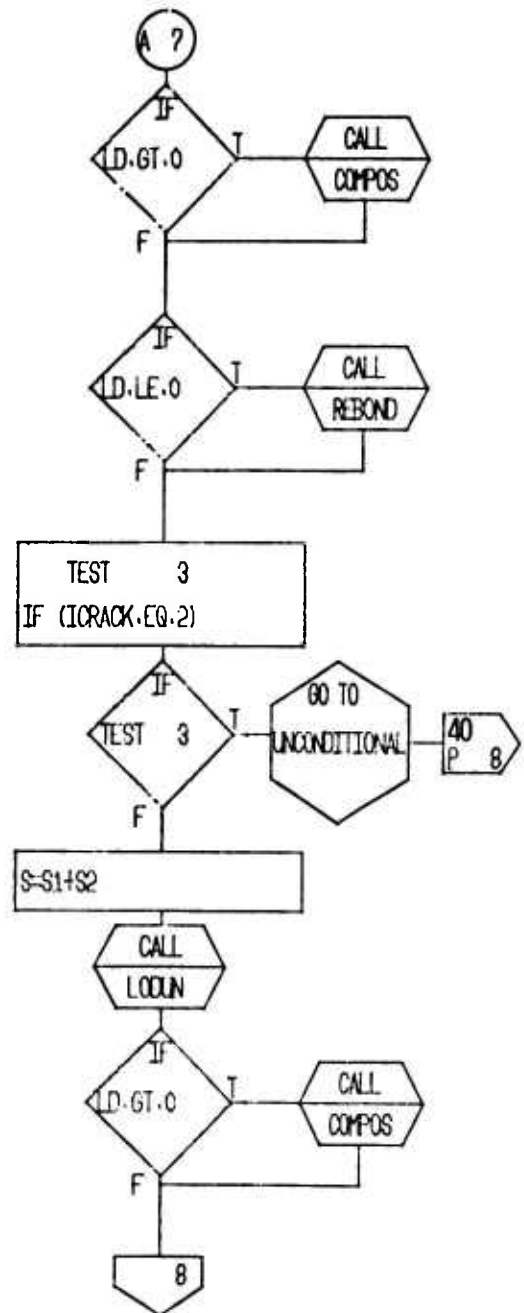
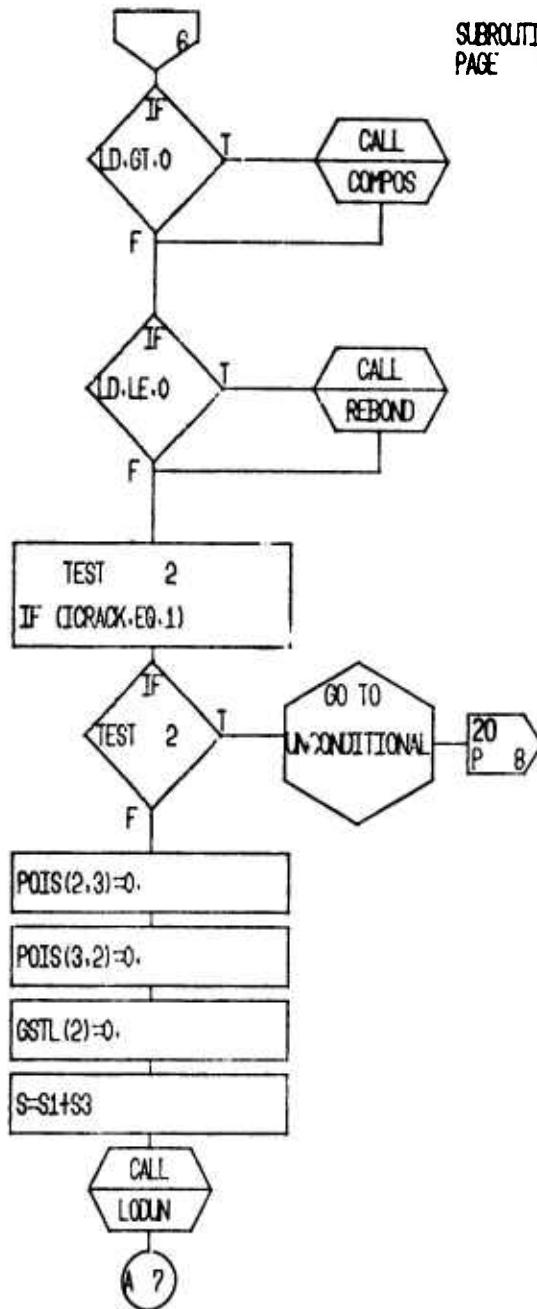




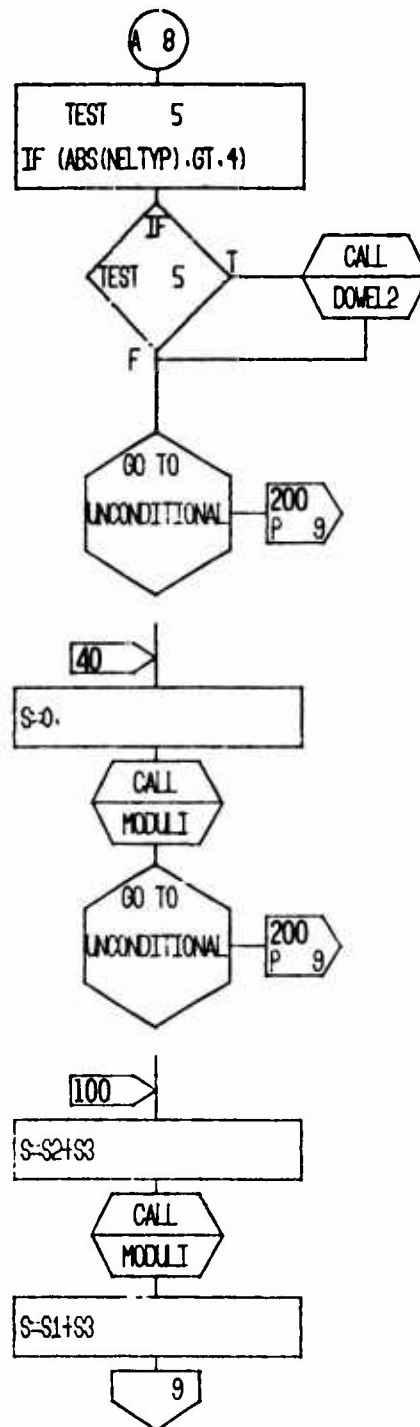
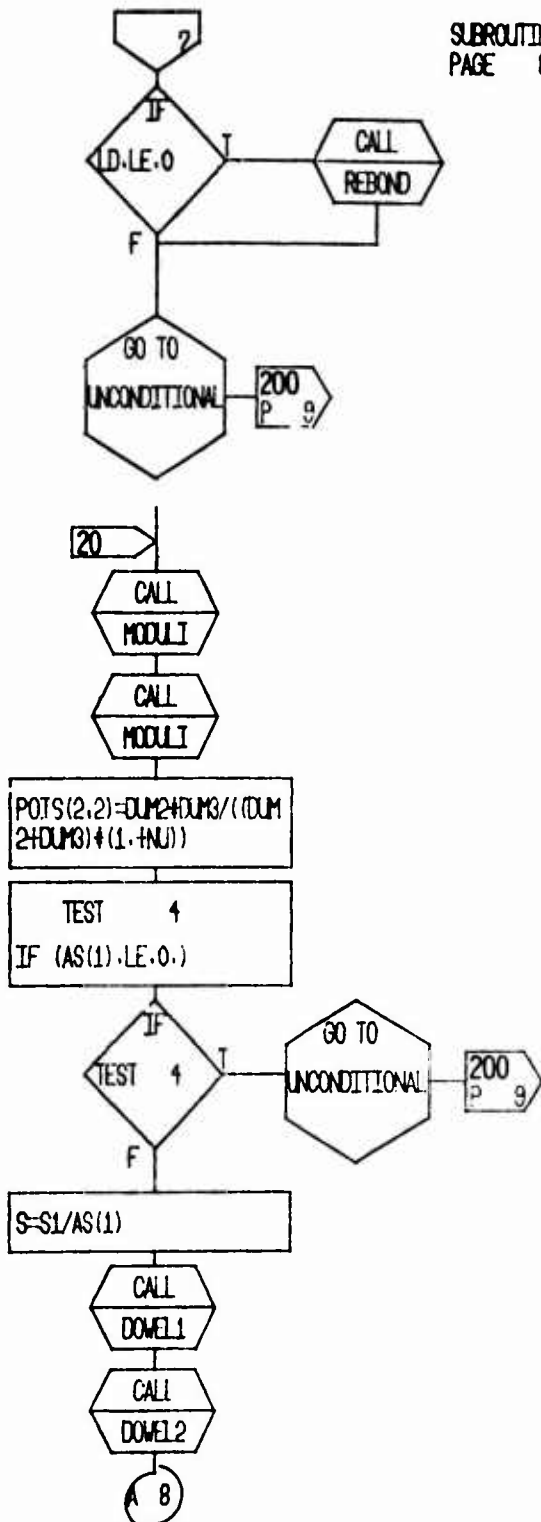
SUBROUTINE VARIO
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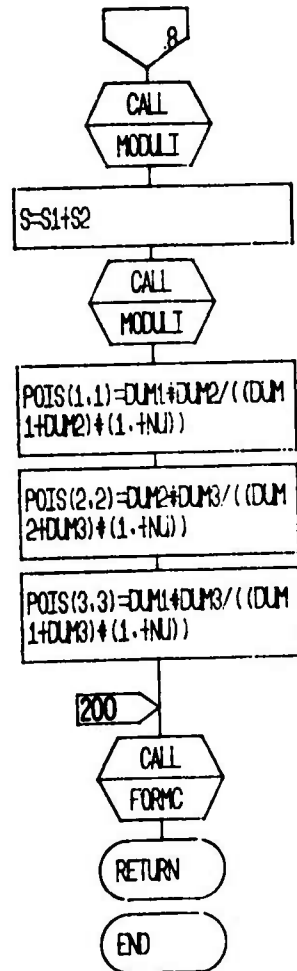
SUBROUTINE VARMOD
PAGE ?



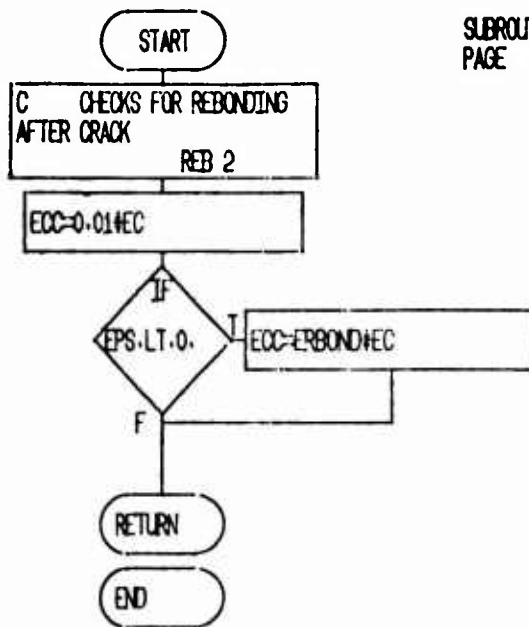
SUBROUTINE VARMOD
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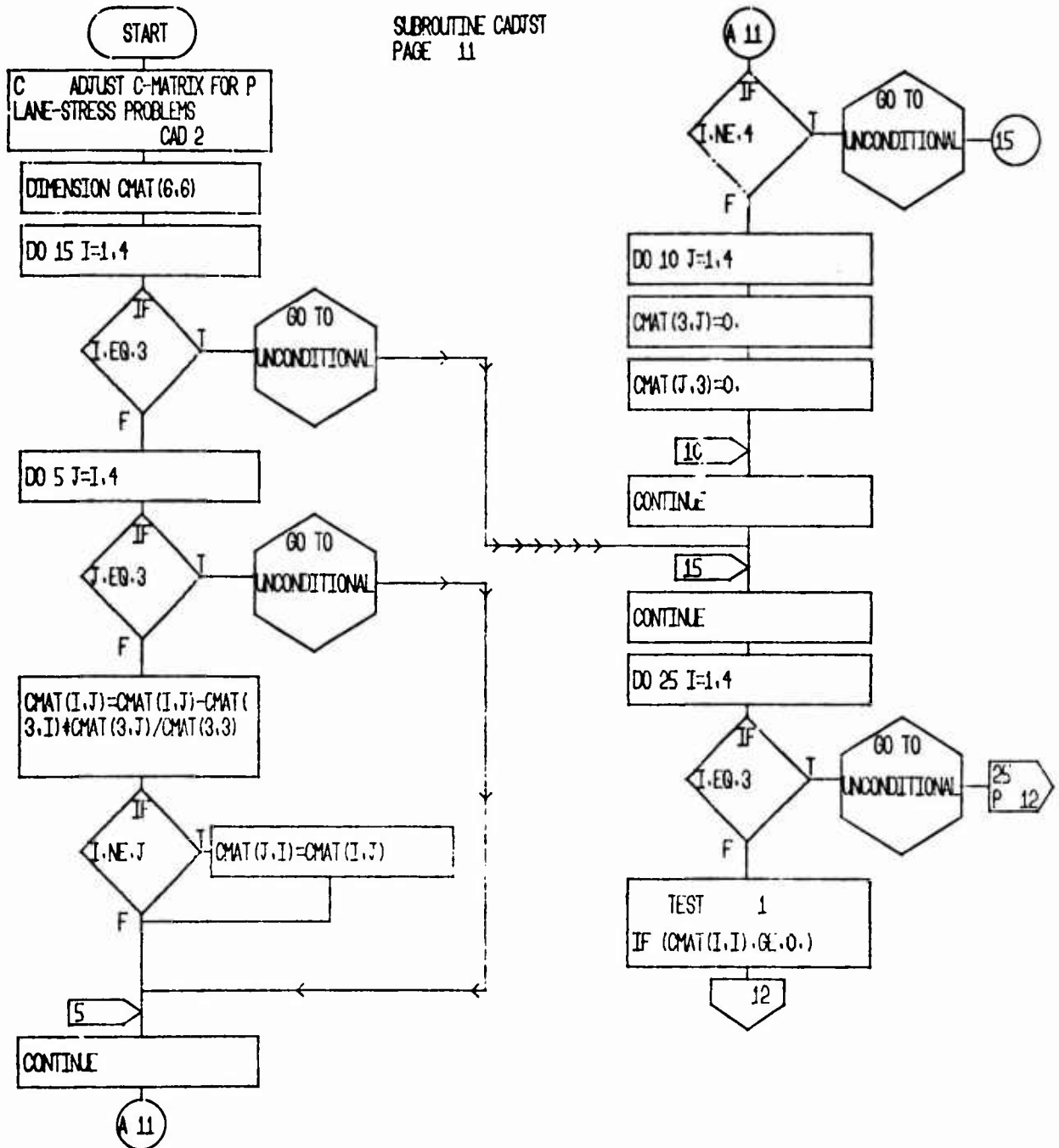
SUBROUTINE VARMOD
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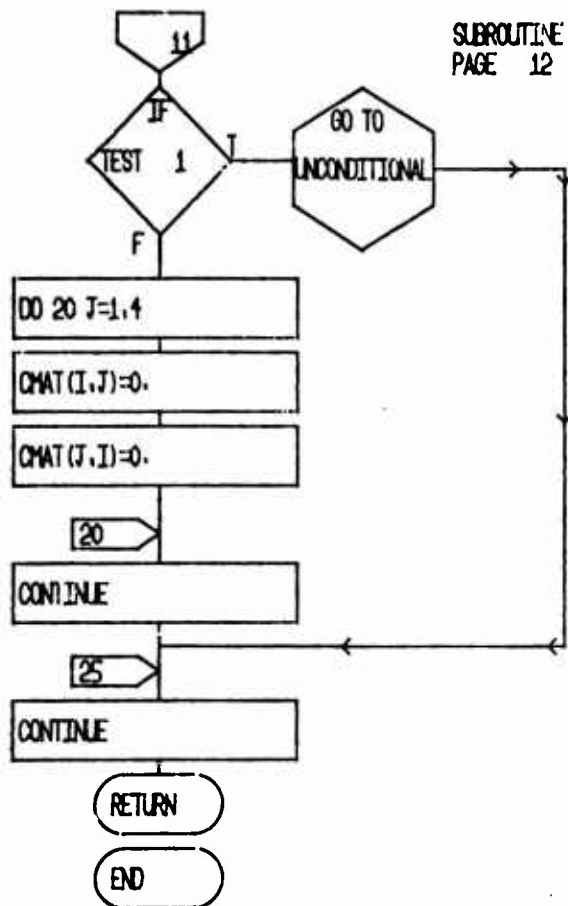
SUBROUTINE REBOND
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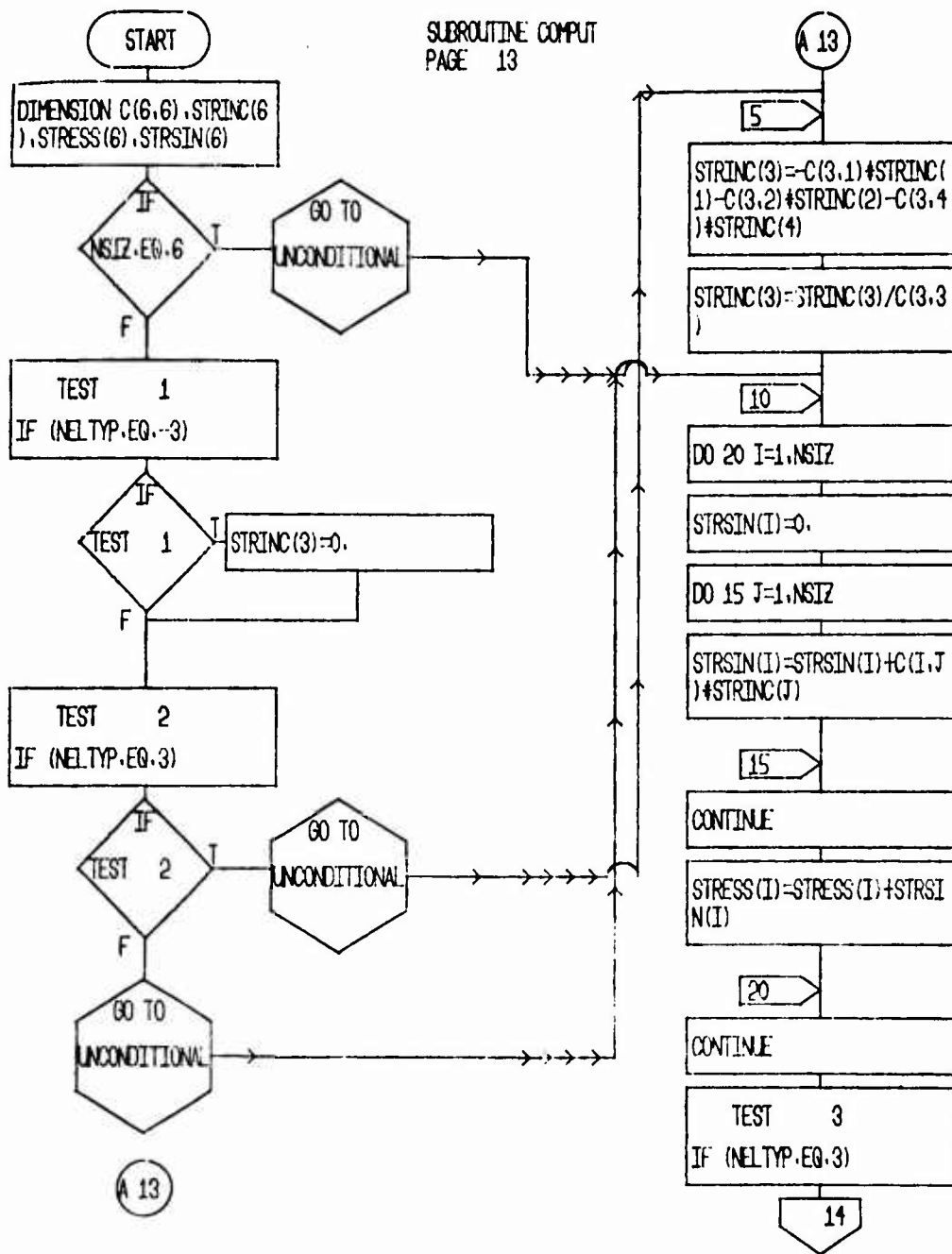


SUBROUTINE CADTST
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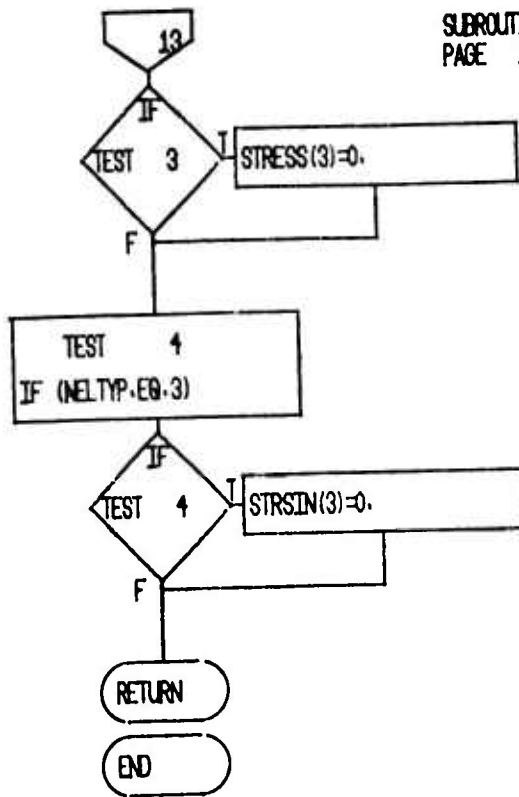


SUBROUTINE CACTST
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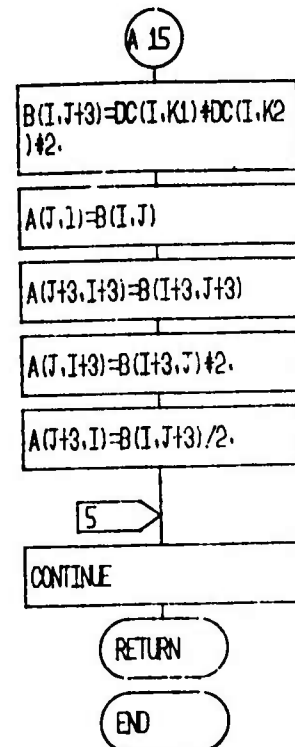
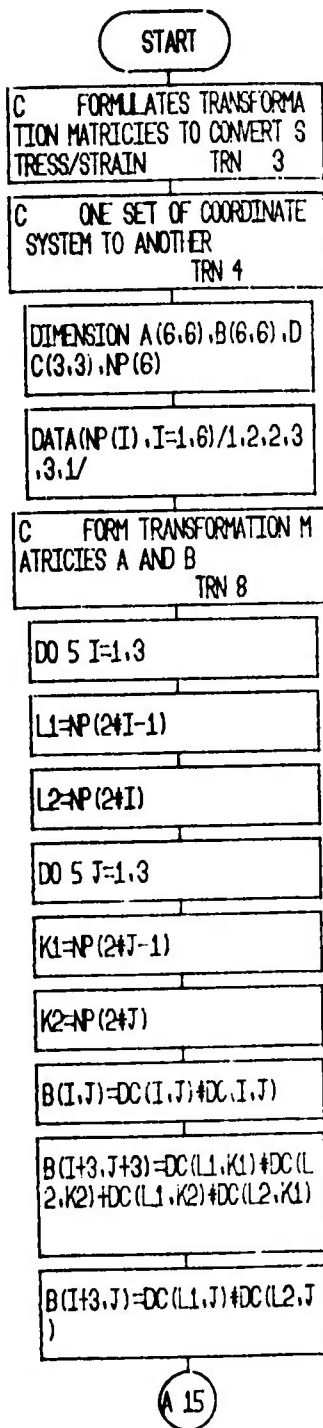




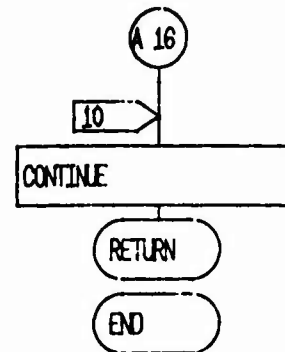
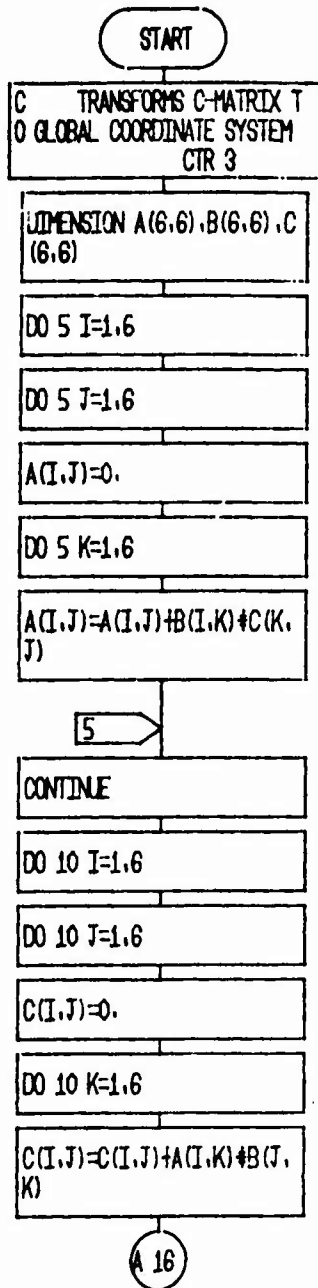
SUBROUTINE COMPUT
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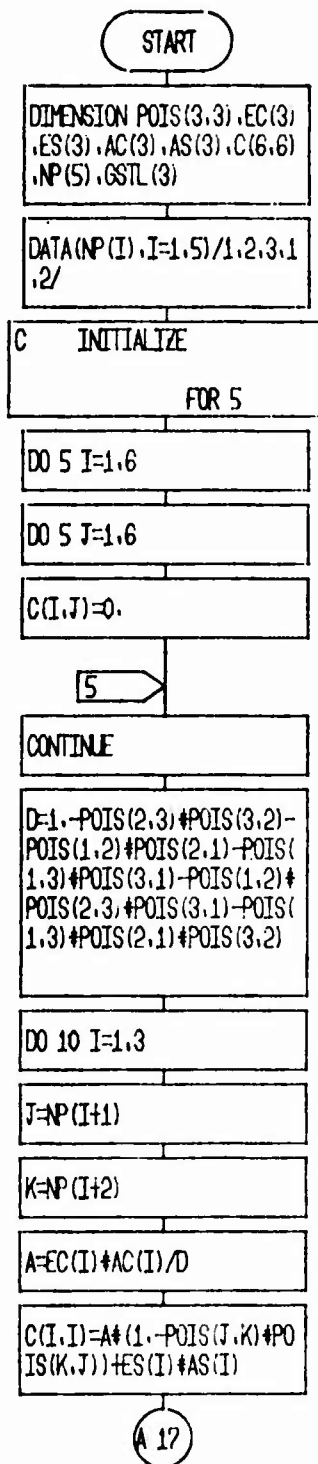


SUBROUTINE TRANSF
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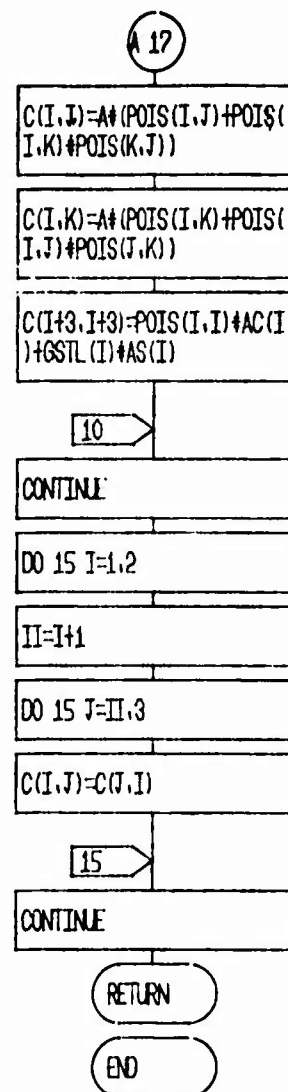


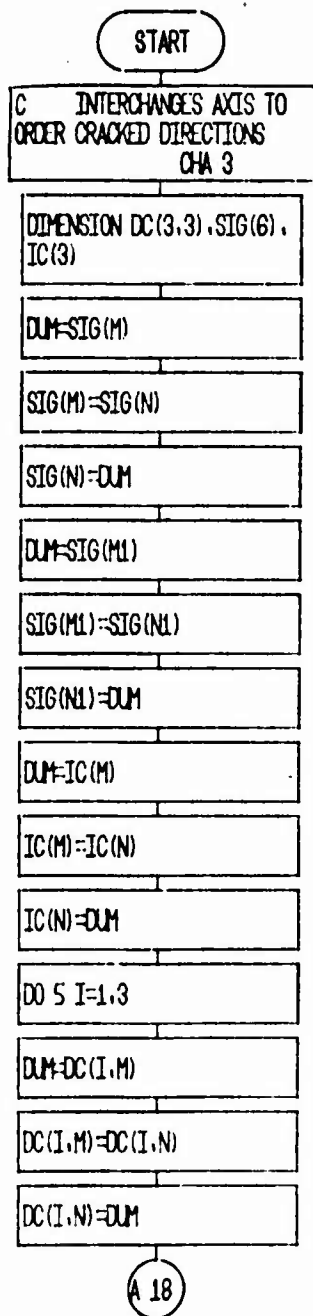
SUBROUTINE CTRANF
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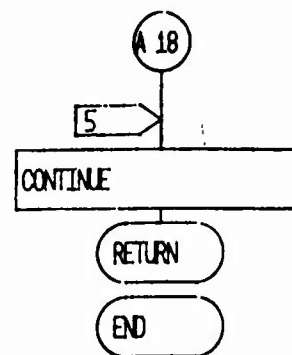


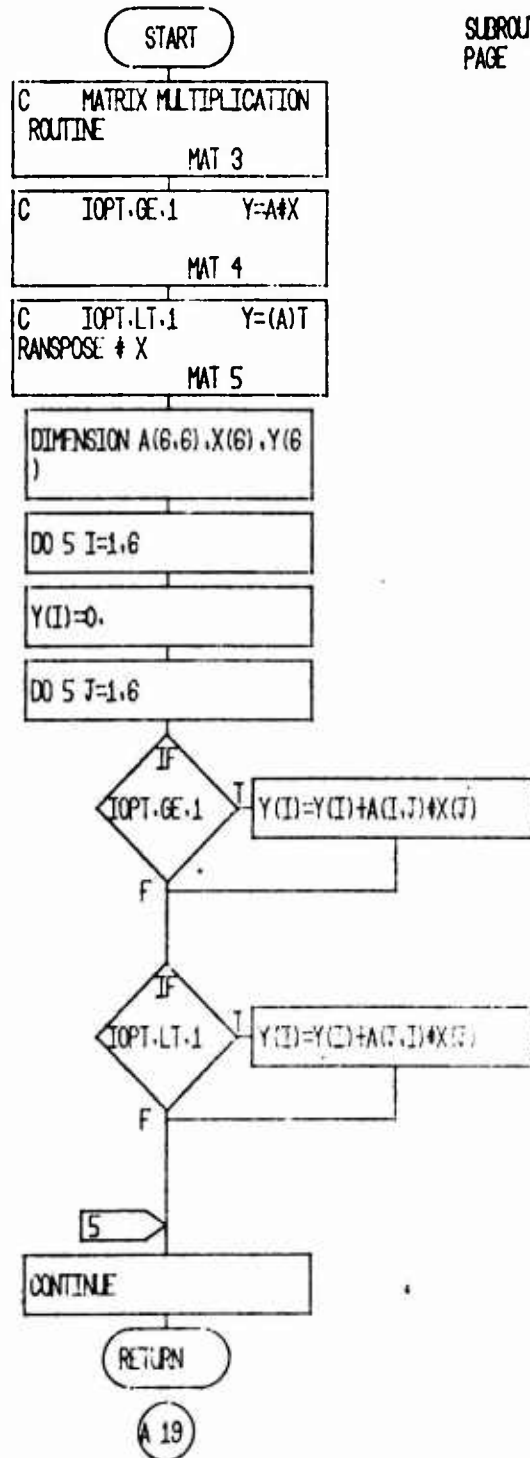
SUBROUTINE FORMC
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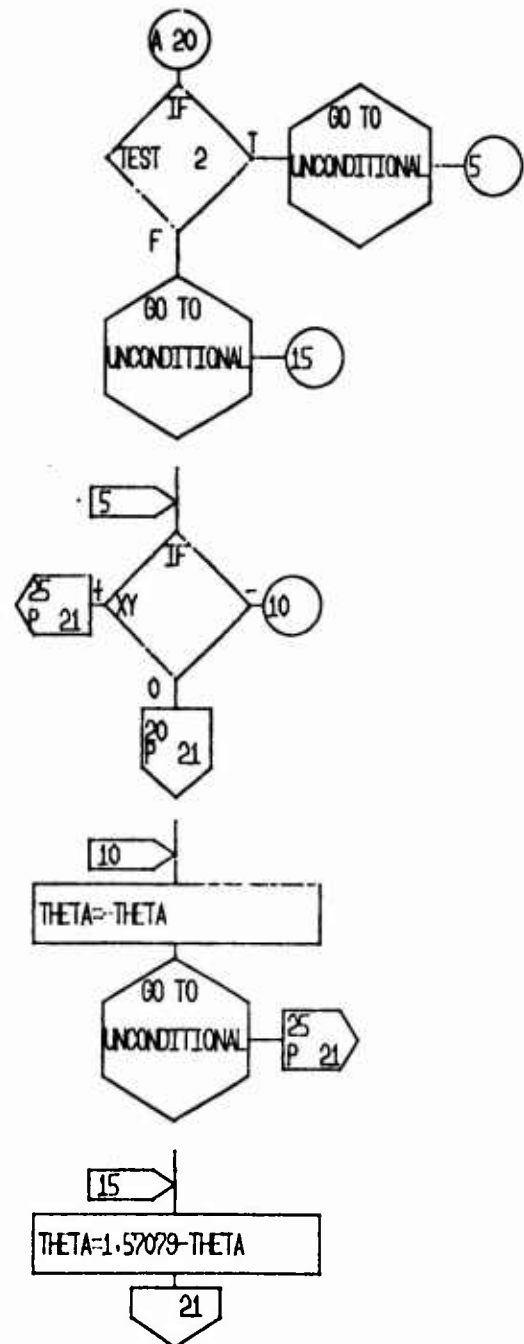
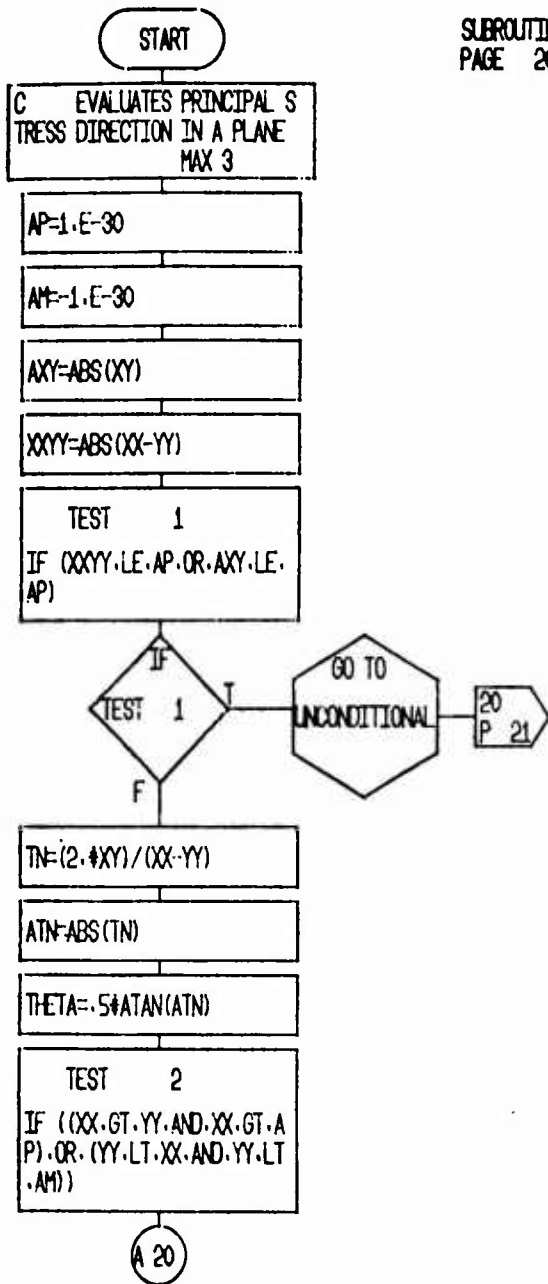


SUBROUTINE CHANGE
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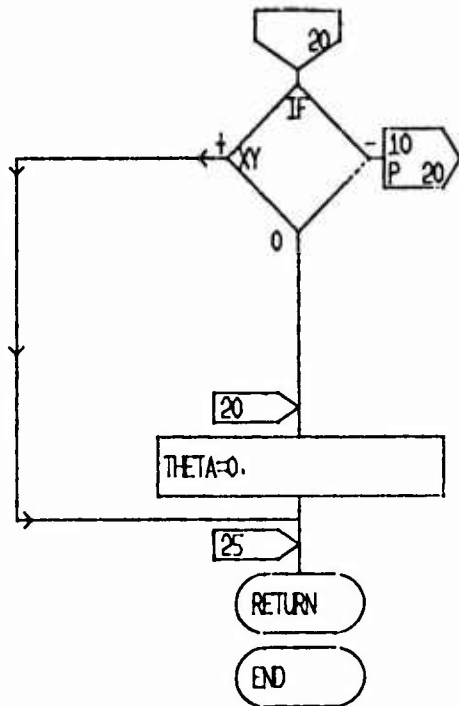




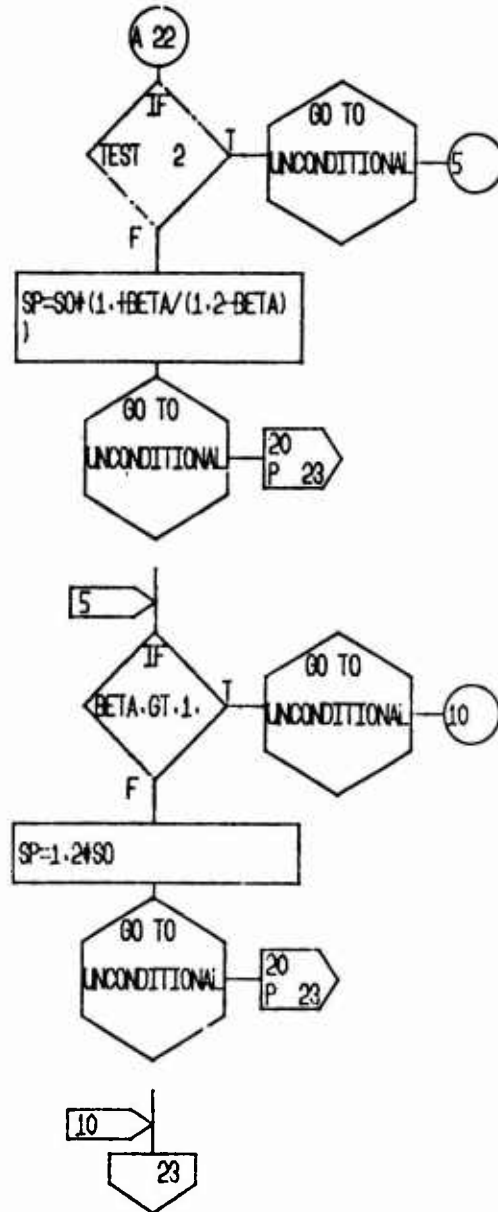
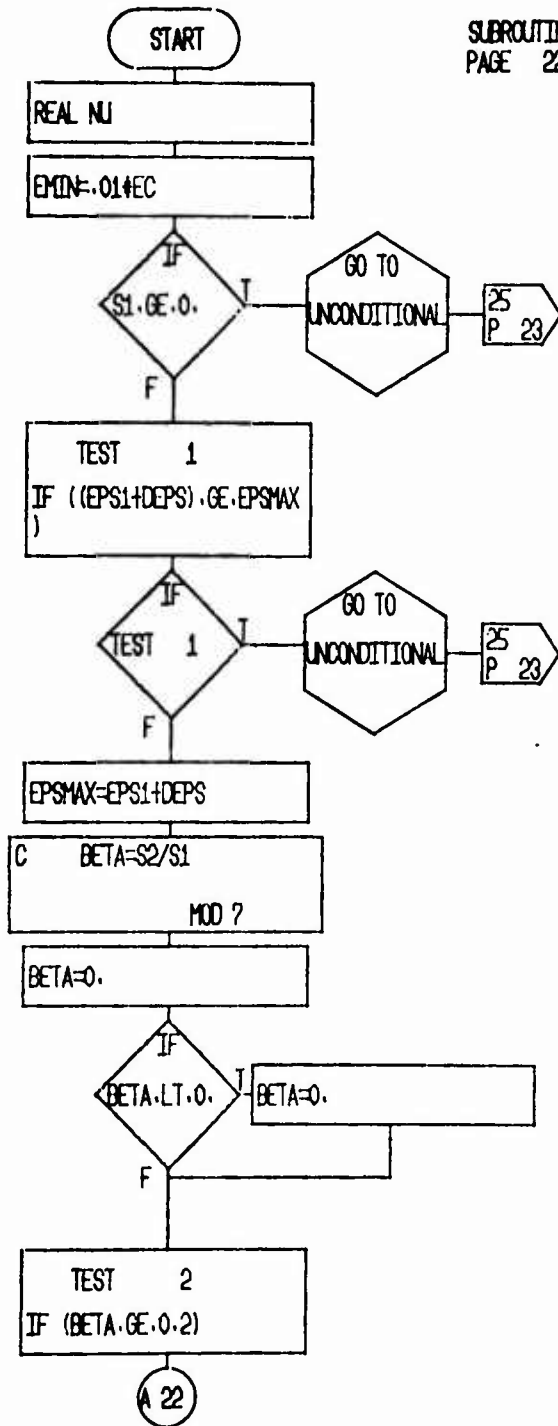
SUBROUTINE MAXMIN
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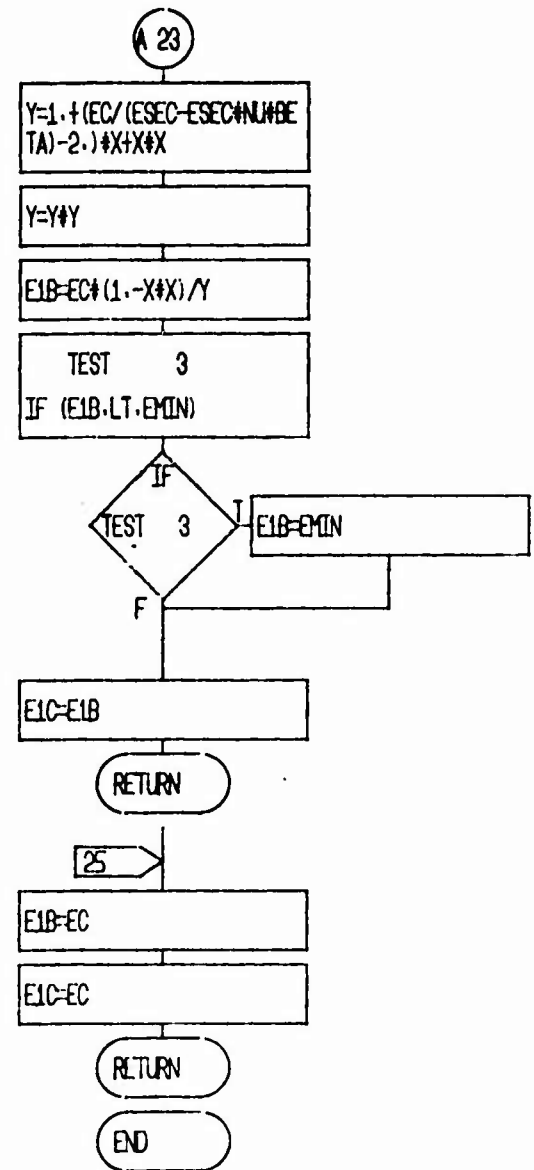
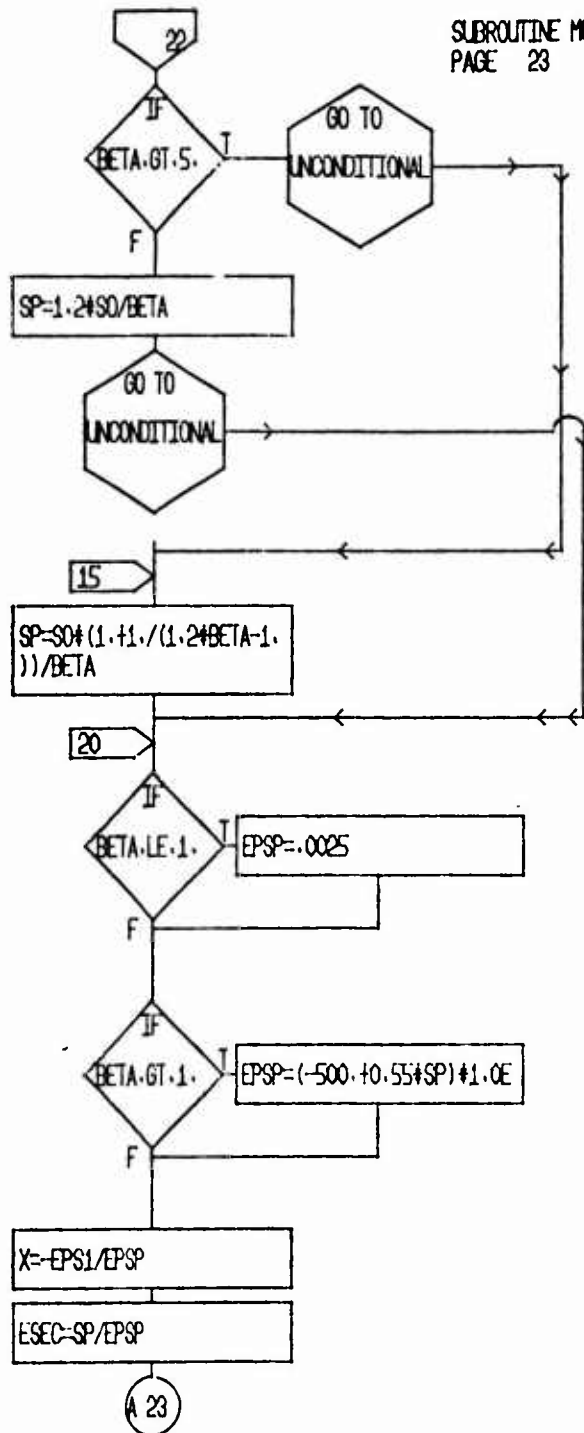
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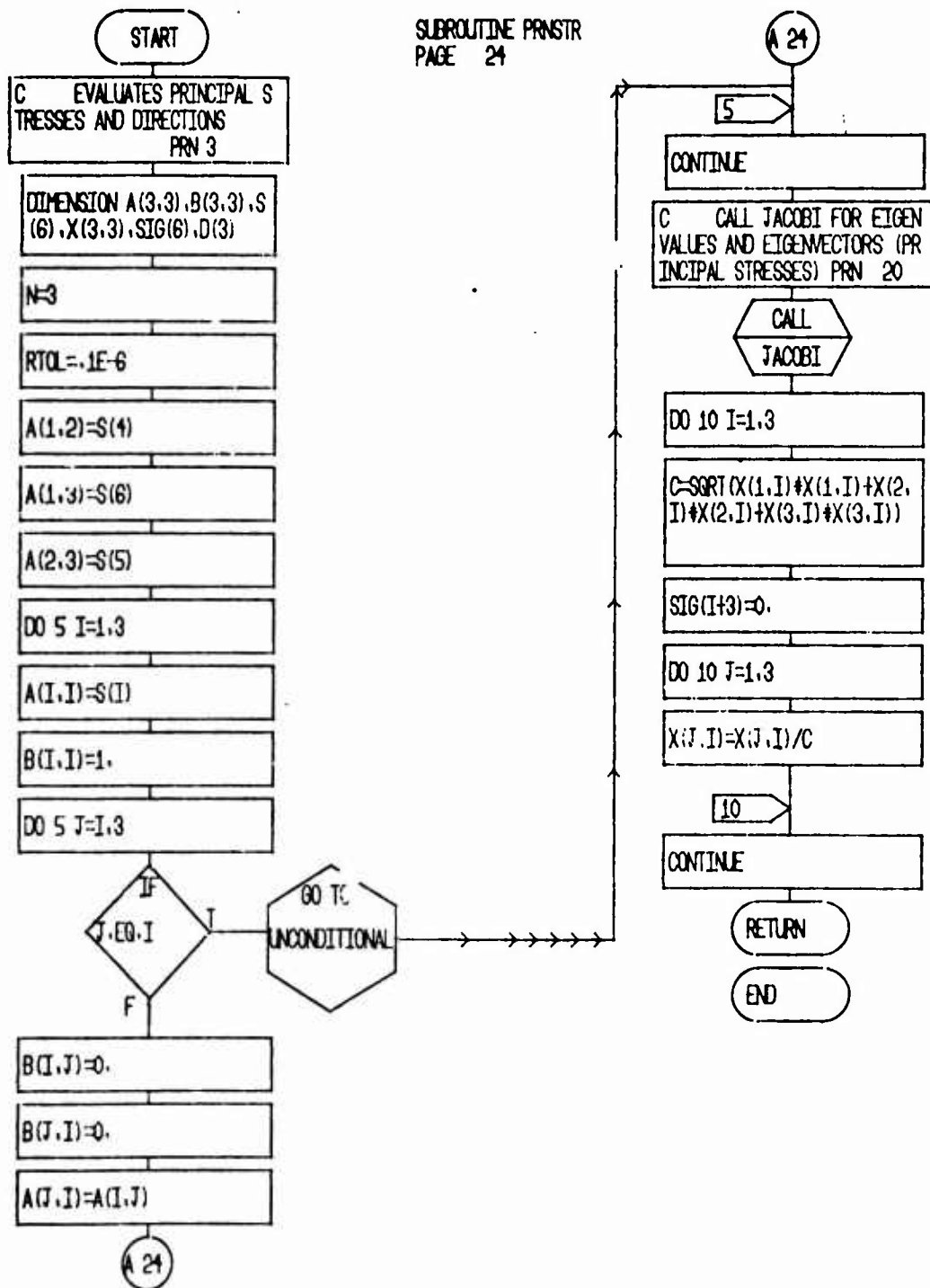


SUBROUTINE MODUL
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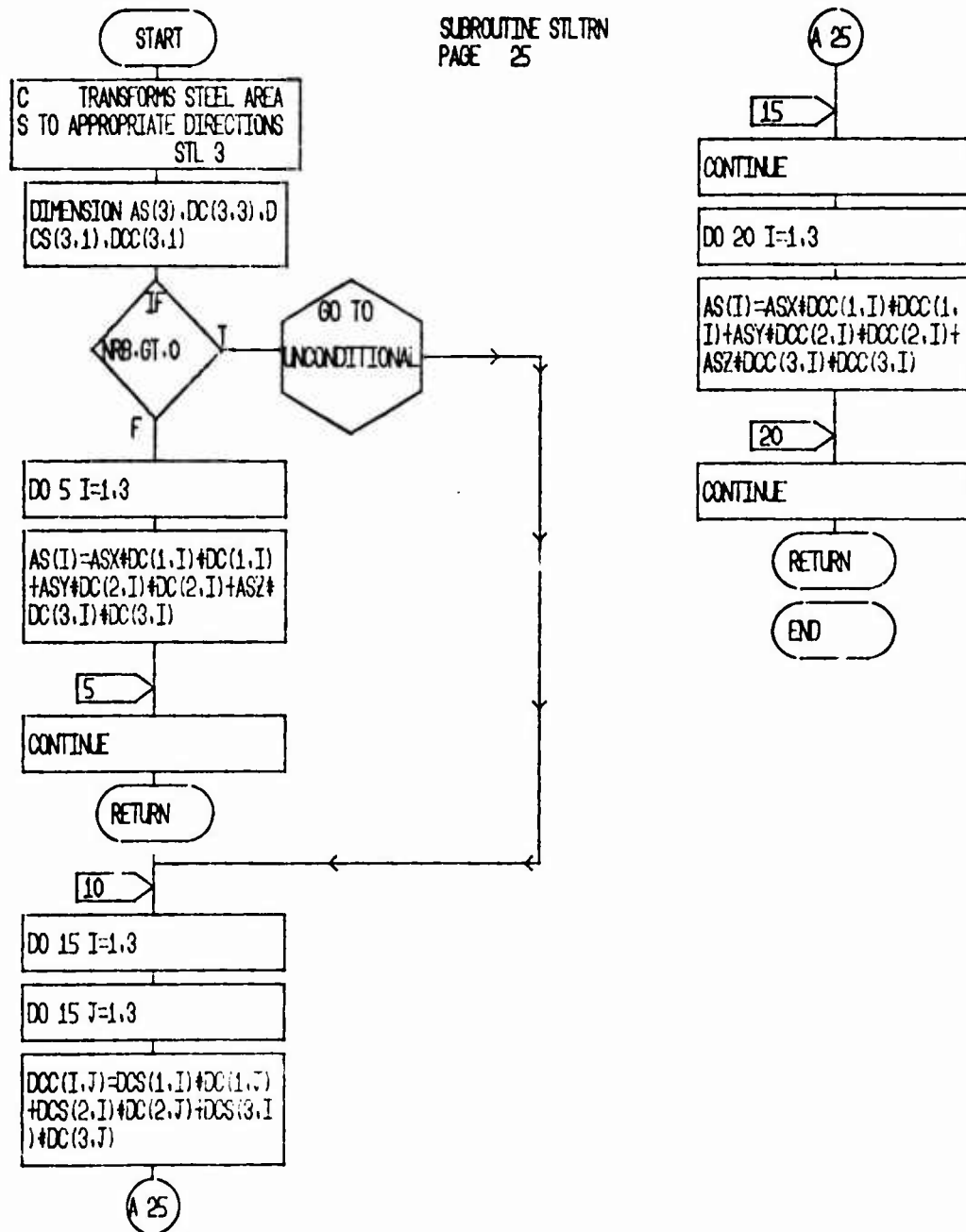


SUBROUTINE MODULI
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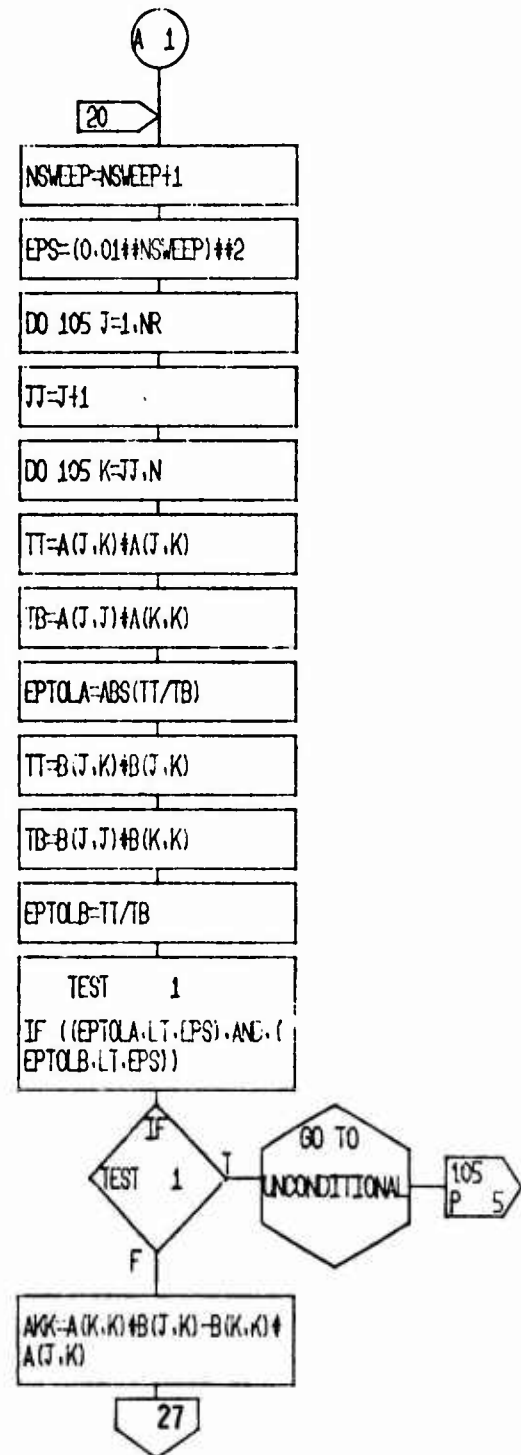
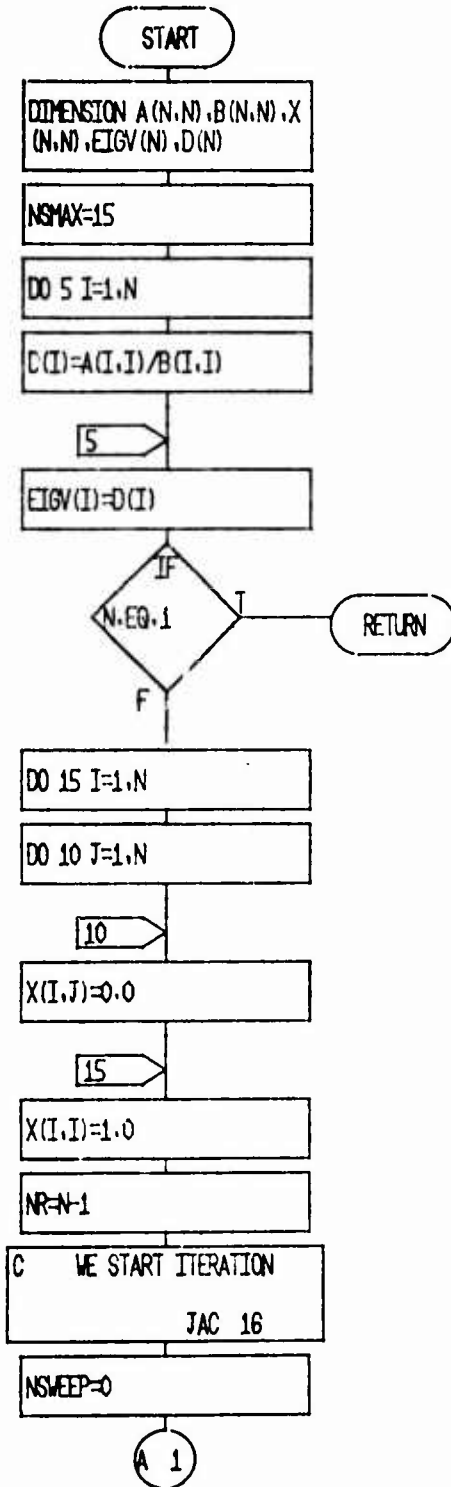




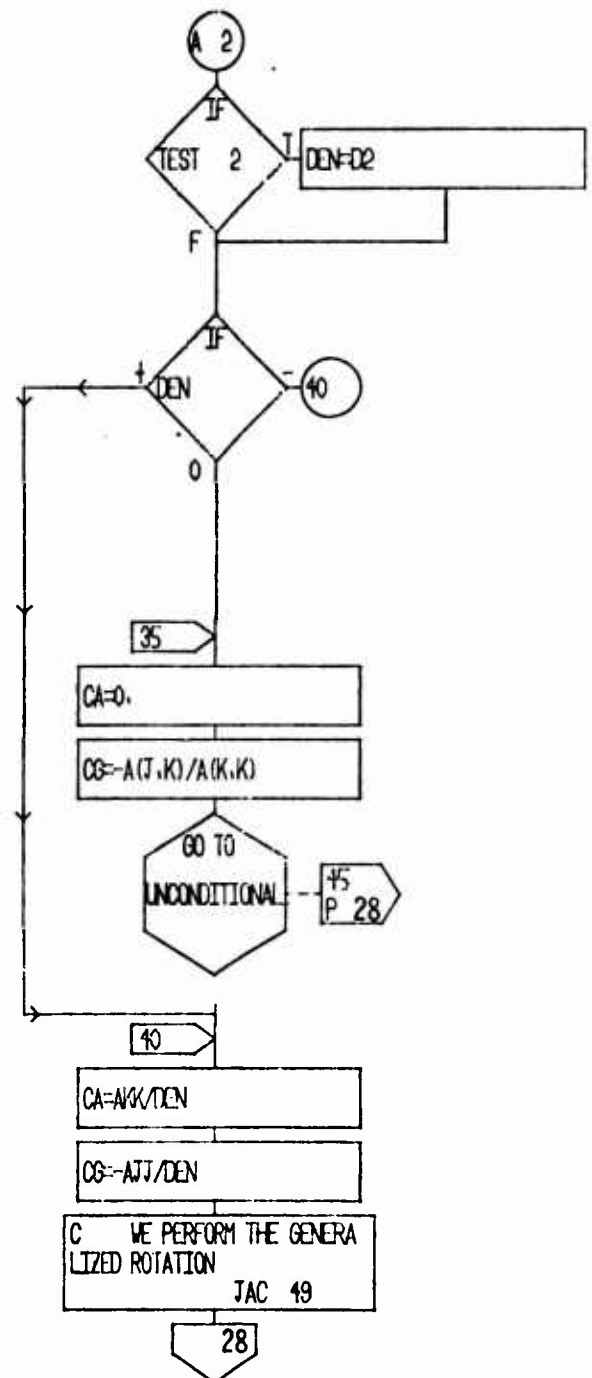
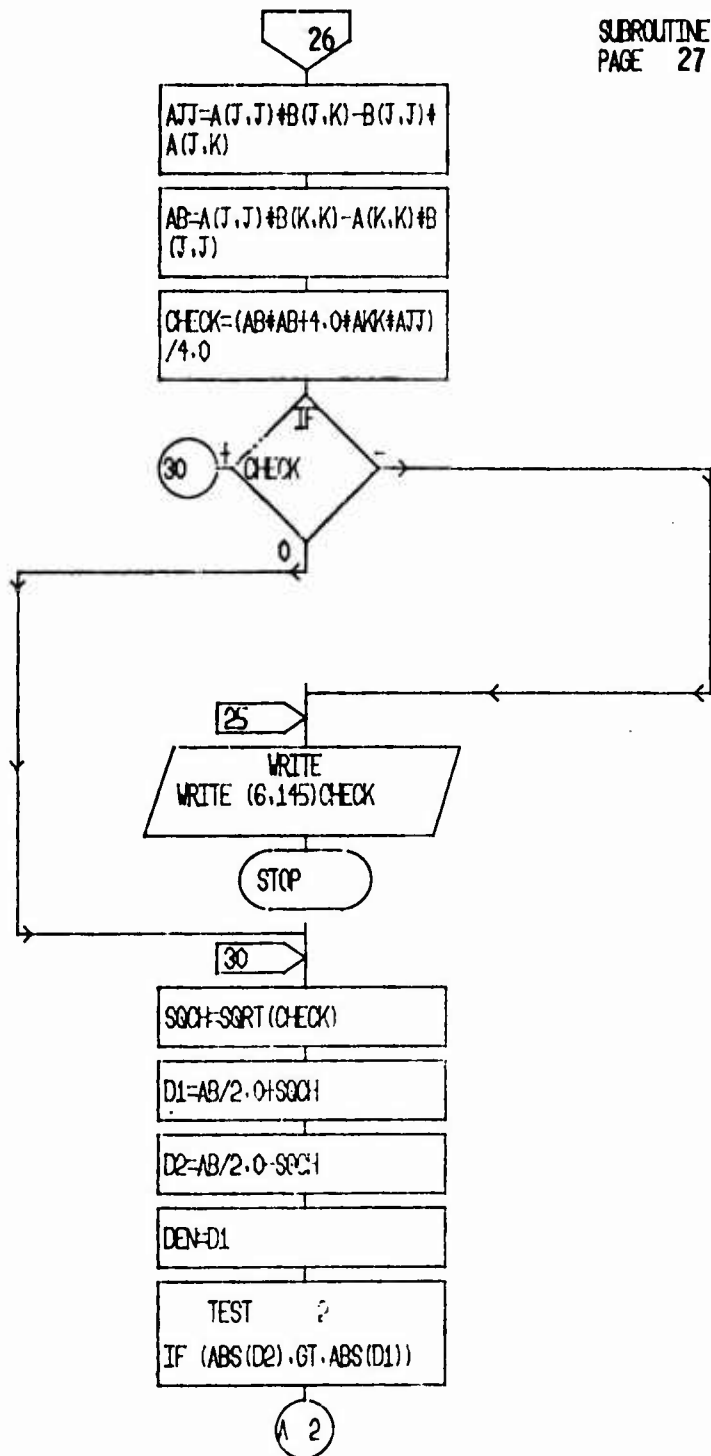
SUBROUTINE STLTRN
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SUBROUTINE JACOBI
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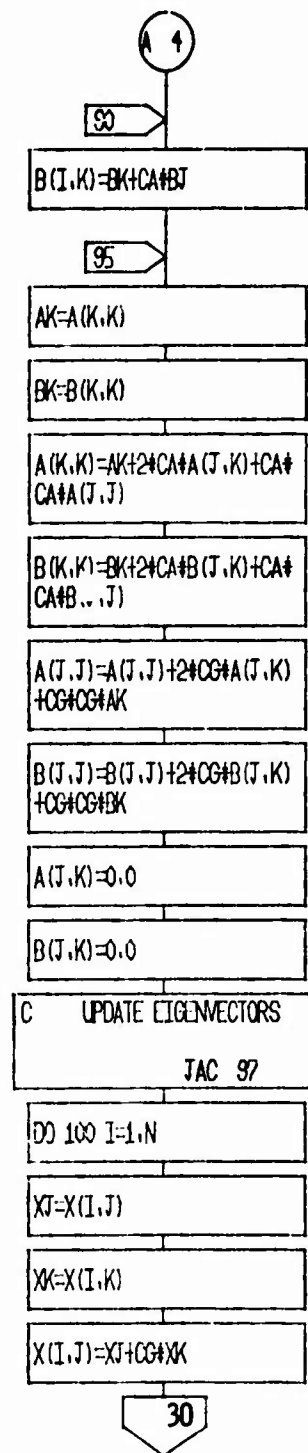
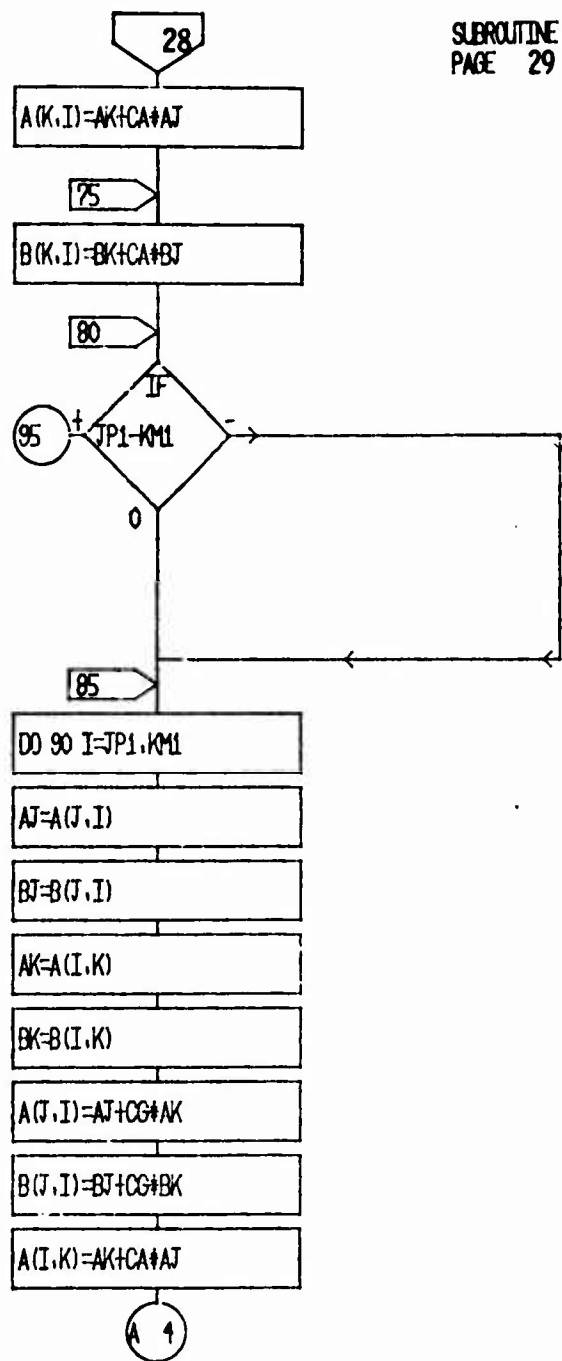
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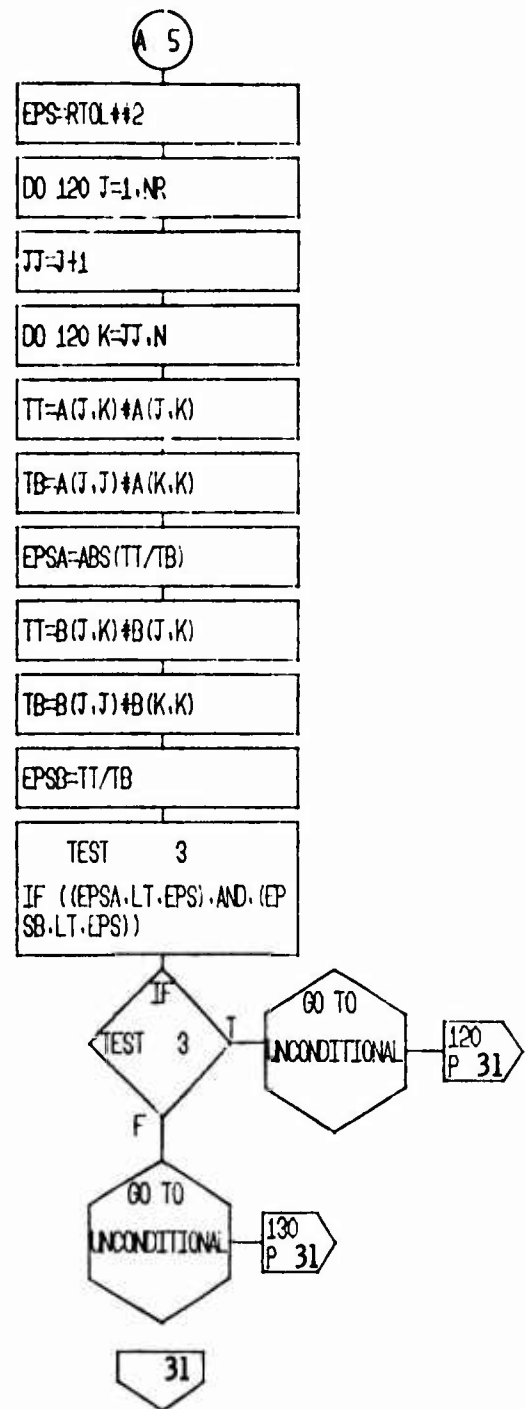
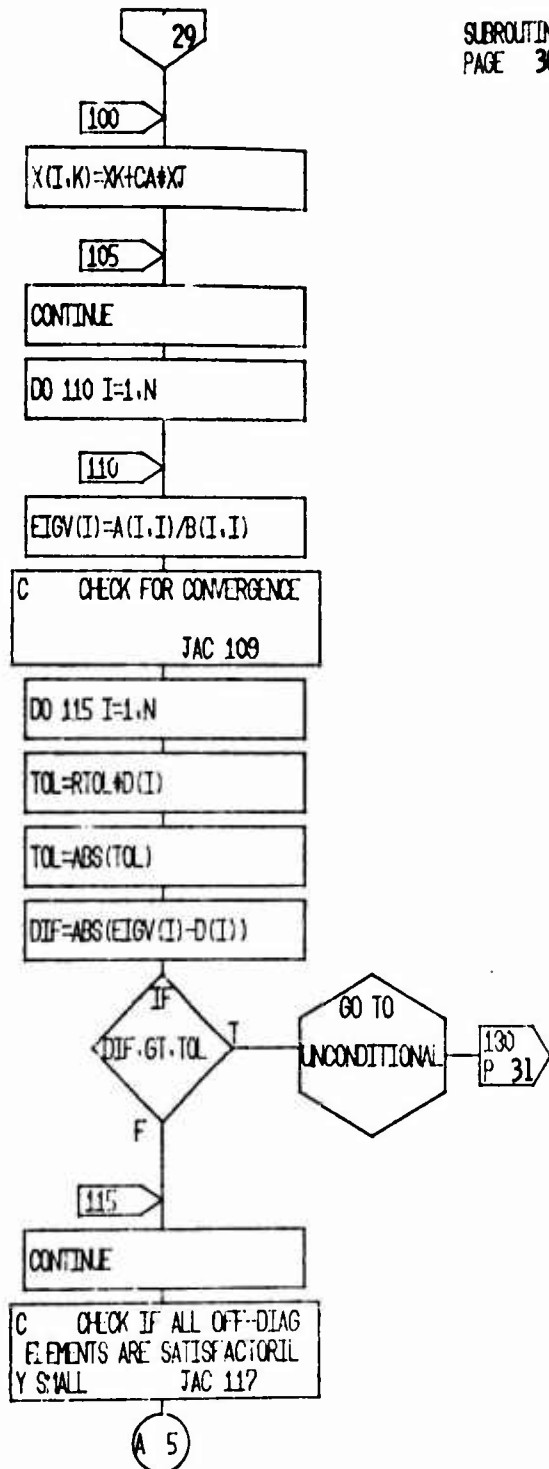
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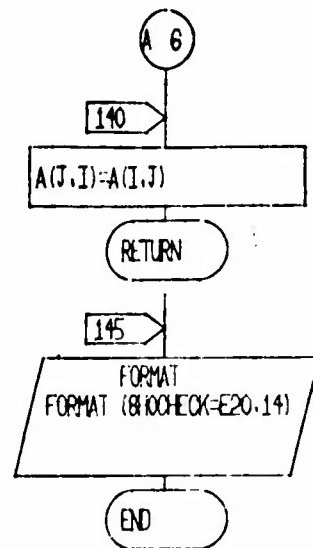
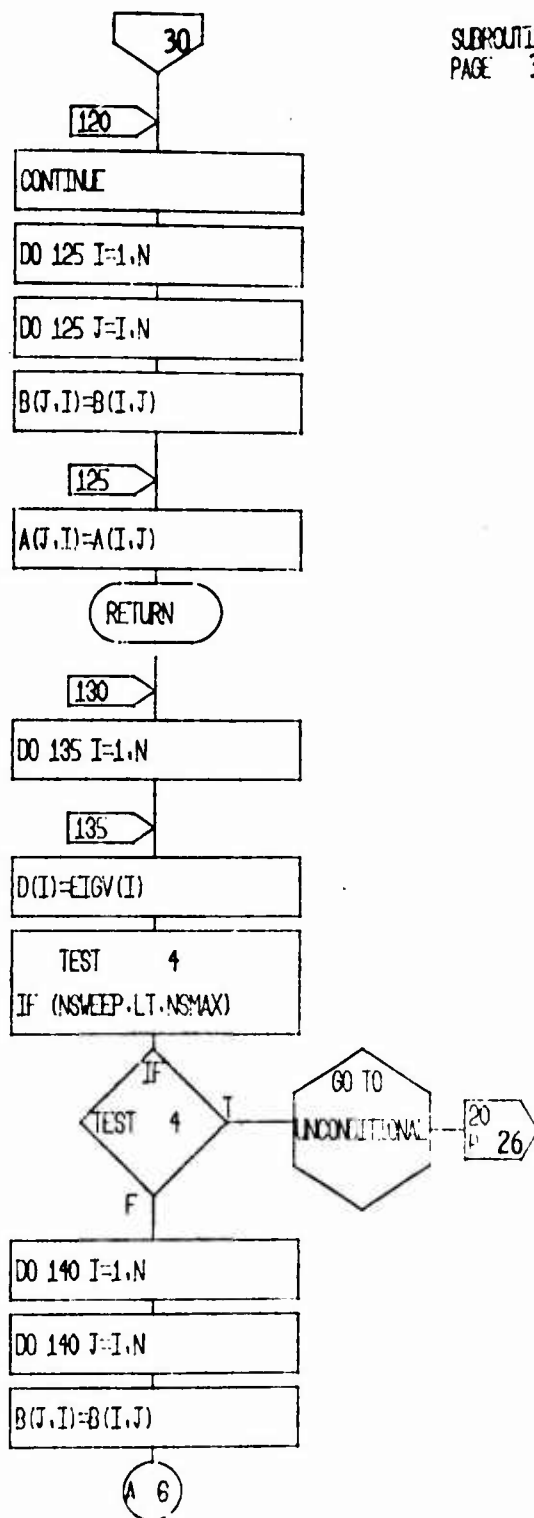
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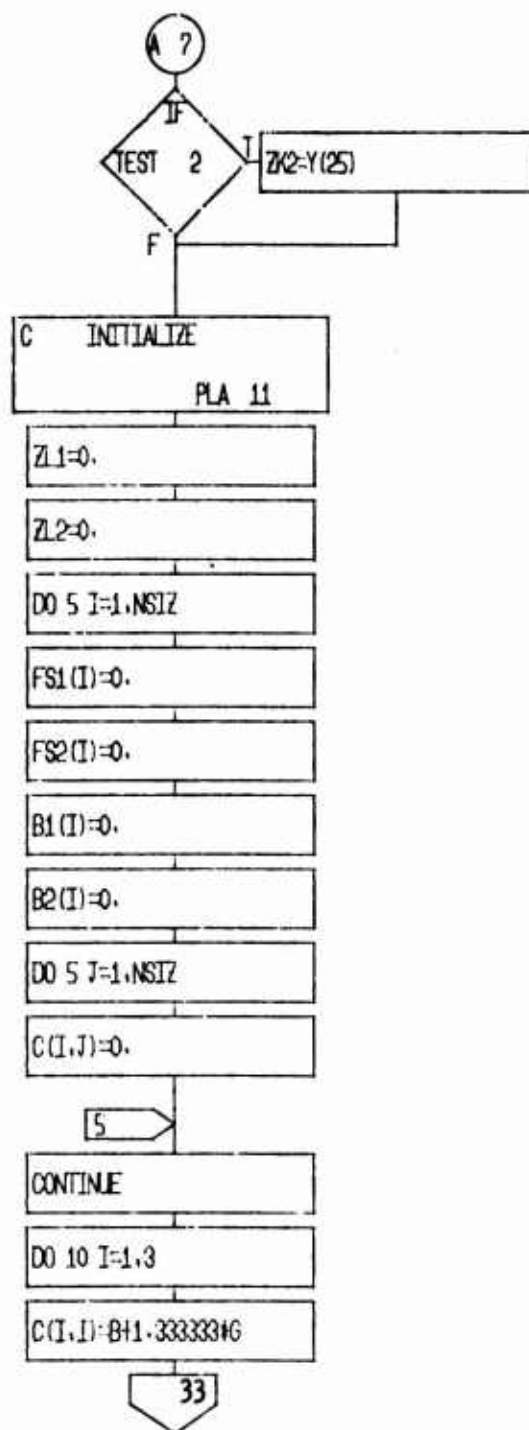
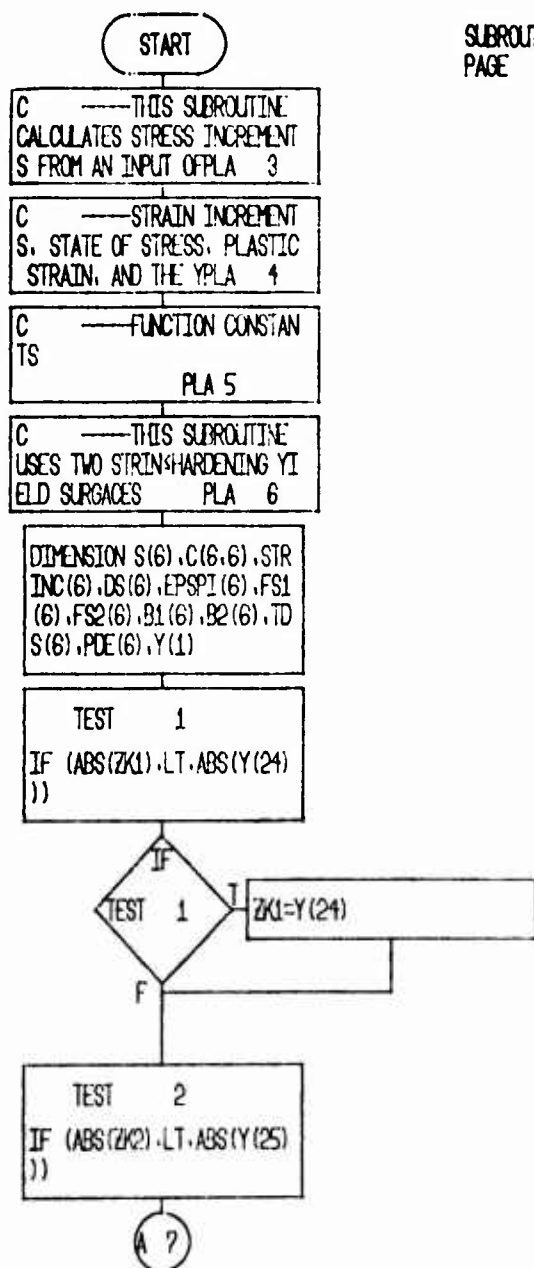
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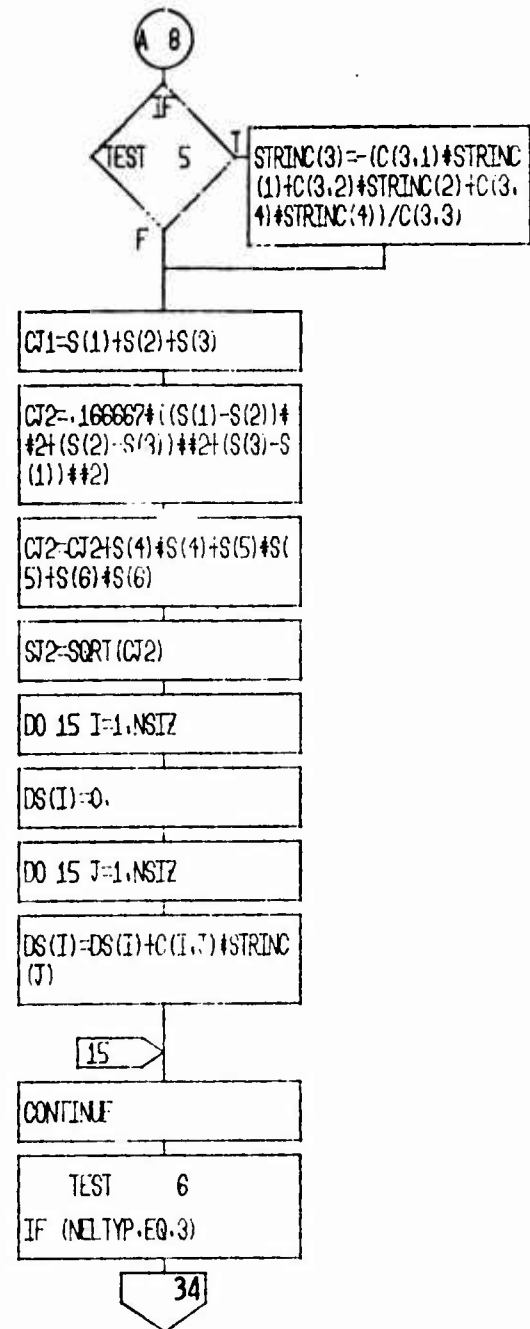
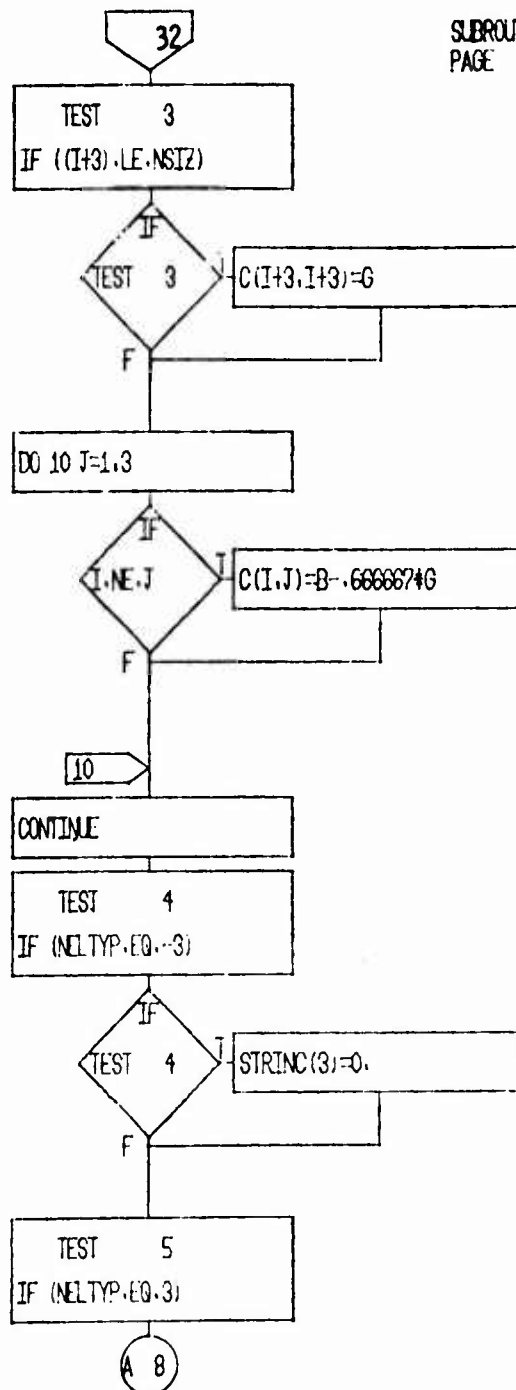
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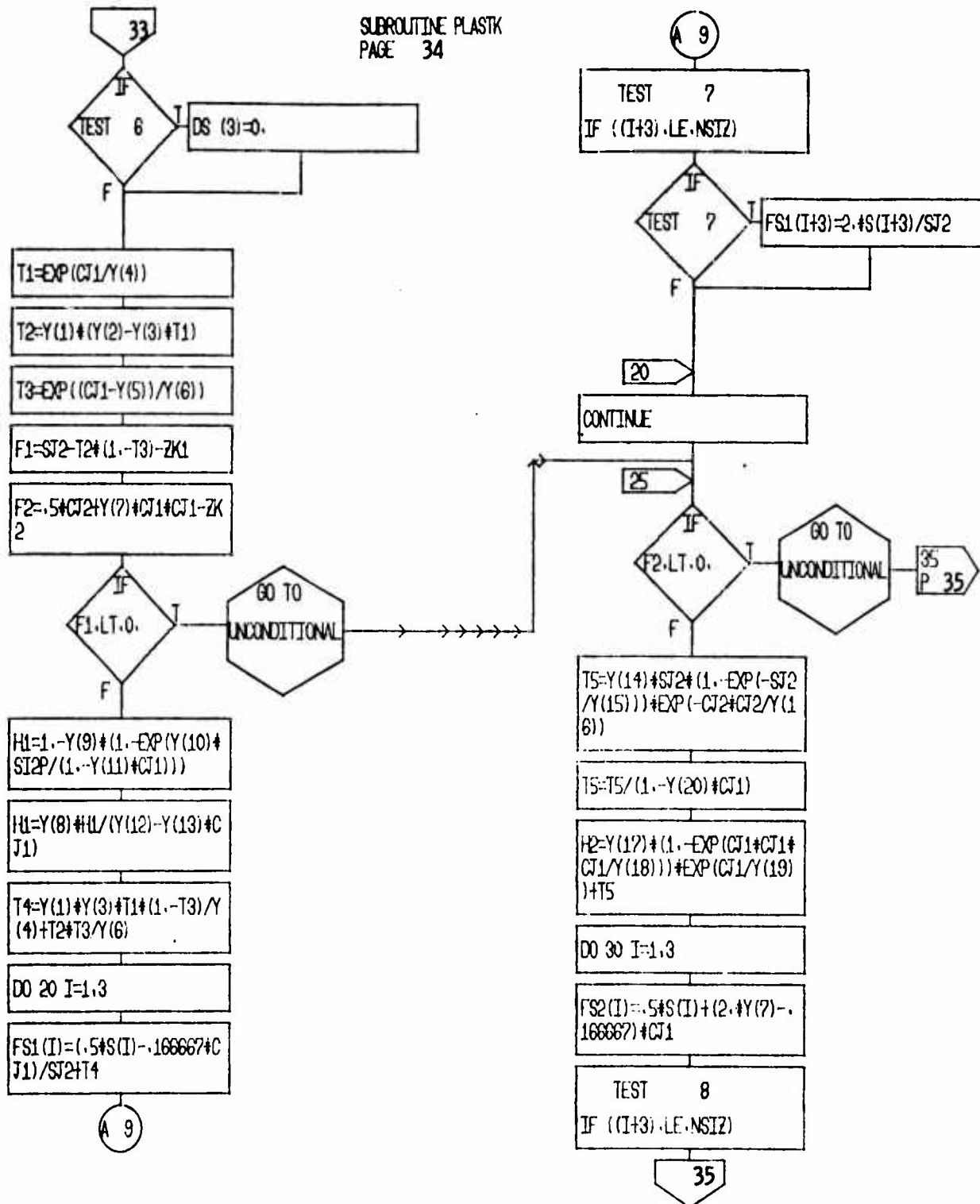
SUBROUTINE PLASTK
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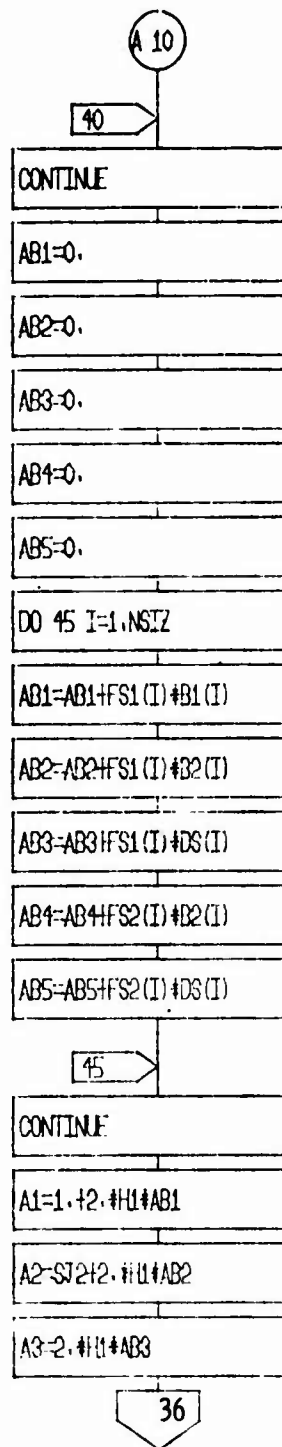
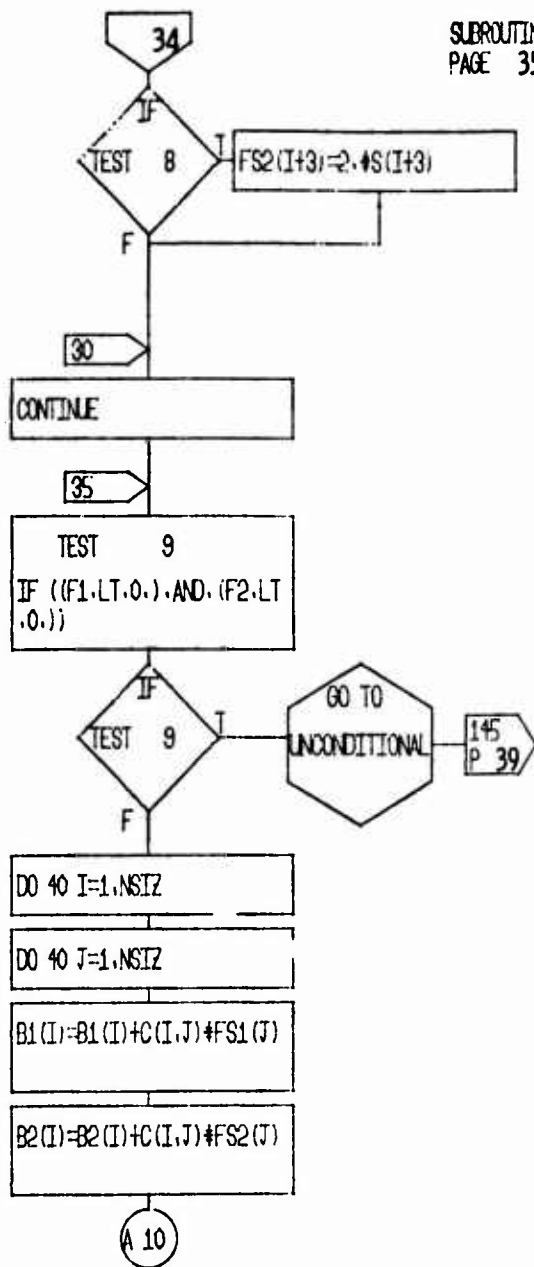
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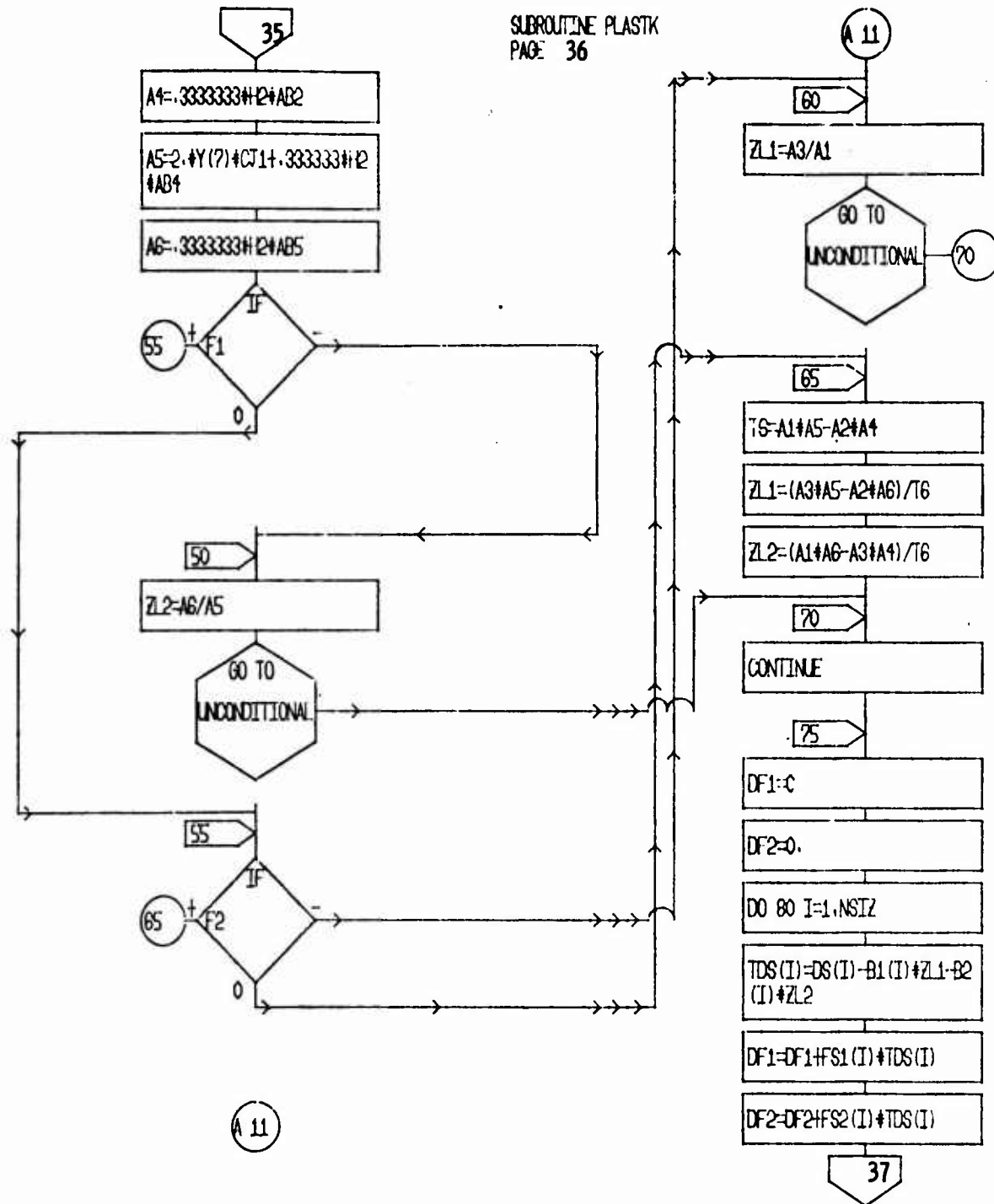
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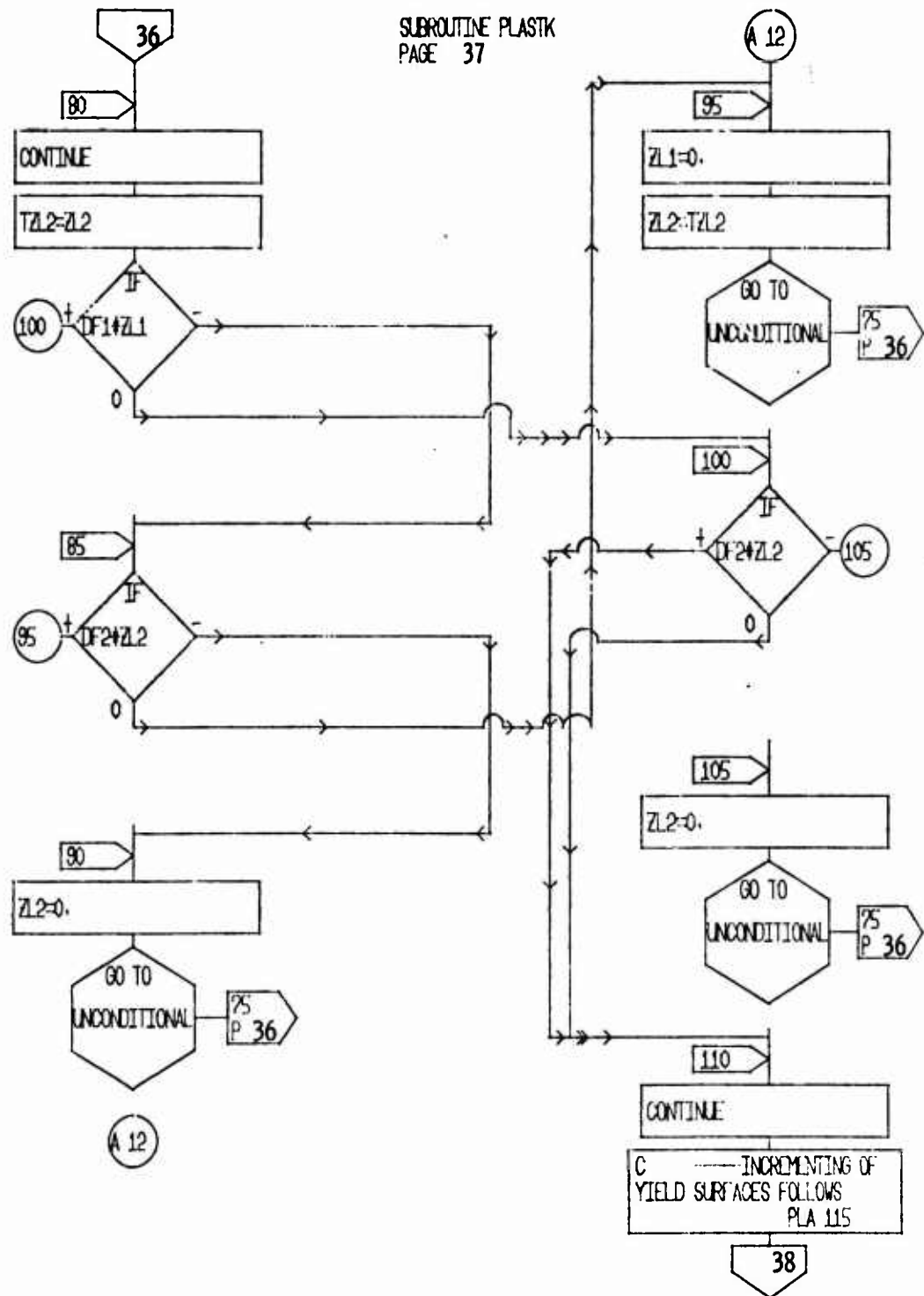
SUBROUTINE PLASTK
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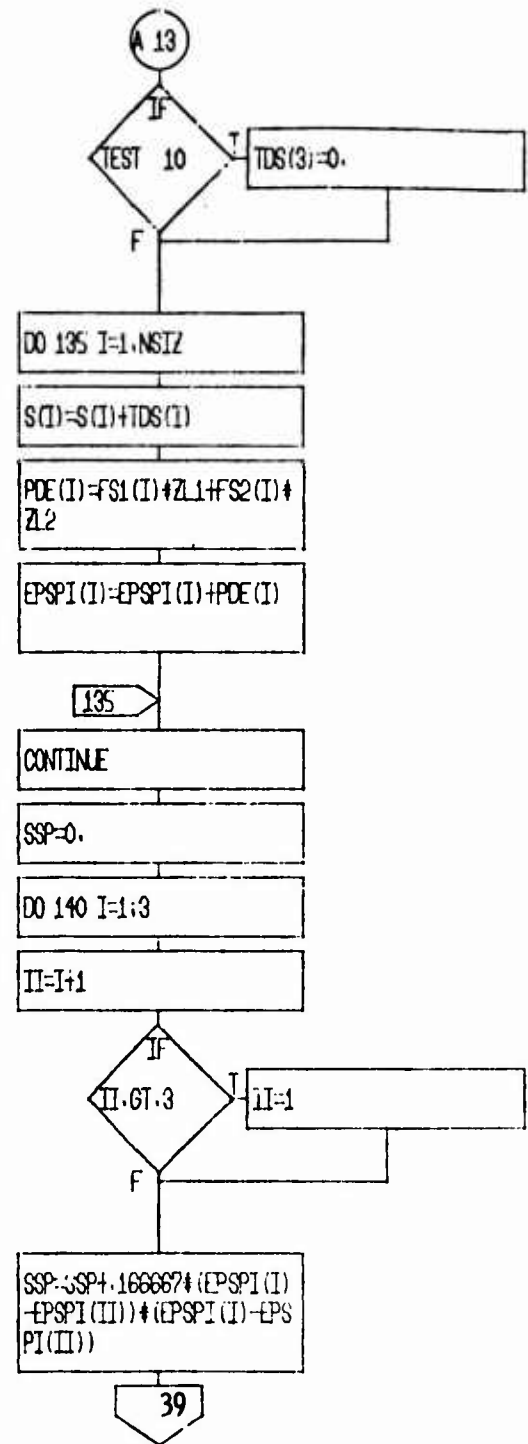
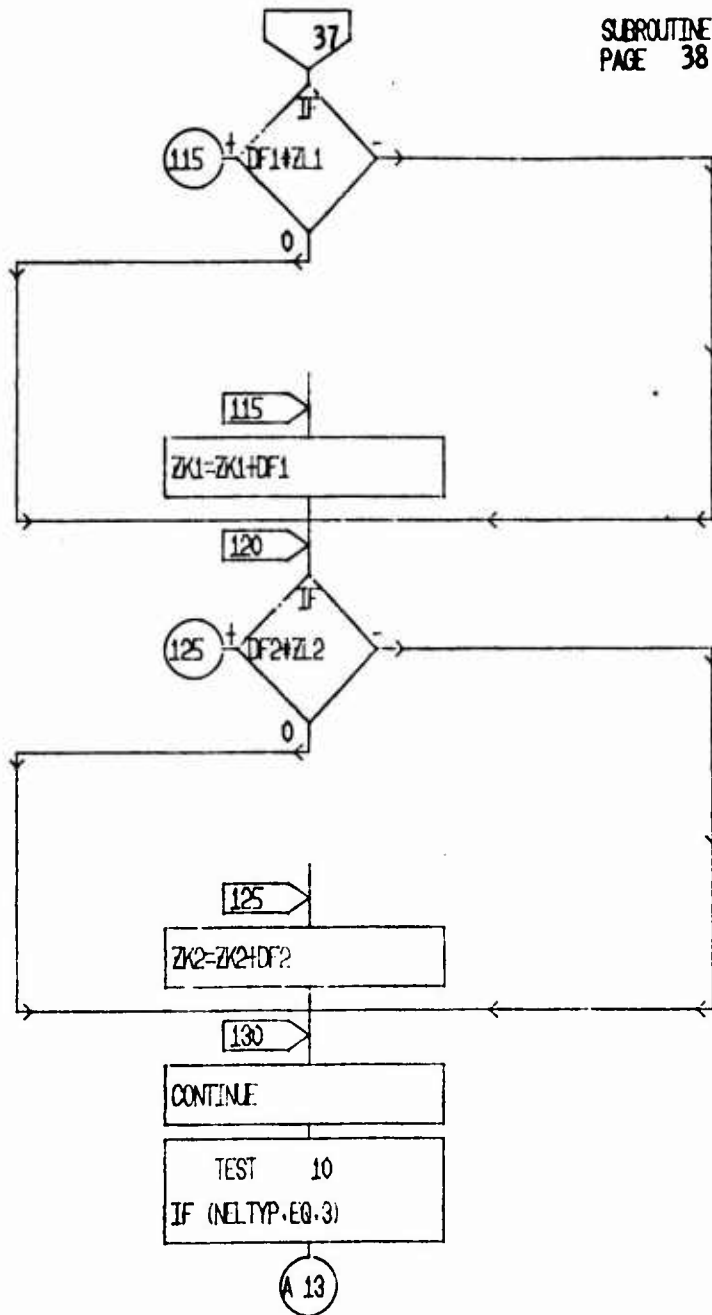
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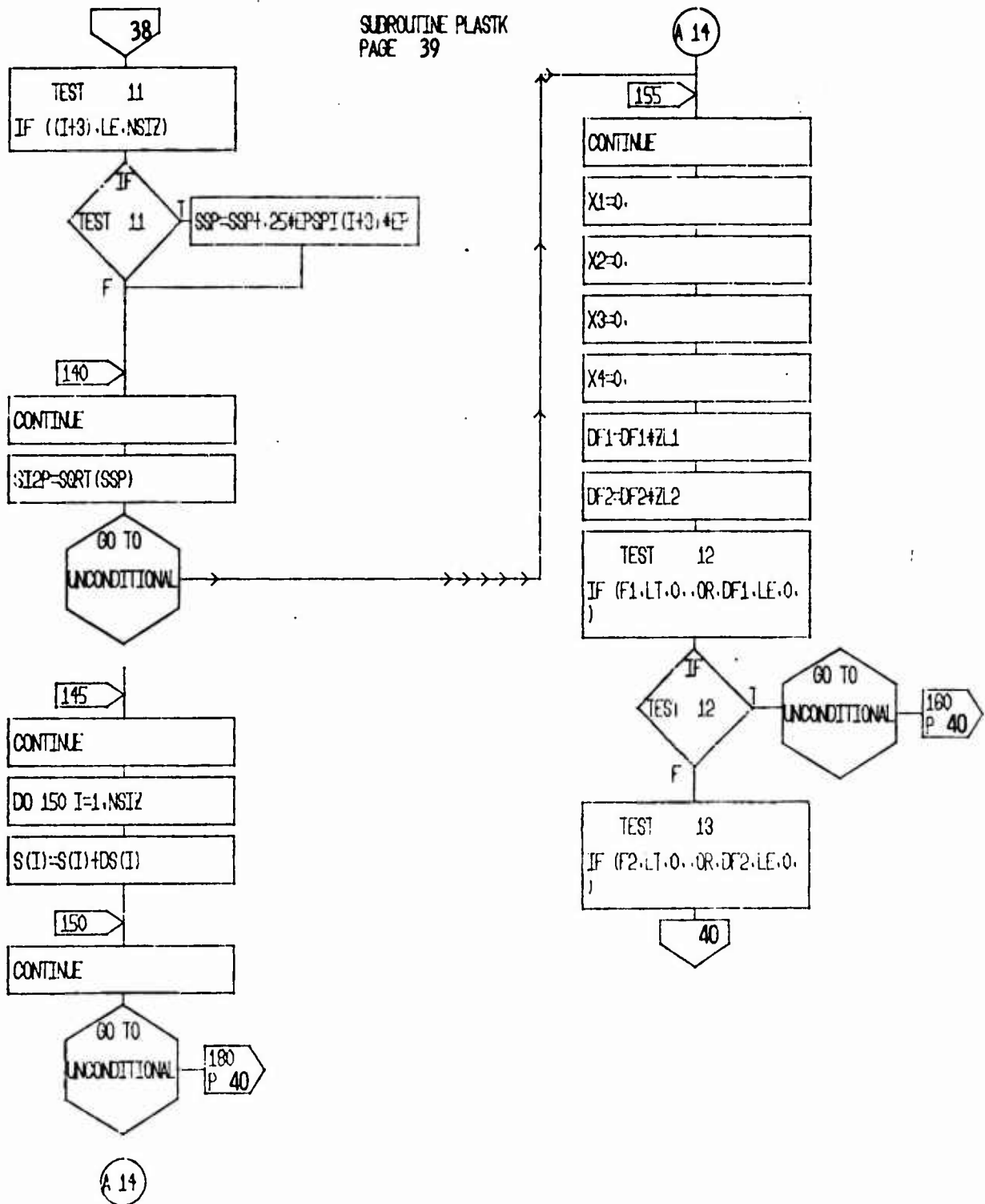
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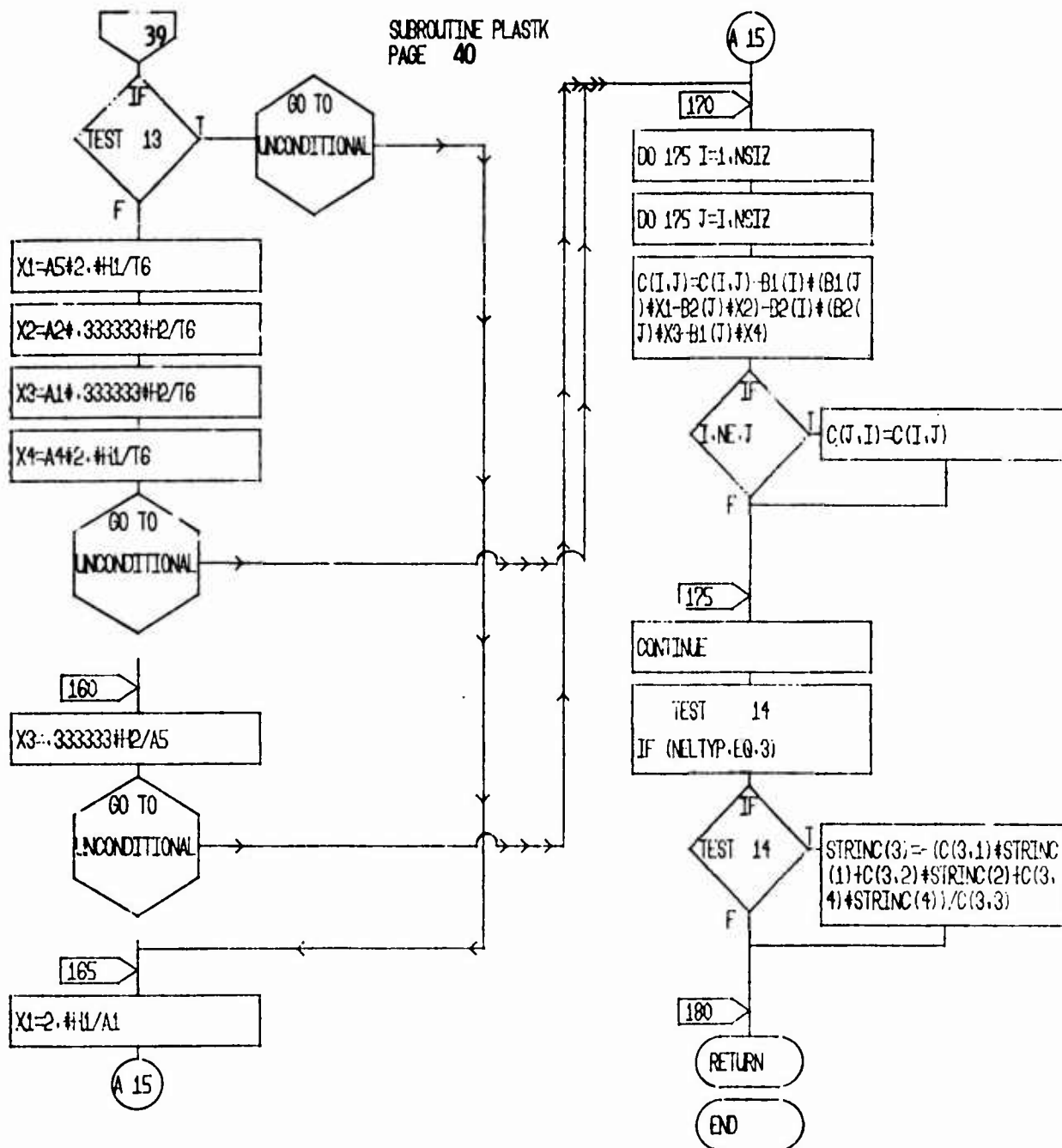
SUBROUTINE PLASTK
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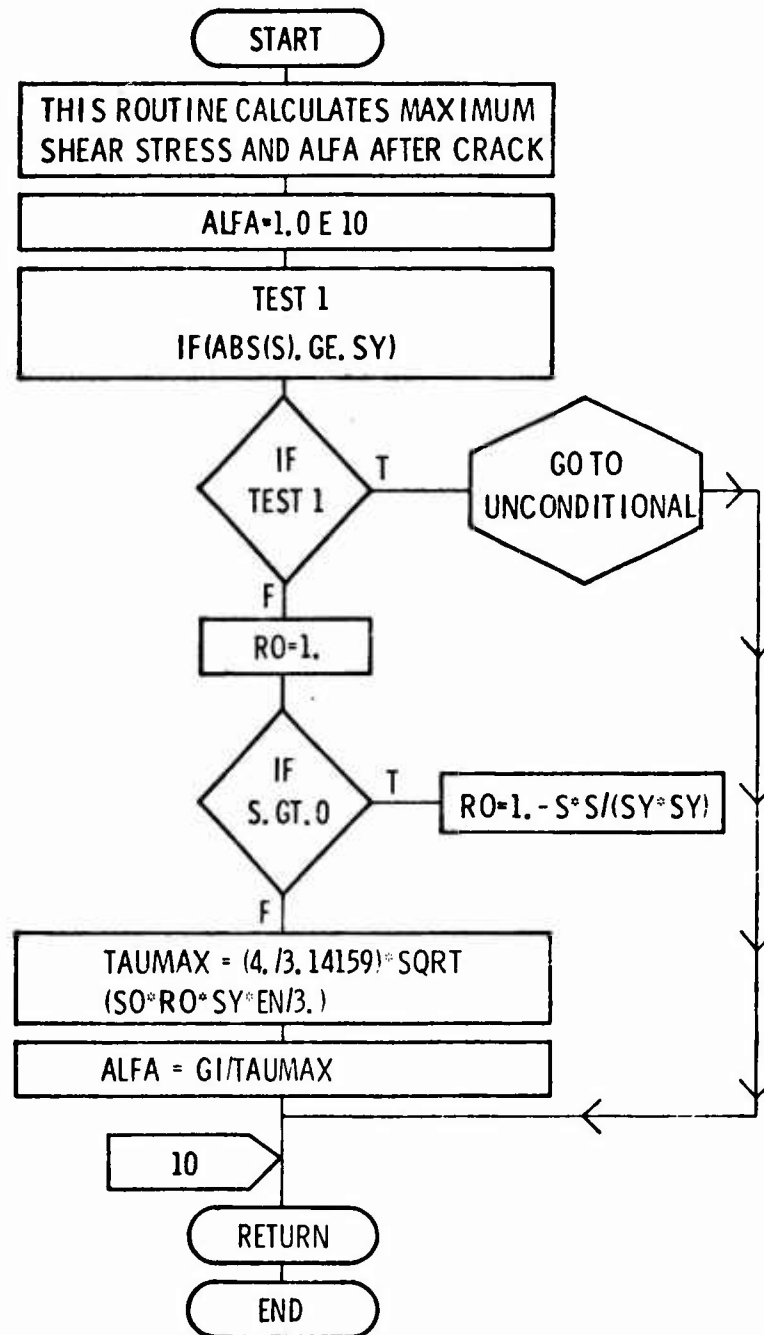
SUBROUTINE PLASTK
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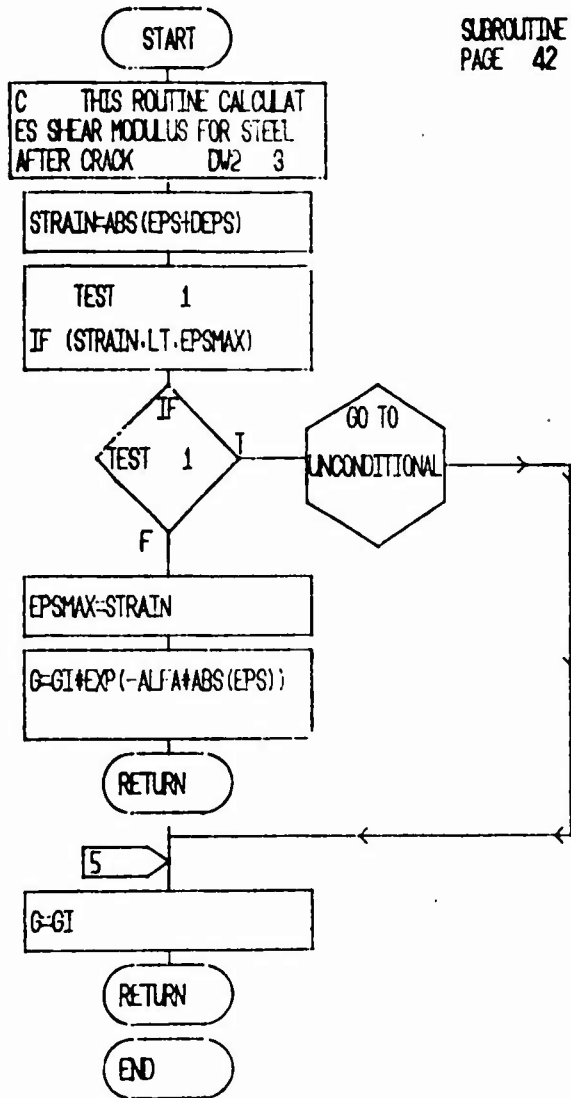


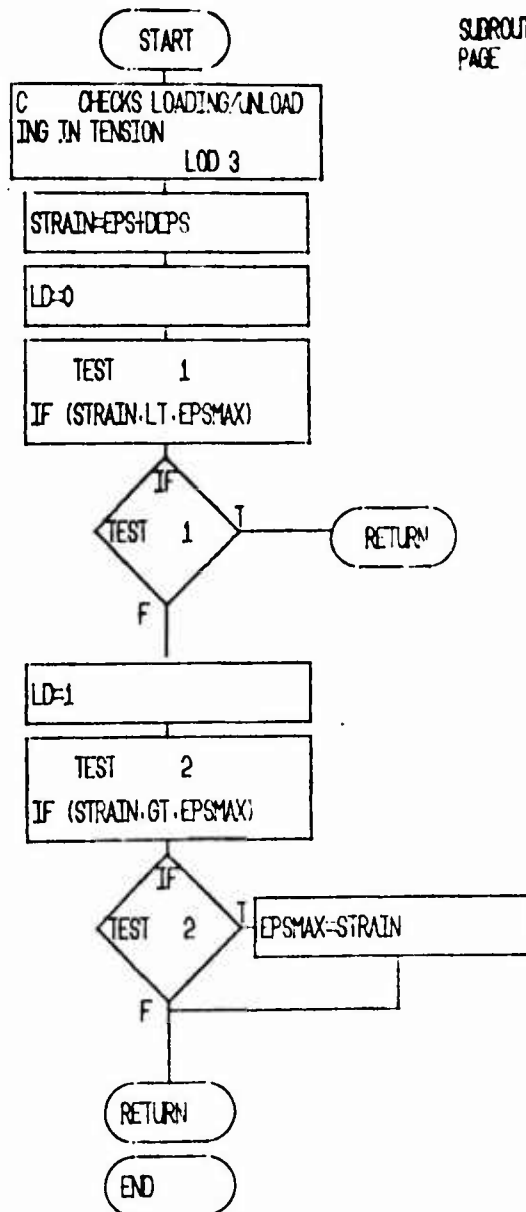
SUBROUTINE PLASTK
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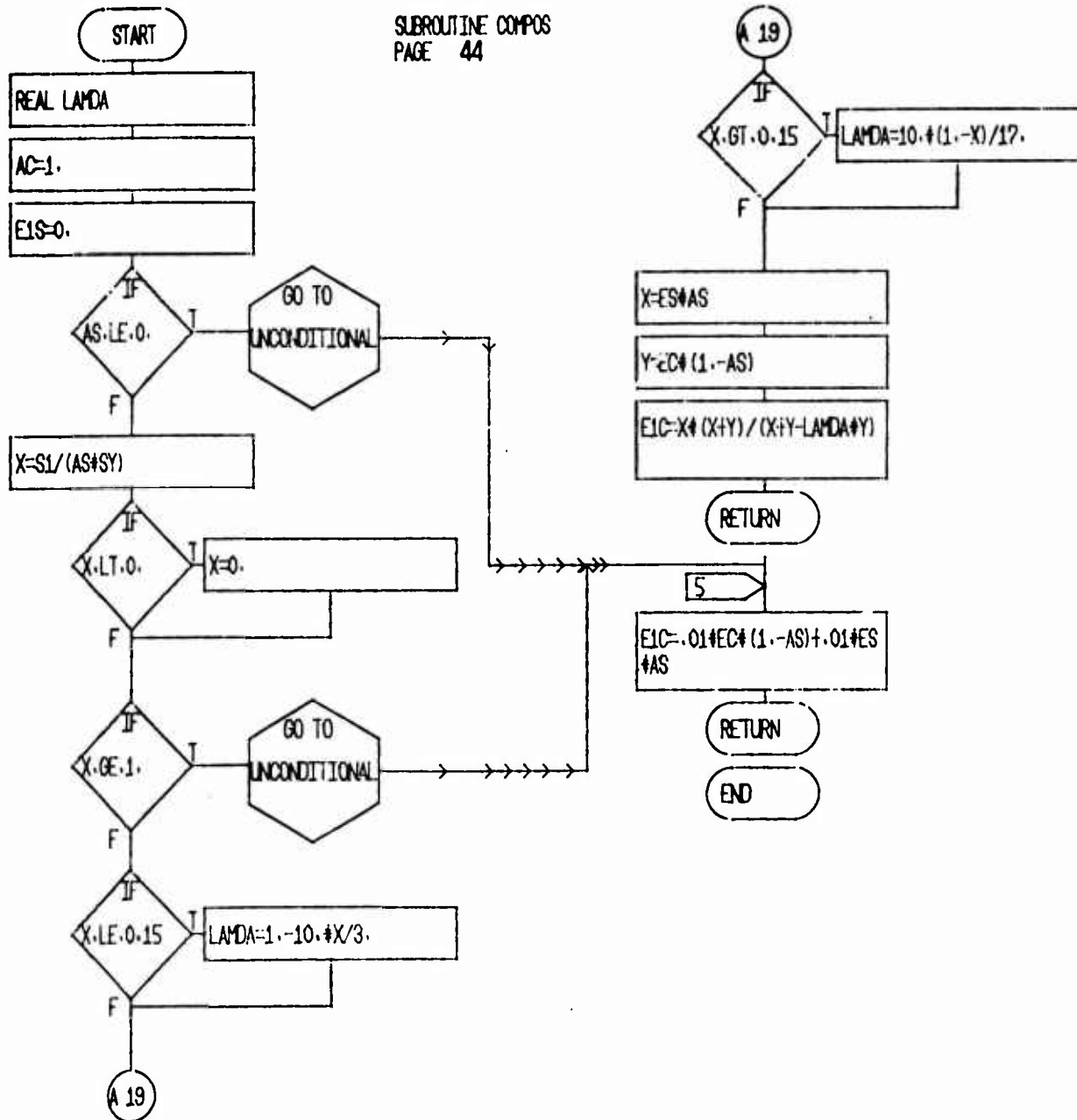
SUBROUTINE DOWEL 1
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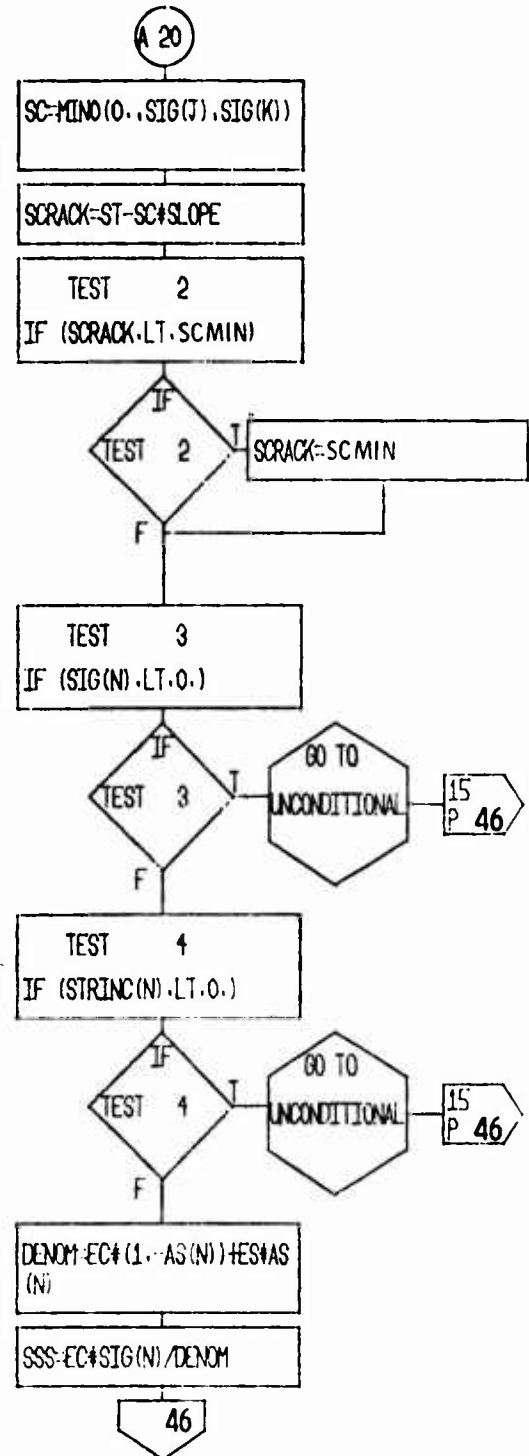
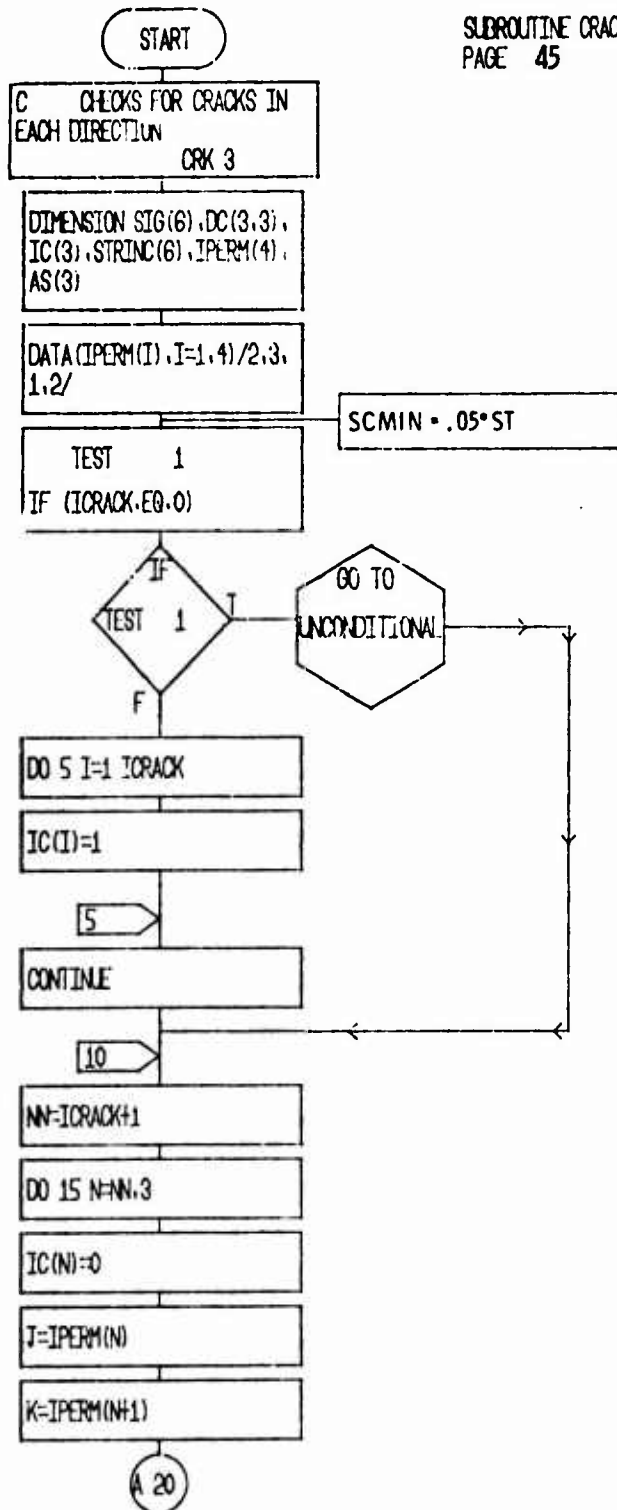




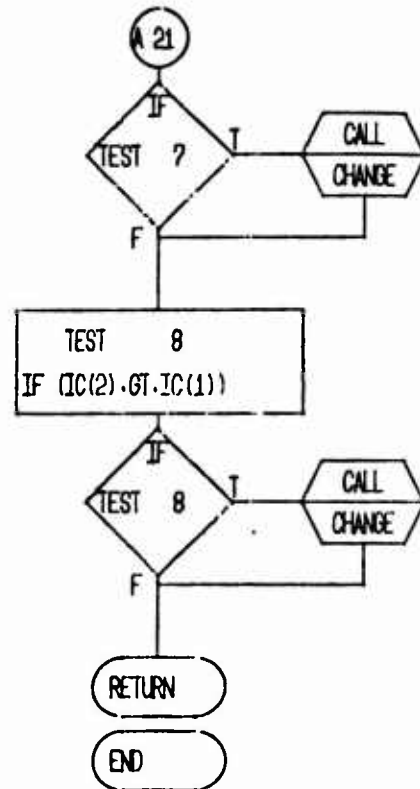
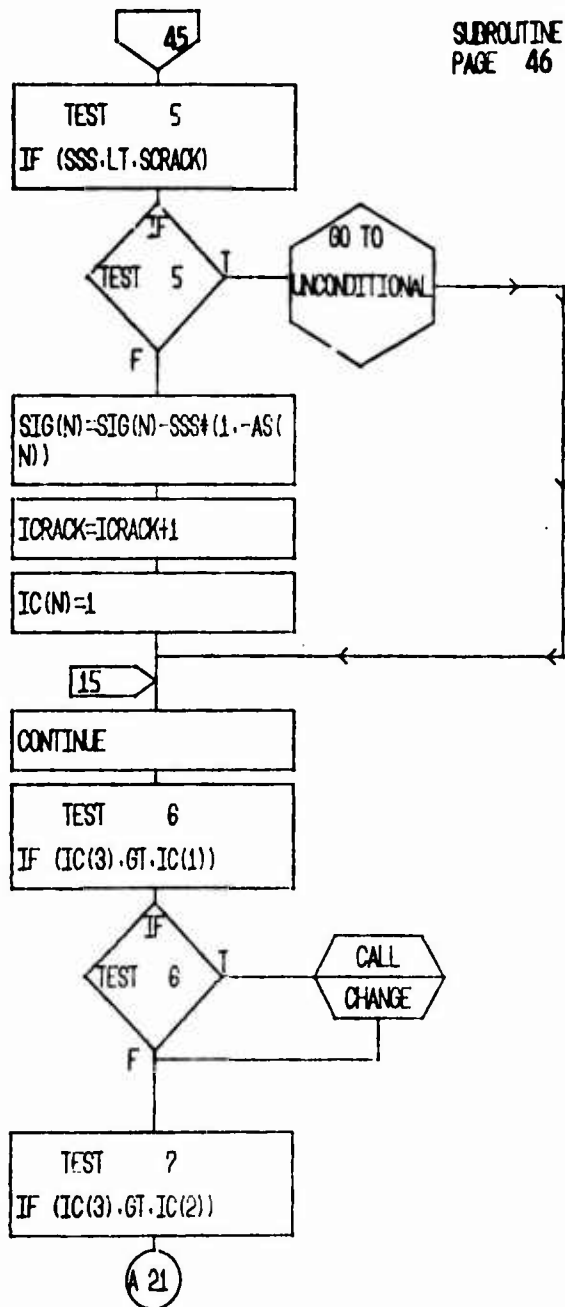
SUBROUTINE COMPOS
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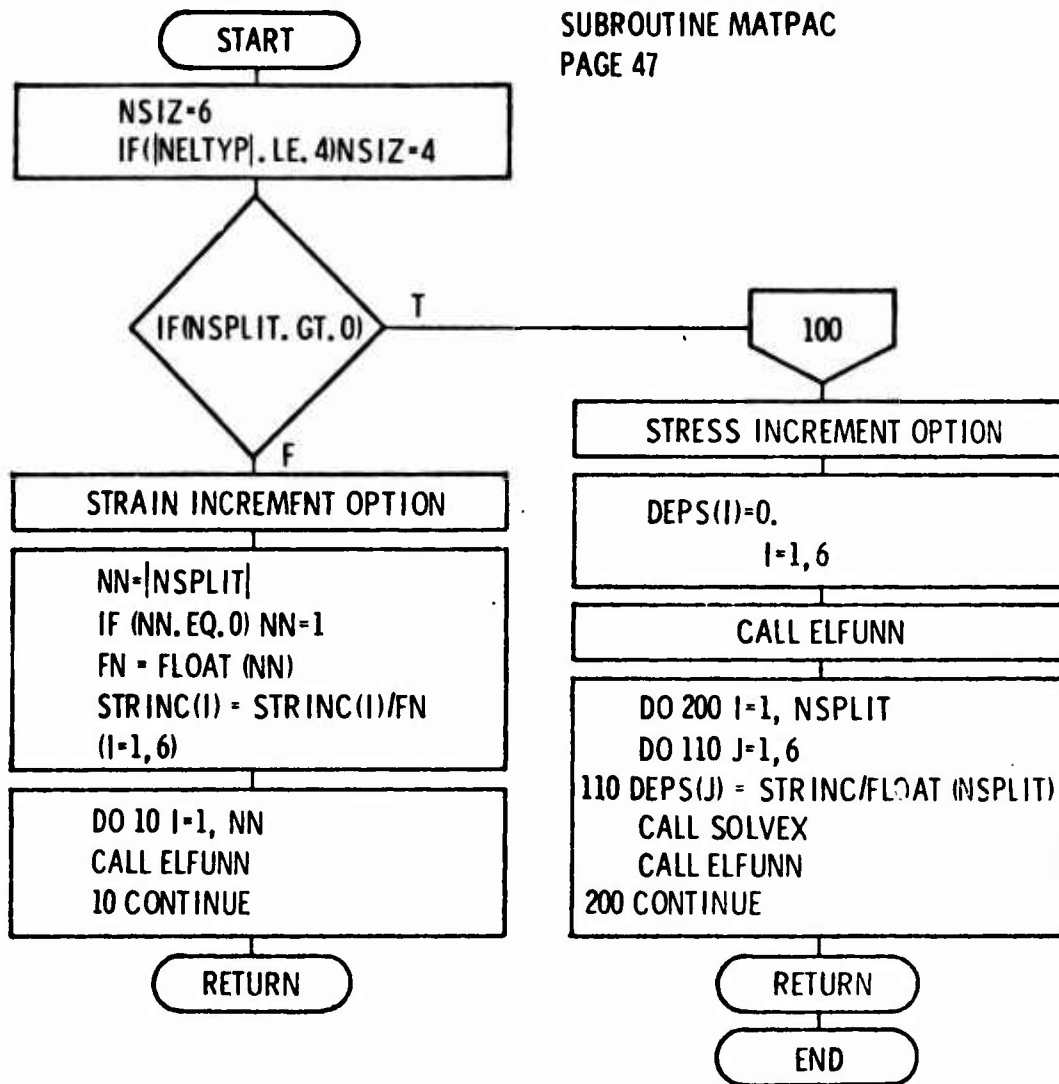
SUBROUTINE CRACKS
PAGE 45



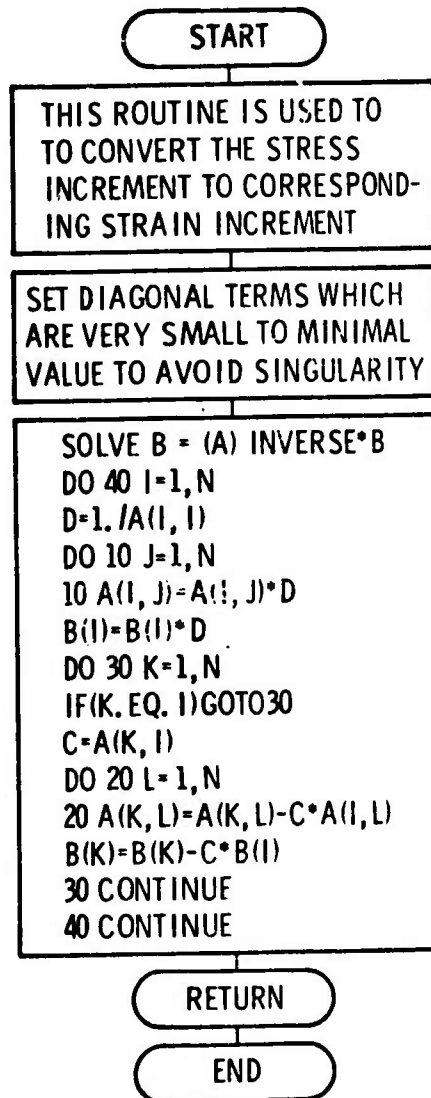
SUBROUTINE CRACKS
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SUBROUTINE MATPAC
PAGE 47



SUBROUTINE SOLVEX
PAGE 48



5. LISTING OF MATPAC PROGRAM

* T I D Y *

SUBROUTINE CADJST (CMAT)

	SUBROUTINE CADJST (CMAT)	CAD	1
C	ADJUST C-MATRIX FOR PLANE-STRESS PROBLEMS	CAD	2
	DIMENSION CMAT(6,6)	CAD	3
C		CAD	4
	DO 15 I=1,4	CAD	5
	IF (I.EQ.3) GO TO 15	CAD	6
	DO 5 J=1,4	CAD	7
	IF (J.EQ.3) GO TO 5	CAD	8
	CMAT(I,J)=CMAT(I,J)-CMAT(3,I)*CMAT(3,J)/CMAT(3,3)	CAD	9
	IF (I.EQ.J) CMAT(I,I)=CMAT(I,I)	CAD	10
5	CONTINUE	CAD	11
	IF (I.NE.4) GO TO 15	CAD	12
	DO 10 J=1,4	CAD	13
	CMAT(3,J)=0.	CAD	14
	CMAT(J,3)=0.	CAD	15
10	CONTINUE	CAD	16
15	CONTINUE	CAD	17
	DO 25 I=1,4	CAD	18
	IF (I.EQ.3) GO TO 25	CAD	19
	IF (CMAT(I,I).GE.0.) GO TO 25	CAD	20
	DO 20 J=1,4	CAD	21
	CMAT(I,J)=0.	CAD	22
	CMAT(J,I)=0.	CAD	23
20	CONTINUE	CAD	24
25	CONTINUE	CAD	25
	RETURN	CAD	26
	END	CAD	27

* T I D Y *

SUBROUTINE CHANGE (DC,SIG,IC,M,N,M1,N1)

	SUBROUTINE CHANGE (DC,SIG,IC,M,N,M1,N1)	CHA	1
C		CHA	2
C	INTERCHANGES AXIS TO ORDER CRACKED DIRECTIONS	CHA	3
C		CHA	4
	DT=ENSTON DC(1,3), SIG(6), IC(5)	CHA	5
	DUM=SIG(4)	CHA	6
	SIG(4)=SIG(4)	CHA	7
	SIG(4)=DUM	CHA	8
	DUM=SIG(4)	CHA	9
	SIG(4)=SIG(4)	CHA	10
	SIG(4)=DUM	CHA	11
	DUM=IC(4)	CHA	12
	IC(4)=IC(4)	CHA	13
	IC(4)=DUM	CHA	14
	ON 5 I=1,3	CHA	15
	DUM=DC(I,M)	CHA	16
	DC(I,M)=DC(I,N)	CHA	17
	DC(I,N)=DUM	CHA	18
5	CONTINUE	CHA	19
	RETURN	CHA	20
	END	CHA	21-

* T I D Y *

SUBROUTINE COMPOS (S1,SY,AC,AS,EC,ES,EIC,EIS)

	SUBROUTINE COMPOS (S1,SY,AC,AS,EC,ES,EIC,EIS)	COM	1
	REAL LAMDA	COM	2
	AC=1.	COM	3
	EIS=0.	COM	4
	IF (AS.LE.0.) GO TO 5	COM	5
	X=S1/(AS*SY)	COM	6
	IF (X.LT.0.) X=0.	COM	7
	IF (X.GE.1.) GO TO 5	COM	8
	IF (X.LE.0.15) LAMDA=1.-10.*X/3.	COM	9
	IF (X.GT.0.15) LAMDA=10.*(1.-X)/17.	COM	10
	X=ES*AS	COM	11
	Y=EC*(1.-AS)	COM	12
	EIC=X*(X+Y)/(X+Y+LAMDA*Y)	COM	13
	RETURN	COM	14
5	EIC=.01*EC*(1.-AS)+.01*ES*AS	COM	15
	RETURN	COM	16
	END	COM	17-

* T I D Y *

SUBROUTINE COMPT (C,STRINC,STRESS,STRSIN,NSIZ,NELTYP)

	SUBROUTINE COMPT (C,STRINC,STRESS,STRSIN,NSIZ,NELTYP)	COM	1
	DIMENSION C(6,6), STRINC(6), STRESS(6), STRSIN(6)	COM	2
	IF (NSIZ.EQ.6) GO TO 10	COM	3
	IF (NELTYP.EQ.3) STRINC(3)=0.	COM	4
	IF (NELTYP.EQ.3) GO TO 5	COM	5
	GO TO 10	COM	6
5	STRINC(3)=C(3,1)*STRINC(1)+C(3,2)*STRINC(2)+C(3,4)*STRINC(4)	COM	7
	STRINC(3)=STRINC(3)/C(3,3)	COM	8
10	DO 20 I=1,NSIZ	COM	9
	STRSIN(I)=0.	COM	10
	DO 15 J=1,NSIZ	COM	11
	STRSIN(I)=STRSIN(I)+C(I,J)*STRINC(J)	COM	12
15	CONTINUE	COM	13
	STRESS(I)=STRESS(I)+STRSIN(I)	COM	14
20	CONTINUE	COM	15
	IF (NELTYP.EQ.3) STRESS(3)=0.	COM	16
	IF (NELTYP.EQ.3) STRSIN(3)=0.	COM	17
	RETURN	COM	18
	END	COM	19

	SUBROUTINE CRACKS (SIG,DC,ST,SLOPE,ICRACK,STRINC,EC,ES,AS)	CRK	1
C		CRK	2
C	CHECKS FOR CRACKS IN EACH DIRECTION	CRK	3
C		CRK	4
	DIMENSION SIG(6), DC(3,3), IC(3), STRINC(6), IPERM(4), AS(3)	CRK	5
	DATA (IPERM(I),I=1,4)/2,3,1,2/	CRK	6
	SCMIN=.05*ST		
	IF (ICRACK.EQ.0) GO TO 10	CRK	7
	DO 5 I=1,ICRACK	CRK	8
	IC(I)=1	CRK	9
5	CONTINUE	CRK	10
10	NN=ICRACK+1	CRK	11
	DO 15 N=NN,3	CRK	12
	IC(N)=0	CRK	13
	J=IPERM(N)	CRK	14
	K=IPERM(N+1)	CRK	15
	SC=MIN(0.,SIG(J),SIG(K))	CRK	16
	SCRACK=ST-SC*SLOPE	CRK	17
	IF (SCRACK.LT.SCMIN) SCRACK=SCMIN		
	IF (SIG(N).LT.0.) GO TO 15	CRK	19
	IF (STRINC(N).LT.0.) GO TO 15	CRK	20
	DENOM=EC*(1.-AS(N))+ES*AS(N)	CRK	21
	SSS=EC*SIG(N)/DENOM	CRK	22
	IF (SSS.LT.SCRACK) GO TO 15	CPK	23
	SIG(N)=SIG(N)-SSS*(1.-AS(N))	CRK	24
	ICRACK=ICRACK+1	CRK	25
	IC(N)=1	CRK	26
15	CONTINUE	CRK	27
	IF (IC(3).GT.IC(1)) CALL CHANGE (DC,SIG,IC,3,1,4,5)	CRK	28
	IF (IC(3).GT.IC(2)) CALL CHANGE (DC,SIG,IC,3,2,4,6)	CPK	29
	IF (IC(2).GT.IC(1)) CALL CHANGE (DC,SIG,IC,2,1,5,6)	CRK	30
	RETURN	CRK	31
	END	CRK	32

	SUBROUTINE CTRANF (A,B,C)	CTR	1
C		CTR	2
C	TRANSFORMS C-MATRIX TO GLOBAL COORDINATE SYSTEM	CTR	3
C		CTR	4
	DIMENSION A(6,6), B(6,6), C(6,6)	CTR	5
	DO 5 I=1,6	CTR	6
	DO 5 J=1,6	CTR	7
	A(I,J)=0.	CTR	8
	DO 5 K=1,6	CTR	9
	A(I,J)=A(I,J)+B(I,K)*C(K,J)	CTR	10
5	CONTINUE	CTR	11
	DO 10 I=1,6	CTR	12
	DO 10 J=1,6	CTR	13
	C(I,J)=0.	CTR	14
	DO 10 K=1,6	CTR	15
	C(I,J)=C(I,J)+A(I,K)*B(K,J)	CTR	16
10	CONTINUE	CTR	17
	DO 15 I=1,6		
	DO 15 J=4,6		
	IF(I.EQ.J) GO TO 15		
	C(I,J)=0.		
	C(J,I)=0.		
15	CONTINUE	CTR	18
	RETURN	CTR	19.
	END		

```

*IT FOR DOBEL1,DOBEL1
SUBROUTINE DOBEL1 (S,SV,SG,ALFA,GAMA,EN,GI)
C
C THIS ROUTINE CALCULATES ALLOWABLE MAXIMUM SHEAR AFTER CRACK
C
ALFA=1.0E+10
IF(AHS(S).GE.SV) GO TO 10
RD=1.
IF(S.GT.0.) RD=1.-S*S/(SV*SV)
TAUMAX=(4./3.14159)*SQRT(SG*RD*SV*EN/3.)
ALFA=GI/TAUMAX
10 RETURN
END
DNI

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* T I D Y *

SUBROUTINE DOWEL2 (GI,ALFA,EPS,DEPS,EPSMAX,G)

C	SUBROUTINE DOWEL2 (GI,ALFA,EPS,DEPS,EPSMAX,G)	DW2	1
C	THIS ROUTINE CALCULATES SHEAR MODULUS FOR STEEL AFTER CRACK	DW2	2
C		DW2	3
	STRAIN=ABS(EPS+DEPS)	DW2	4
	IF (STRAIN,LT,EPSMAX) GO TO 5	DW2	5
	EPSMAX=STRAIN	DW2	6
	G=GI*EXP(-ALFA*ABS(EPS))	DW2	7
	RETURN	DW2	8
S	G=GI	DW2	9
	RETURN	DW2	10
	END	DW2	11
		DW2	12=

```

      SUBROUTINE ELFUNN (STRESS, STRAIN, STRINC, EPSSEL, COEFF, RLKMOD, SHRMOD,
      IYCOF, DC, DCS, AST, VRH, ZK1, ZK2, SIPP, ICOMP, NELTYP, C, ICRACK)
C
C   CALCULATES STRESS FROM STRAIN INCREMENTS FOR COMPOSITE MATERIALS
C
      DIMENSION STRESS(6), STRAIN(6), STRINC(6), EPSSEL(6), COEFF(15),
      IYCOF(25), DC(3,3), DCS(3,3), AST(3), C(6,6), SIG(4), EPS(6), DEPS(6),
      2      AS(4), STRSIN(4), A(6,6), R(6,6), EPSPL(6), FS1(6), FS2(6),
      3R1(6), R2(6), TDS(6), PDE(6), BUFFER(40)
      REAL R1, R2
      EQUIVALENCE (PDE, SIG), (TDS, EPS), (B2, DEPS), (B1, BUFFER), (FS2, AS),
      1(FS1, A), (EPSPL, R)
      NSIZ=6
      IF (IAHS(NELTYP), LE, 4) NSIZ=4
      IF (ICOMP, EQ, 2) GO TO 200
      EC=COEFF(1)
      ES=COEFF(2)
      SC=COEFF(3)
      SY=COEFF(4)
      NU=COEFF(5)
      ST=COEFF(6)
      SLOPF=COEFF(7)
      EA=COEFF(8)
      GST=COEFF(9)
      ERHOND=COEFF(10)
      IF (ICRACK, EQ, 0)
      1CALL PRSTR (BUFFER(1), BUFFER(10), STRESS, DC, SIG, BUFFER(19))
C   CRACKED SECTION CONVERT TO PRINCIPAL ORTHOTROPIC DIRECTIONS
      CALL TRANSF (A, B, DC)
      CALL MATPLY (A, STRESS, SIG, 1)
      IF (NSIZ, EQ, 4, OR, ICRACK, GE, 2) GO TO 20
C   CRACK NORMAL TO 1-AXIS, FIND EXTREMUM STRESSES IN 2-3 PLANE
      CALL MAXMIN (SIG(2), SIG(3), SIG(5), THETA)
C   ADJUST DIRECTIONS
      CC=COS(THETA)
      SS=SIN(THETA)
      DO 10 J=1,3
      DCM=DC(J,2)*CC+DC(J,3)*SS
      DC(J,3)=-DC(J,2)*SS+DC(J,3)*CC
      DC(J,2)=DCM
10    CONTINUE
      CALL TRANSF (A, R, DC)
      CALL MATPLY (A, STRESS, SIG, 1)
20    CALL MATPLY (R, STRINC, DEPS, -1)
C   CHECK FOR ADDITIONAL CRACKS
      CALL STLTHN (AS, DC, DCS, BUFFER, VRH, AST(1), AST(2), AST(3))
      CALL CRACKS (SIG, DC, ST, SLOPF, ICRACK, DEPS, EC, FS, AS)
      CALL TRANSF (A, H, DC)
      CALL MATPLY (R, STRAIN, EPS, -1)
      CALL MATPLY (R, STRINC, DEPS, -1)
      CALL MATPLY (R, SIG, STRESS, 1)
      CALL STLTHN (AS, DC, DCS, BUFFER, VRH, AST(1), AST(2), AST(3))
C
      CALL VARMOD (SIG, EPS, SO, SY, EC, ES, NU, AS, C, ICRACK, NELTYP, DEPS, EPSSEL,
      ALFA, GAMA, PV, GST, ERHOND)
      CALL CTRNF (BUFFER, H, C)
      CALL COMPT (C, STRINC, STRESS, STRSIN, NSIZ, NELTYP)
      IF (NELTYP, EQ, 3) CALL CADJUST (C)
      DO 50 I=1, NSIZ

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          STRAIN(I)=STRAIN(I)+STRINC(I)
50      CONTINUE
          IF(ICRACK.LE.0) RETURN
          CALL MATPLY (A,STRESS,SIG,1)
          DO 60 I=1,ICRACK
          IF (SIG(I).LE.0.) GO TO 55
          S=SV*AS(I)
          IF(SIG(I).GT.S) SIG(I)=S
          GO TO 60
55      CONTINUE
          S=SV*AS(I)
          IF(EPS(I).GT.0.) SIG(I)=AMAX1(S,SIG(I))
60      CONTINUE
          CALL MATPLY (B,SIG,STRESS,1)
          RETURN

C
C      USE TERFER MODEL
C
200     DO 250 I=1,NSIZ
          EPSPL(I)=STRAIN(I)-EPSEL(I)
250     CONTINUE
          CALL PLASTK(STRESS,C,VCOF,STRINC,STRSIN,EPSP,FS1,FS2,B1,B2,
          .TDS,PDF,7K1,7K2,S1ZP,HKKMOD,SHRMOD,NSIZ,NELTYP)
          DO 260 I=1,NSIZ
          STRAIN(I)=STRAIN(I)+STRINC(I)
          EPSEL(I)=STRAIN(I)-EPSP(I)
260     CONTINUE
          IF(NELTYP.EQ.3) CALL CADJST (C)
          RETURN
          END

```

* T I D Y *

SUBROUTINE FORMC (POIS,FC,ES,AC,AS,C,GSTL)

	SUBROUTINE FORMC (POIS,FC,ES,AC,AS,C,GSTL)	FOR	1
	DIMENSION POIS(3,3), FC(3), ES(3), AC(3), AS(3), C(6,6), NP(5), GSTL(3)	FOR	2
	DATA (NP(I),I=1,5)/1,2,3,1,2/	FOR	3
	INITIALIZE	FOR	4
C	DO 5 I=1,6	FOR	5
	DO 5 J=1,6	FOR	6
	C(I,J)=0.	FOR	7
5	CONTINUE	FOR	8
	D=1.-POIS(2,3)*POIS(3,2)-POIS(1,2)*POIS(2,1)-POIS(1,3)*POIS(3,1)-P	FOR	9
	10IS(1,2)*POIS(2,3)*POIS(3,1)-POIS(1,3)*POIS(2,1)*POIS(3,2)	FOR	10
	DO 10 I=1,3	FOR	11
	J=NP(I+1)	FOR	12
	K=NP(I+2)	FOR	13
	A=EC(I)*AC(I)/D	FOR	14
	C(I,I)=A*(1.-POIS(J,K)*POIS(K,J))+ES(I)*AS(I)	FOR	15
	C(I,J)=A*(POIS(I,J)+POIS(I,K)*POIS(K,J))	FOR	16
	C(I,K)=A*(POIS(I,K)+POIS(I,J)*POIS(J,K))	FOR	17
	C(I+3,I+3)=POIS(I,I)*AC(I)+GSTL(I)*AB(I)	FOR	18
10	CONTINUE	FOR	19
	DO 15 I=1,2	FOR	20
	II=I+1	FOR	21
	DO 15 J=II,3	FOR	22
	C(I,J)=C(J,I)	FOR	23
15	CONTINUE	FOR	24
	RETURN	FOR	25
	END	FOR	26
		FOR	27

* T I D Y *

SUBROUTINE JACOBI (A,B,X,EIGV,D,N,RTOL)

	SUBROUTINE JACOBI (A,B,X,EIGV,D,N,RTOL)	JAC	1
C	DIMENSION A(N,N), B(N,N), X(N,N), EIGV(N), D(N)	JAC	2
C	NSMAX=15	JAC	3
	DO 5 I=1,N	JAC	4
	D(I)=A(I,I)/B(I,I)	JAC	5
5	EIGV(I)=D(I)	JAC	6
	IF (N.EQ.1) RETURN	JAC	7
	DO 15 I=1,N	JAC	8
	DO 10 J=1,N	JAC	9
10	X(I,J)=0.0	JAC	10
15	X(I,I)=1.0	JAC	11
	NR=N+1	JAC	12
C	WE START ITERATION	JAC	13
C	NS=0	JAC	14
20	NS=NS+1	JAC	15
	EPS=(0.01**NSWEEP)**2	JAC	16
	DO 105 J=1,NR	JAC	17
	JJ=J+1	JAC	18
	DO 105 K=JJ,N	JAC	19
	TT=A(J,K)*A(J,K)	JAC	20
	TH=A(J,J)*A(K,K)	JAC	21
	EPTOLA=ABS(TT/TH)	JAC	22
	TT=B(J,K)*B(J,K)	JAC	23
	TH=B(J,J)*B(K,K)	JAC	24
	EPTOLB=ABS(TT/TH)	JAC	25
	IF ((EPTOLA.LT.EPS).AND.(EPTOLB.LT.EPS)) GO TO 105	JAC	26
	AKK=A(K,K)*B(J,K)-B(K,K)*A(J,K)	JAC	27
	AJJ=A(J,J)*B(J,K)-B(J,J)*A(J,K)	JAC	28
	ARR=A(J,J)*B(K,K)-A(K,K)*B(J,J)	JAC	29
	CHECK=(ARR+4.0*AKK*AJJ)/4.0	JAC	30
	IF (CHECK) 25,30,30	JAC	31
25	WRITE (6,145) CHECK	JAC	32
	STOP	JAC	33
30	SCCH=SQRT(CHECK)	JAC	34
	D1=ARR/2.0+SCCH	JAC	35
	D2=ARR/2.0-SCCH	JAC	36
	DEN=D1	JAC	37
	IF (ABS(D2).GT.ABS(D1)) DEN=D2	JAC	38
	IF (DEN) 40,35,40	JAC	39
35	CA=0.	JAC	40
	CG=A(J,K)/A(K,K)	JAC	41
	GO TO 45	JAC	42
40	CA=AKK/DEN	JAC	43
	CG=AJJ/DEN	JAC	44
C	WE PERFORM THE GENERALIZED ROTATION	JAC	45
C	IF (N=2) 50,95,50	JAC	46
45		JAC	47
		JAC	48
		JAC	49
		JAC	50

* T I D Y *

SUBROUTINE JACOBI (A,R,X,EIGV,D,N,RTOL)

50	JP1=J+1	JAC	51
	JM1=J-1	JAC	52
	KP1=K+1	JAC	53
	KM1=K-1	JAC	54
C		JAC	55
	IF (JM1=1) 65,55,55	JAC	56
55	DO 60 I=1,JM1	JAC	57
	AJ=A(I,J)	JAC	58
	BJ=B(I,J)	JAC	59
	AK=A(I,K)	JAC	60
	BK=B(I,K)	JAC	61
	A(I,J)=AJ+CG*AK	JAC	62
	B(I,J)=BJ+CG*BK	JAC	63
	A(I,K)=AK+CA*AJ	JAC	64
60	B(I,K)=BK+CA*BJ	JAC	65
C		JAC	66
65	IF (KP1=N) 70,70,80	JAC	67
70	DO 75 I=KP1,N	JAC	68
	AJ=A(J,I)	JAC	69
	BJ=B(J,I)	JAC	70
	AK=A(K,I)	JAC	71
	BK=B(K,I)	JAC	72
	A(J,I)=AJ+CG*AK	JAC	73
	B(J,I)=BJ+CG*BK	JAC	74
	A(K,I)=AK+CA*AJ	JAC	75
75	B(K,I)=BK+CA*BJ	JAC	76
C		JAC	77
80	IF (JP1=KM1) 85,85,95	JAC	78
85	DO 90 I=JP1,KM1	JAC	79
	AJ=A(J,I)	JAC	80
	BJ=B(J,I)	JAC	81
	AK=A(I,K)	JAC	82
	BK=B(I,K)	JAC	83
	A(J,I)=AJ+CG*AK	JAC	84
	B(J,I)=BJ+CG*BK	JAC	85
	A(I,K)=AK+CA*AJ	JAC	86
90	B(I,K)=BK+CA*BJ	JAC	87
95	AK=A(K,K)	JAC	88
	BK=B(K,K)	JAC	89
	A(K,K)=AK+2*CA*AJ+CA*CA*AJ	JAC	90
	B(K,K)=BK+2*CA*BJ+CA*CA*BJ	JAC	91
	A(J,J)=AJ+2*CG*AK+CG*CG*AK	JAC	92
	B(J,J)=BJ+2*CG*BK+CG*CG*BK	JAC	93
	A(J,K)=0.0	JAC	94
	B(J,K)=0.0	JAC	95
C		JAC	96
C	UPDATE EIGENVECTORS	JAC	97
	DO 100 I=1,N	JAC	98
	XJ=X(I,J)	JAC	99
	XK=X(I,K)	JAC	100

* T I D Y *

SUBROUTINE JACOBI (A,B,X,EIGV,D,N,RTOL)

	X(I,J)=XJ+CG*XX	JAC 101
100	X(I,K)=XK+CA*XJ	JAC 102
C		JAC 103
105	CONTINUE	JAC 104
C		JAC 105
	DO 110 I=1,N	JAC 106
110	FIGV(I)=A(I,I)/H(I,I)	JAC 107
C		JAC 108
C	CHECK FOR CONVERGENCE	JAC 109
	DO 115 I=1,N	JAC 110
	TOL=RTOL*0(I)	JAC 111
	TOL=ABS(TOL)	JAC 112
	DTF=ABS(FIGV(I)-0(I))	JAC 113
	IF (DTF.GT.TOL) GO TO 130	JAC 114
115	CONTINUE	JAC 115
C		JAC 116
C	CHECK IF ALL OFF-DIAG ELEMENTS ARE SATISFACTORILY SMALL	JAC 117
	EPS=RTOL**2	JAC 118
	DO 120 J=1,NR	JAC 119
	JJ=J+1	JAC 120
	DO 120 K=JJ,N	JAC 121
	TT=A(J,K)*A(J,K)	JAC 122
	TH=A(J,J)*A(K,K)	JAC 123
	EPSA=ABS(TT/TH)	JAC 124
	TT=B(J,K)*B(J,K)	JAC 125
	TH=B(J,J)*B(K,K)	JAC 126
	EPSB=TT/TH	JAC 127
	IF ((EPSA.LT.EPS).AND.(EPSB.LT.EPS)) GO TO 120	JAC 128
	GO TO 130	JAC 129
120	CONTINUE	JAC 130
C		JAC 131
	DO 125 I=1,N	JAC 132
	DO 125 J=I,N	JAC 133
	H(J,I)=R(I,J)	JAC 134
125	A(J,I)=A(I,J)	JAC 135
	RETURN	JAC 136
C		JAC 137
130	DO 135 I=1,N	JAC 138
135	D(I)=FIGV(I)	JAC 139
	IF (NS-EEP.LT.NSMAX) GO TO 20	JAC 140
	DO 140 I=1,N	JAC 141
	DO 140 J=I,N	JAC 142
	B(J,I)=B(I,J)	JAC 143
140	A(J,I)=A(I,J)	JAC 144
	RETURN	JAC 145
C		JAC 146
C		JAC 147
145	FORMAT (8HOCHECK =E20.14)	JAC 148
	END	JAC 149

* T I D Y *

SUBROUTINE LODUN (EPS,DEPS,EPSMAX,LO)

C	SUBROUTINE LODUN (EPS,DEPS,EPSMAX,LO)	LOD	1
C	CHECKS LOADING/UNLOADING IN TENSION	LOD	2
C		LOD	3
	STRAIN=EPS+DEPS	LOD	4
	LO=0	LOD	5
	IF (STRAIN.LT.EPSMAX) RETURN	LOD	6
	LO=1	LOD	7
	IF (STRAIN.GT.EPSMAX) EPSMAX=STRAIN	LOD	8
	RETURN	LOD	9
	END	LOD	10
		LOD	11

```

01T FOR MATPAC,MATPAC
      SUBROUTINE MATPAC (STRESS,STRAIN,STRINC,EPSEL,COEFF,RLKMOD,SHRMOD,
      YCOF,DC,DCS,AST,NRH,ZK1,ZK2,SIZP,ICOMPS,NELTYP,C,ICRACK,NSPLIT)
C
C      IF(NSPLIT,LT,0) STRINC =STRAIN INCREMENT
C      IF(NSPLIT,GE,0) STRINC =STRESS INCREMENT
C      IABS(NSPLIT)= NUMBER OF SPLITS FOR STRESS OR STRAIN INCREMENTS
C
      DIMENSION STRESS(6),STRAIN(6),STRINC(6),EPSEL(6),COEFF(15),
      YCOF(25),DC(3,3),DCS(3,3),AST(3),C(6,6),DEPS(6)
C
      NSIZ=6
      IF(NELTYP,LE,4) NSIZ=4
      IF(NSPLIT,GT,0) GO TO 100
C
C      STRAIN INCREMENT OPTION
C
      NN=IABS(NSPLIT)
      IF(NN,EQ,0) NN=1
      FN=FLOAT(NN)
      DO 5 I=1,6
5      STRINC(I)=STRINC(I)/FN
      DO 10 I=1,NN
      CALL ELFEMM (STRESS,STRAIN,STRINC,EPSEL,COEFF,RLKMOD,SHRMOD,YCOF,
      DC,DCS,AST,NRH,ZK1,ZK2,SIZP,ICOMPS,NELTYP,C,ICRACK)
10      CONTINUE
      RETURN
C
C      STRESS INPUT OPTION
C
100      CONTINUE
      DO 105 I=1,6
105      DEPS(I)=0.
      CALL ELFEMM (STRESS,STRAIN,DEPS,EPSEL,COEFF,RLKMOD,SHRMOD,YCOF,
      DC,DCS,AST,NRH,ZK1,ZK2,SIZP,ICOMPS,NELTYP,C,ICRACK)
      FN=FLOAT(NSPLIT)
      DO 200 I=1,NSPLIT
      DO 110 J=1,6
110      DEPS(J)=STRINC(I)/FN
      CALL SOLVEX (NSIZ,C,DEPS)
      CALL ELFEMM (STRESS,STRAIN,DEPS,EPSEL,COEFF,RLKMOD,SHRMOD,YCOF,
      DC,DCS,AST,NRH,ZK1,ZK2,SIZP,ICOMPS,NELTYP,C,ICRACK)
200      CONTINUE
      RETURN
      END

```

* T I D Y *

SUBROUTINE MATPLY (A,X,Y,IOPT)

```

C      SUBROUTINE MATPLY (A,X,Y,IOPT)
C      MATRIX MULTIPLICATION ROUTINE
C      IOPT,GE.1      Y=A*X
C      IOPT,LT.1      Y=(A)TRANSPOSE * X
C
      DIMENSION A(6,6), X(6), Y(6)
      DO 5 I=1,6
      Y(I)=0.
      DO 5 J=1,6
      IF (IOPT,GE.1) Y(I)=Y(I)+A(I,J)*X(J)
      IF (IOPT,LT.1) Y(I)=Y(I)+A(J,I)*X(J)
5      CONTINUE
      RETURN
      END

```

```

MAT 1
MAT 2
MAT 3
MAT 4
MAT 5
MAT 6
MAT 7
MAT 8
MAT 9
MAT 10
MAT 11
MAT 12
MAT 13
MAT 14
MAT 15

```

* T I D Y *

SUBROUTINE MAXMIN (XX,YY,XY,THETA)

	SUBROUTINE MAXMIN (XX,YY,XY,THETA)	MAX	1
C		MAX	2
C	EVALUATES PRINCIPAL STRESS DIRECTION IN A PLANE	MAX	3
C		MAX	4
	AP=1,E=30	MAX	5
	AM=1,E=30	MAX	6
	AXY=ABS(XY)	MAX	7
	XXYY=ABS(XX-YY)	MAX	8
	IF (XXYY,1E,AP,OR,AXY,LE,AP) GO TO 20	MAX	9
	TN=(2,XY)/(XX+YY)	MAX	10
	ATN=ABS(TN)	MAX	11
	THETA=.5*ATAN(ATN)	MAX	12
	IF ((XX,GT,YY,AND,XX,GT,AP),OR,(YY,LT,XX,AND,YY,LT,AM)) GO TO 5	MAX	13
	GO TO 15	MAX	14
5	IF (XY) 10,20,25	MAX	15
10	THETA=-THETA	MAX	16
	GO TO 25	MAX	17
15	THETA=1.57079-THETA	MAX	18
	IF (XY) 10,20,25	MAX	19
20	THETA=0.	MAX	20
25	RETURN	MAX	21
	END	MAX	22-

* T I D Y *

SUBROUTINE MODULI (S1,S2,S0,EPS1,EC,ES,E1C,E1B,NU,DEPS,EPSMAX)

	SUBROUTINE MODULI (S1,S2,S0,EPS1,EC,ES,E1C,E1B,NU,DEPS,EPSMAX)	MOD	1
	REAL NU	MOD	2
	EMIN=.01*FC	MOD	3
	IF (S1,GE,0.) GO TO 25	MOD	4
	IF ((EPS1+DEPS),GE,EPSMAX) GO TO 25	MOD	5
	EPSMAX=EPS1+DEPS	MOD	6
C	BETA=S2/S1	MOD	7
	BETA=0.	MOD	8
	IF (BETA,LT,0.) BETA=0.	MOD	9
	IF (BETA,GE,0.2) GO TO 5	MOD	10
	SP=S0*(1.+BETA/(1.2-BETA))	MOD	11
	GO TO 20	MOD	12
5	IF (BETA,GT,1.) GO TO 10	MOD	13
	SP=1.2*S0	MOD	14
	GO TO 20	MOD	15
10	IF (BETA,GT,5.) GO TO 15	MOD	16
	SP=1.2*S0/BETA	MOD	17
	GO TO 20	MOD	18
15	SP=S0*(1.+1./(1.2*BETA-1.))/BETA	MOD	19
20	IF (BETA,LE,1.) EPSP=.0025	MOD	20
	IF (BETA,GT,1.) EPSP=(-500.+0.55*SP)*1.0E+6	MOD	21
	X=-EPS1/EPSP	MOD	22
	ESEC=SP/EPSP	MOD	23
	Y=1.+(EC/(ESEC-FSEC*NU*BETA)-2.)*X+X*X	MOD	24
	Y=Y*Y	MOD	25
	E1B=FC*(1.-X*X)/Y	MOD	26
	IF (E1B,LT,EMIN) E1B=EMIN	MOD	27
	E1C=E1B	MOD	28
	RETURN	MOD	29
25	E1B=EC	MOD	30
	E1C=EC	MOD	31
	RETURN	MOD	32
	END	MOD	33

* T I D Y *

```

SUBROUTINE PLASTK (S,C,Y,STRINC,DS,EPSP1,FS1,FS2,B1,B2,TDS,PDE,ZK1
SUBROUTINE PLASTK (S,C,Y,STRINC,DS,EPSP1,FS1,FS2,B1,B2,TDS,PDE,ZK1PLA 1
1,ZK2,S12P,B,G,NSIZ,NFLTYP) PLA 2
C -----THIS SUBROUTINE CALCULATES STRESS INCREMENTS FROM AN INPUT OF PLA 3
C -----STRAIN INCREMENTS, STATE OF STRESS, PLASTIC STRAIN, AND THE YPLA 4
C -----FUNCTION CONSTANTS PLA 5
C -----THIS SUBROUTINE USES TWO STRAINHARDENING YIELD SURFACES PLA 6
DIMENSION S(6), C(6,6), STRINC(6), DS(6), EPSP1(6), FS1(6), FS2(6) PLA 7
1, B1(6), B2(6), TDS(6), PDE(6), Y(1) PLA 8
IF (ABS(ZK1),LT,ABS(Y(24))) ZK1=Y(24) PLA 9
IF (ABS(ZK2),LT,ABS(Y(25))) ZK2=Y(25) PLA 10
C INITIALIZE PLA 11
ZL1=0. PLA 12
ZL2=0. PLA 13
DO 5 I=1,NSIZ PLA 14
FS1(I)=0. PLA 15
FS2(I)=0. PLA 16
B1(I)=0. PLA 17
B2(I)=0. PLA 18
DO 5 J=1,NSIZ PLA 19
C(I,J)=0. PLA 20
CONTINUE PLA 21
DO 10 I=1,3 PLA 22
C(I,I)=R+1.333333*G PLA 23
IF ((I+3),LE,NSIZ) C(I+3,I+3)=0 PLA 24
DO 10 J=1,3 PLA 25
IF (I,NE,J) C(I,J)=R+.666667*G PLA 26
CONTINUE PLA 27
10 IF (NFLTYP,EQ,3) STRINC(3)=0. PLA 28
IF (NFLTYP,EQ,3) STRINC(3)=(C(3,1)*STRINC(1)+C(3,2)*STRINC(2)+C(3,3) PLA 29
1,4)*STRINC(4))/C(3,3) PLA 30
CJ1=S(1)+S(2)+S(3) PLA 31
CJ2=.166667*((S(1)-S(2))**2+(S(2)-S(3))**2+(S(3)-S(1))**2) PLA 32
CJ2=CJ2+S(4)+S(4)+S(5)+S(5)+S(6)+S(6) PLA 33
S12=SQRT(CJ2) PLA 34
DO 15 I=1,NSIZ PLA 35
DS(I)=0. PLA 36
DO 15 J=1,NSIZ PLA 37
DS(I)=DS(I)+C(I,J)*STRINC(J) PLA 38
15 CONTINUE PLA 39
IF (NFLTYP,EQ,3) DS(3)=0. PLA 40
T1=EXP(CJ1/Y(4)) PLA 41
T2=Y(1)*(Y(2)-Y(3)*T1) PLA 42
T3=EXP((CJ1-Y(5))/Y(6)) PLA 43
P1=SJ2-T2*(1.-T3)-ZK1 PLA 44
P2=.5*CJ2+Y(7)*CJ1*ZK2 PLA 45
IF (P1,LT,0.) GO TO 25 PLA 46
H1=1.-Y(9)*(1.-EXP(Y(10)*S12P/(1.-Y(11)*CJ1))) PLA 47
H1=Y(8)*H1/(Y(12)-Y(13)*CJ1) PLA 48
T4=Y(1)*Y(3)*T1*(1.-T3)/Y(4)+T2*T3/Y(6) PLA 49
DO 20 I=1,3 PLA 50

```


* T I D Y *

SUBROUTINE PLASTK (S,C,Y,STRINC,DS,FPSP1,FS1,FS2,H1,H2,TDS,PDF,ZK1

	FS1(I)=(.5*S(I)+.166667*CJ1)/SJ2+T4	PLA 51
	IF ((I+3).LE.NSIZ) FS1(I+3)=2.*S(I+3)/SJ2	PLA 52
20	CONTINUE	PLA 53
25	IF (F2.LT.0.) GO TO 35	PLA 54
	T5=Y(14)*SJ2*(1.-EXP(-SJ2/Y(15)))*EXP(-CJ2*CJ2/Y(16))	PLA 55
	T5=T5/(1.-Y(20)*CJ1)	PLA 56
	H2=Y(17)*(1.-EXP(CJ1*CJ1/Y(18)))*EXP(CJ1/Y(19))+T5	PLA 57
	DO 30 I=1,3	PLA 58
	FS2(I)=.5*S(I)+(2.*Y(7)+.166667)*CJ1	PLA 59
	IF ((I+3).LE.NSIZ) FS2(I+3)=2.*S(I+3)	PLA 60
30	CONTINUE	PLA 61
35	IF ((F1.LT.0.).AND.(F2.LT.0.)) GO TO 145	PLA 62
	DO 40 I=1,NSIZ	PLA 63
	DO 40 J=1,NSIZ	PLA 64
	B1(I)=H1(I)+C(I,J)*FS1(J)	PLA 65
	B2(I)=H2(I)+C(I,J)*FS2(J)	PLA 66
40	CONTINUE	PLA 67
	AB1=0.	PLA 68
	AB2=0.	PLA 69
	AB3=0.	PLA 70
	AB4=0.	PLA 71
	AB5=0.	PLA 72
	DO 45 I=1,NSIZ	PLA 73
	AB1=AB1+FS1(I)*B1(I)	PLA 74
	AB2=AB2+FS1(I)*B2(I)	PLA 75
	AB3=AB3+FS1(I)*DS(I)	PLA 76
	AB4=AB4+FS2(I)*B2(I)	PLA 77
	AB5=AB5+FS2(I)*DS(I)	PLA 78
45	CONTINUE	PLA 79
	A1=1.+2.*H1*AB1	PLA 80
	A2=SJ2+2.*H1*AB2	PLA 81
	A3=2.*H1*AB3	PLA 82
	A4=.3333333*(2*AB2	PLA 83
	A5=2.*Y(7)*CJ1+.3333333*H2*AB4	PLA 84
	A6=.3333333*(2*AB5	PLA 85
	IF (F1) 50,55,55	PLA 86
50	ZL2=A6/A5	PLA 87
	GO TO 70	PLA 88
55	IF (F2) 60,65,65	PLA 89
60	ZL1=A3/A1	PLA 90
	GO TO 70	PLA 91
65	T6=A1*A5-A2*A4	PLA 92
	ZL1=(A3*A5-A2*A6)/T6	PLA 93
	ZL2=(A1*A6-A3*A4)/T6	PLA 94
70	CONTINUE	PLA 95
75	DF1=0.	PLA 96
	DF2=0.	PLA 97
	DO 80 I=1,NSIZ	PLA 98
	TDS(I)=DS(I)+B1(I)*ZL1+B2(I)*ZL2	PLA 99
	DF1=DF1+FS1(I)*TDS(I)	PLA 100

* T I D Y *

SUBROUTINE PLASTK (S,C,Y,STRING,DS,EPSP1,FS1,FS2,B1,B2,TDS,PDE,ZK1

	DF2=DF2+FS2(I)*TDS(I)	PLA 101
80	CONTINUE	PLA 102
	TZL2=ZL2	PLA 103
	IF (DF1*ZL1) A5,100,100	PLA 104
85	IF (DF2*ZL2) 90,95,95	PLA 105
90	ZL2=0.	PLA 106
	GO TO 75	PLA 107
95	ZL1=0.	PLA 108
	ZL2=TZL2	PLA 109
	GO TO 75	PLA 110
100	IF (DF2*ZL2) 105,110,110	PLA 111
105	ZL2=0.	PLA 112
	GO TO 75	PLA 113
110	CONTINUE	PLA 114
C	-----INCREMENTING OF YIELD SURFACES FOLLOWS	PLA 115
	IF (DF1*ZL1) 120,120,115	PLA 116
115	ZK1=ZK1+DF1	PLA 117
120	IF (DF2*ZL2) 130,130,125	PLA 118
125	ZK2=ZK2+DF2	PLA 119
130	CONTINUE	PLA 120
	IF (AELTYP,FQ,3) TDS(3)=0.	PLA 121
	DO 135 I=1,NSIZ	PLA 122
	S(I)=S(I)+TDS(I)	PLA 123
	PDE(I)=FS1(I)*ZL1+FS2(I)*ZL2	PLA 124
	EPSP1(I)=EPSP1(I)+PDE(I)	PLA 125
135	CONTINUE	PLA 126
	SSP=0.	PLA 127
	DO 140 I=1,3	PLA 128
	II=I+1	PLA 129
	IF (II,GT,3) II=1	PLA 130
	SSP=SSP+.166667*(EPSP1(I)-EPSP1(II))*(EPSP1(I)-EPSP1(II))	PLA 131
	IF ((I+3).LE,NSIZ) SSP=SSP+.25*EPSP1(I+3)*EPSP1(I+3)	PLA 132
140	CONTINUE	PLA 133
	SI2P=SQRT(SSP)	PLA 134
	GO TO 155	PLA 135
145	CONTINUE	PLA 136
	DO 150 I=1,NSIZ	PLA 137
	S(I)=S(I)+DS(I)	PLA 138
150	CONTINUE	PLA 139
	GO TO 180	PLA 140
155	CONTINUE	PLA 141
	X1=0.	PLA 142
	X2=0.	PLA 143
	X3=0.	PLA 144
	X4=0.	PLA 145
	DF1=DF1*ZL1	PLA 146
	DF2=DF2*ZL2	PLA 147
	IF (F1,LT,0.,OR,DF1,LE,0.) GO TO 160	PLA 148
	IF (F2,LT,0.,OR,DF2,LE,0.) GO TO 165	PLA 149
	X1=A5*2.*W1/T6	PLA 150

* T I D Y *

SJHROUTINE PLASTK (S,C,Y,STRINC,DS,EPSP1,FS1,FS2,B1,B2,IDS,P02,ZK1

	X2=A2*.333333*H2/T6	PLA 151
	X3=A1*.333333*H2/T6	PLA 152
	X4=A4*2.*H1/T6	PLA 153
	GO TO 170	PLA 154
160	X3=.333333*H2/A5	PLA 155
	GO TO 170	PLA 156
165	X1=2.*H1/A1	PLA 157
170	DO 175 I=1,NSIZ	PLA 158
	DO 175 J=1,NSTZ	PLA 159
	C(I,J)=C(I,J)+B1(I)*(B1(J)*X1+B2(J)*X2)+B2(I)*(B2(J)*X3+B1(J)*X4)	PLA 160
	IF (I.NE.J) C(J,I)=C(I,J)	PLA 161
175	CONTINUE	PLA 162
	IF (NELTYP.EQ.3) STRINC(3)=(C(3,1)*STRINC(1)+C(3,2)*STRINC(2)+C(3,3)*STRINC(4))/C(3,3)	PLA 163
180	RETURN	PLA 164
	END	PLA 165
		PLA 166=

* T I D Y *

SUBROUTINE PRNSTR (A,B,S,X,SIG,D)

	SUBROUTINE PRNSTR (A,B,S,X,SIG,D)	PRN 1
		PRN 2
C	EVALUATES PRINCIPAL STRESSES AND DIRECTIONS	PRN 3
C		PRN 4
C	DIMENSION A(3,3), B(3,3), S(6), X(3,3), SIG(6), D(3)	PRN 5
	N=3	PRN 6
	RTOL=.1E-4	PRN 7
	A(1,2)=S(4)	PRN 8
	A(1,3)=S(6)	PRN 9
	A(2,3)=S(5)	PRN 10
	DO 5 I=1,3	PRN 11
	A(I,1)=S(I)	PRN 12
	B(I,1)=1.	PRN 13
	DO 5 J=1,3	PRN 14
	IF (J,FO,I) GO TO 5	PRN 15
	B(I,J)=0.	PRN 16
	B(J,I)=0.	PRN 17
	A(J,I)=A(I,J)	PRN 18
5	CONTINUE	PRN 19
C	CALL JACOBI FOR EIGENVALUES AND EIGENVECTORS (PRINCIPAL STRESSES)	PRN 20
	CALL JACOBI (A,B,X,SIG,D,N,RTOL)	PRN 21
	DO 10 I=1,3	PRN 22
	C=SQRT(X(1,I)*X(1,I)+X(2,I)*X(2,I)+X(3,I)*X(3,I))	PRN 23
	SIG(I+3)=0.	PRN 24
	DO 10 J=1,3	PRN 25
	X(J,I)=X(J,I)/C	PRN 26
10	CONTINUE	PRN 27
	RETURN	PRN 28
	END	PRN 29

* T I D Y *

SUBROUTINE REBOND (EPS,ECC,EC,ERBOND;

C SUBROUTINE REBOND (EPS,ECC,EC,ERBOND)
CHECKS FOR REBONDING AFTER CRACK
ECC=0.01*EC
IF (EPS.LT.0.) ECC=ERBOND*EC
RETURN
END

RFB 1
REB 2
REB 3
REB 4
REB 5
REB 6-

```

017 FOR SOLVEX,SOLVEX
      SUBROUTINE SOLVEX (N,A,H)
      DIMENSION A(6,6),B(6)
C
C      B=A*INVERSE * H
C
      TRACE=0.
      DO 5 I=1,N
5      TRACE =TRACE+ABS(A(I,I))
      TRACE=.001*TRACE
      DO 6 I=1,N
      IF(ABS(A(I,I)).LT,TRACE)A(I,I)=TRACE
6      CONTINUE
C
      DO 40 I=1,N
      IF(A(I,I).EQ.0.) GO TO 40
      D=1./A(I,I)
      DO 10 J=1,N
10     A(I,J)=A(I,J)*D
      B(I)=B(I)*D
      DO 30 K=1,N
      IF(K.EQ.I) GO TO 30
      C=A(K,I)
      IF(C.EQ.0.) GO TO 30
      DO 20 L=1,N
20     A(K,L)=A(K,L)-C*A(I,L)
      B(K)=B(K)-C*B(I)
30     CONTINUE
40     CONTINUE
      RETURN
      END

```

* T I D Y *

SUBROUTINE STLTN (AS,DC,DCS,DCC,NRB,ASX,ASY,ASZ)

	SUBROUTINE STLTN (AS,DC,DCS,DCC,NRB,ASX,ASY,ASZ)	STL	1
C		STL	2
C	TRANSFORMS STEEL AREAS TO APPROPRIATE DIRECTIONS	STL	3
C		STL	4
C	DIMENSION AS(3), DC(3,3), DCS(3,1), DCC(3,1)	STL	5
C		STL	6
	IF (NRB.GT.0) GO TO 10	STL	7
	DO 5 I=1,3	STL	8
	AS(I)=ASX*DC(1,I)+DC(1,I)+ASY*DC(2,I)+DC(2,I)+ASZ*DC(3,I)+DC(3,I)	STL	9
5	CONTINUE	STL	10
	RETURN	STL	11
C		STL	12
10	DO 15 I=1,3	STL	13
	DO 15 J=1,3	STL	14
	DCC(I,J)=DCS(1,I)*DC(1,J)+DCS(2,I)*DC(2,J)+DCS(3,I)*DC(3,J)	STL	15
15	CONTINUE	STL	16
	DO 20 I=1,3	STL	17
	AS(I)=ASX*DCC(1,I)+DCC(1,I)+ASY*DCC(2,I)+DCC(2,I)+ASZ*DCC(3,I)+DCC(3,I)	STL	18
	1(3,I)	STL	19
20	CONTINUE	STL	20
	RETURN	STL	21
C		STL	22
	END	STL	23-

* T I D Y *

SUBROUTINE TRANSF (A,B,DC)

C	SUBROUTINE TRANSF (A,B,DC)	TRN	1
C	FORMULATES TRANSFORMATION MATRICIES TO CONVERT STRESS/STRAIN	TRN	2
C	ONE SET OF COORDINATE SYSTEM TO ANOTHER	TRN	3
C		TRN	4
	DIMENSION A(6,6), B(6,6), DC(3,3), NP(6)	TRN	5
	DATA (NP(I),I=1,6)/1,2,2,3,3,1/	TRN	6
C	FORM TRANSFORMATION MATRICIES A AND B	TRN	7
	DO 5 I=1,3	TRN	8
	L1=NP(2*I-1)	TRN	9
	L2=NP(2*I)	TRN	10
	DO 5 J=1,3	TRN	11
	K1=NP(2*J-1)	TRN	12
	K2=NP(2*J)	TRN	13
	B(I,J)=DC(I,J)*DC(I,J)	TRN	14
	B(I+3,J+3)=DC(L1,K1)*DC(L2,K2)+DC(L1,K2)*DC(L2,K1)	TRN	15
	H(I+3,J)=DC(L1,J)*DC(L2,J)	TRN	16
	B(I,J+3)=DC(I,K1)*DC(I,K2)*2.	TRN	17
	A(J,I)=B(I,J)	TRN	18
	A(J+3,I+3)=H(I+3,J+3)	TRN	19
	A(J,I+3)=B(I+3,J)*2.	TRN	20
	A(J+3,I)=H(I,J+3)/2.	TRN	21
5	CONTINUE	TRN	22
	RETURN	TRN	23
	END	TRN	24
		TRN	25.


```

217 FOR VARNOD,VARNOD
  SUBROUTINE VARNOD (SIG,EPS,SO,SY,EC,FS,NU,AS,C,ICRACK,NELTYP,DEPS,
    ,EPSEL,ALFA,GAMA,EN,GST,ERROND)
  DIMENSION SIG(1),EPS(1),AS(1),C(6,6),ECC(3),FSS(3),AC(3),POIS(3,3)
  1,DEPS(1),EPSEL(1),GSTL(3)
  REAL NU
  S1=SIG(1)
  S2=SIG(2)
  S3=SIG(3)
  E1=EPS(1)
  E2=EPS(2)
  E3=EPS(3)
  G1=GST
  DO 5 I=1,3
    GSTL(I)=GST
    POIS(I,1)=0.
    AC(I)=1.-AS(I)
    FSS(I)=ES
  DO 5 J=1,3
    IF(I.NE.J) POIS(I,J)=NU
5  CONTINUE
    IF(ICRACK.EQ.0) GO TO 100
    POIS(2,1)=0.
    POIS(3,1)=0.
    POIS(1,2)=0.
    POIS(1,3)=0.
    GSTL(1)=0.
    GSTL(3)=0.
    S=S2+S3
    CALL LODUN (EPS(1),DEPS(1),EPSEL(1),LD)
    IF (LD.GT.0 ) CALL COMPOS (S1,SY,AC(1),AS(1),EC,ES,ECC(1),FSS(1))
    IF(LD.LE.0) CALL REBOND (EPS(1),ECC(1),EC,ERROND)
    IF(ICRACK.EQ.1) GO TO 20
    POIS(2,3)=0.
    POIS(3,2)=0.
    GSTL(2)=0.
    S=S1+S3
    CALL LODUN (EPS(2),DEPS(2),EPSEL(2),LD)
    IF (LD.GT.0 ) CALL COMPOS (S2,SY,AC(2),AS(2),EC,ES,ECC(2),FSS(2))
    IF(LD.LE.0) CALL REBOND (EPS(2),ECC(2),EC,ERROND)
    IF(ICRACK.EQ.2) GO TO 40
    S=S1+S2
    CALL LODUN (EPS(3),DEPS(3),EPSEL(3),LD)
    IF (LD.GT.0 ) CALL COMPOS (S3,SY,AC(3),AS(3),EC,ES,ECC(3),FSS(3))
    IF(LD.LE.0) CALL REBOND (EPS(3),ECC(3),EC,ERROND)
    GO TO 200
20  CALL MODULI (S2,S3,SO,E2,FC,FS,ECC(2),DUM2,NU,DEPS(2),EPSEL(2))
    CALL MODULI (S3,S2,SO,E3,EC,ES,ECC(3),DUM3,NU,DEPS(3),EPSEL(3))
    POIS(2,2)=DUM2*DUM3/((DUM2+DUM3)*(1.+NU))
    IF (AS(1).LE.0.) GO TO 200
    S=S1/AS(1)
    CALL DDEL1 (S,SY,SO,ALFA,GAMA,EN,G1)
    CALL DDEL2 (G1,ALFA,EPSEL(4),DEPS(4),EPSEL(4),GSTL(1))
    IF(ABS(NELTYP).GT.4) CALL DDEL2 (G1,ALFA,EPSEL(6),DEPS(6),EPSEL(6),
    ,GSTL(3))
    GO TO 200
40  S=0.
    CALL MODULI (S3,S,SO,E3,FC,FS,ECC(3),DUM3,NU,DEPS(3),EPSEL(3))
    GO TO 200
100 S=S2+S3

```

```

CALL MODULT (S1,S ,SD,E1,EC,ES,ECC(1),DUM1,NU,DEPS(1),EPSEL(1))
S=S1+S3
CALL MODULT (S2,S ,SD,E2,EC,FS,FCC(2),DUM2,NU,DEPS(2),EPSEL(2))
S=S1+S2
CALL MODULT (S3,S,SD,E3,EC,ES,FCC(3),DUM3,NU,DEPS(3),EPSEL(3))
POIS(1,1)=DUM1*DUM2/((DUM1+DUM2)*(1.+NU))
POIS(2,2)=DUM2*DUM3/((DUM2+DUM3)*(1.+NU))
POIS(3,3)=DUM1*DUM3/((DUM1+DUM3)*(1.+NU))
200 CALL FORMC (POIS,ECC,ESS,AC,AS,C,GSTL)
RETURN
END

```

6. DEMONSTRATION PROBLEMS

Two simple one-dimensional demonstration cases are included in the following. The first problem is used to demonstrate the capability of the material model to use strain as input to calculate stress. The second problem is used to demonstrate the capability of the material model to accept stress input. The stress calculated from the first example was used as input to the material model in the second example.

a. Demonstration Problem 1

This example has been run to compute the stresses corresponding to several specified strain increments. The input parameters were:

NRB = 0; ICOMPS = 1; NELTYP = 4 (plane strain problem);
NSTEPS = 12; NSPLIT = -1 (indicating strain input)

NRB equal to zero implies that steel is oriented in the global coordinate directions. Thus it is not necessary to provide steel orientation defined by DCS.

CØEFF(1)	=	3×10^6 psi ($20,685 \times 10^6$ N/m ²)	=	E_c
CØEFF(2)	=	30×10^6 psi ($20,685 \times 10^7$ N/m ²)	=	E_s
CØEFF(3)	=	6×10^3 psi ($41,370 \times 10^3$ N/m ²)	=	f'_c
CØEFF(4)	=	4.8×10^4 psi ($33,096 \times 10^4$ N/m ²)	=	f_y
CØEFF(5)	=	0.15	=	ν
CØEFF(6)	=	500 psi (34.5×10^5 N/m ²)	=	f_t
CØEFF(7)	=	-0.083	=	η (slope of cracking envelope)
CØEFF(8)	=	4.0	=	n (dowel shear coefficient)

CØEFF(9) = 1.2×10^7 psi (8274×10^7 N/m²) = G_s

CØEFF(10) = 0.5 = Rebonding coefficient

COEFF(11 through 15) = 0.0 (not necessary for this example)

AST(1) = 0.02

AST(2) = 0.0

AST(3) = 0.0

DCS: 0, ..., 0

The element is subjected to compressive loading, then unloading, and then reloading.

Strain increments: 1 through 7 are -5×10^{-4} ; 8, 9 are $+5 \times 10^{-4}$; 10 through 14 are -5×10^{-4} .

The resulting stress/strain curve is shown in figure 108(a).

b. Demonstration Problem 2

Using the stress increment shown in figure 108(a) as input, the material program was run again with the following parameters:

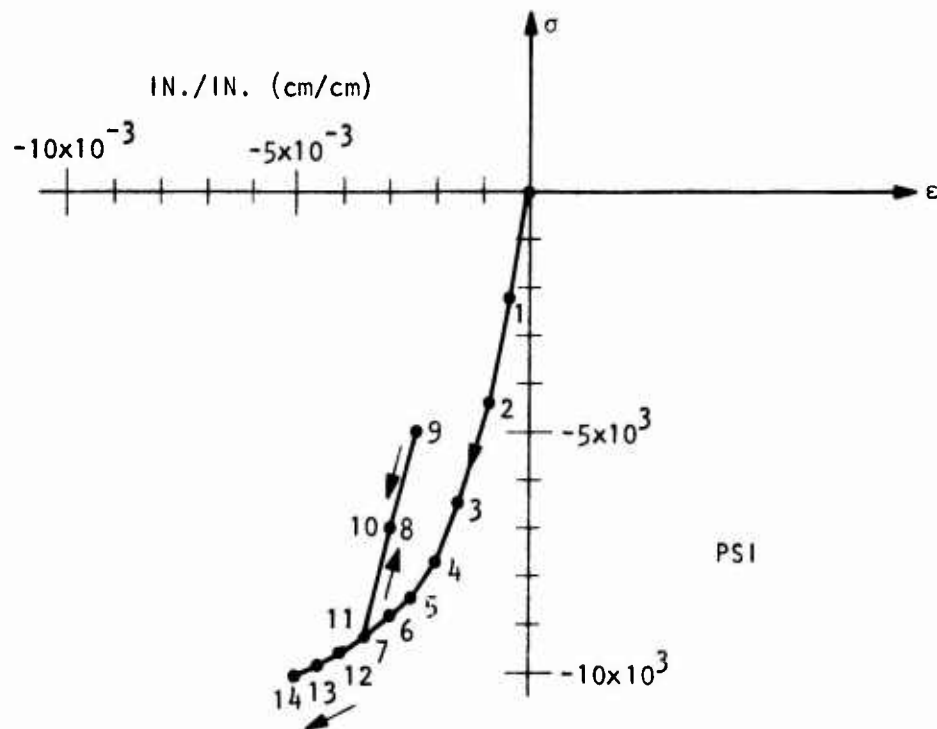
NRB = 0; ICOMPS = 1; NELTYP = 4; NSTEPS = 11;

NSPLIT = 5 (indicating stress input)

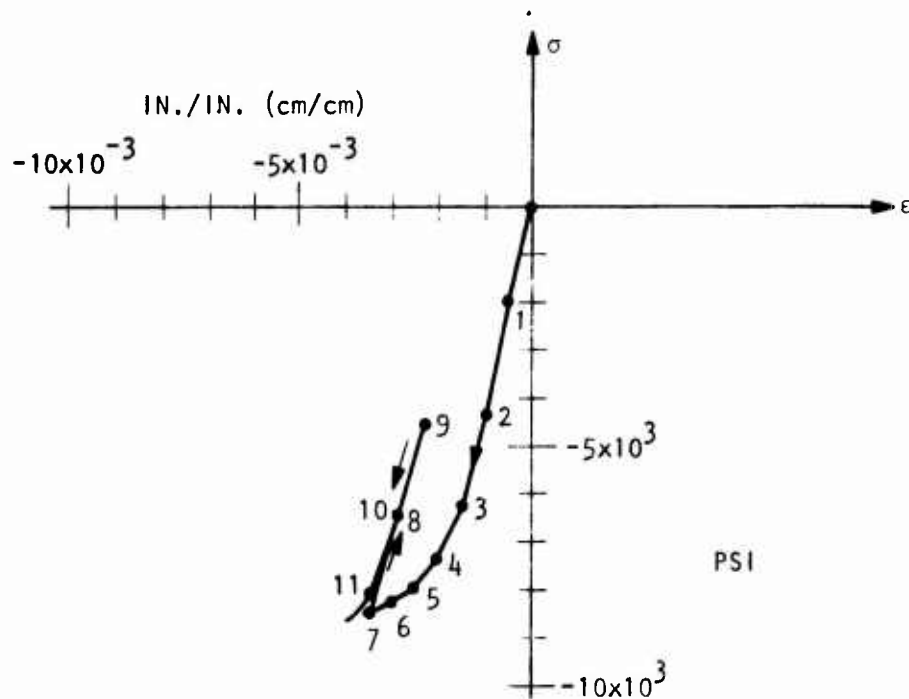
CØEFF(1 through 15) = Same as for example 1

AST(1 through 3) = Same as for example 1

DCS = Same as for example 1



(a) Stress corresponding to specified strain increments



(b) Strain corresponding to specified stress increments

Figure 108. Stress/Strain diagram for Examples 1 and 2
(English Units)

Stress increments:

$$\begin{aligned} & -2.110 \times 10^3, \quad -2.303 \times 10^3, \quad -2.009 \times 10^3, \quad -1.382 \times 10^3 \\ & -7.478 \times 10^2, \quad -3.181 \times 10^2, \quad -3.181 \times 10^2, \quad +2.111 \times 10^3 \\ & +2.111 \times 10^3, \quad -2.111 \times 10^3, \quad -2.111 \times 10^3 \end{aligned}$$

The stress increments for components 2 and 3 are 1 through 7: -3.26×10^2 ; and 8,9: $+3.26 \times 10^2$; 10,11: -3.26×10^2 .

The resulting stress-strain diagram is shown in figure 108(b).

Results of figure 108(b) are in good agreement with those of figure 108(a). A listing of the input and output from these two examples is included in the next pages.

EXAMPLE 1 UNIAXIAL STRAIN (2 PERCENT STEEL) STRAIN INPUT OPTION

NRB 0 ICOMPS 1 NELTYP 4 NSTEPS 30 NSPLIY 1

COEFF	.35000+07	.30000+08	.60000+08	.48000+03	.15000+00	.50000+03	.40000+01	.12000+08	.50000+00
AST	.00000	.00000	.00000	.00000	.00000				
	.20000+01	.00000	.00000						
DCS	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
STEP= 1									
STRING	.50000+03	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
STRESS	.21179+04	.32609+03	.32609+03	.00000	.00000	.00000	.00000	.00000	.00000
STRAIN	.50000+03	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
EPSEL	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
ICRACK	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
DIRECTS	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
CMATRIX									
	.42217+07	.65217+06	.65217+06	.00000	.00000	.00000	.00000	.00000	.00000
	.65217+06	.36957+07	.65217+06	.00000	.00000	.00000	.00000	.00000	.00000
	.65217+06	.65217+06	.36957+07	.00000	.00000	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.17313+07	.00000	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.00000	.15217+07	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.00000	.00000	.15217+07	.00000	.00000	.00000
STEP= 2									
STRING	.50000+03	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
STRESS	.44137+04	.65217+03	.65217+03	.00000	.00000	.00000	.00000	.00000	.00000
STRAIN	.10000+02	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
EPSEL	.10000+02	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
ICRACK	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
DIRECTS	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
CMATRIX									
	.46050+07	.65217+06	.65217+06	.00000	.00000	.00000	.00000	.00000	.00000
	.65217+06	.36957+07	.65217+06	.00000	.00000	.00000	.00000	.00000	.00000
	.65217+06	.65217+06	.36957+07	.00000	.00000	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.18064+07	.00000	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.00000	.15217+07	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.00000	.00000	.15983+07	.00000	.00000	.00000
STEP= 3									
STRING	.50000+03	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
STRESS	.64230+04	.97826+03	.97826+03	.00000	.00000	.00000	.00000	.00000	.00000
STRAIN	.15000+02	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
EPSEL	.15000+02	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
ICRACK	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
DIRECTS	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
CMATRIX									
	.40187+07	.65217+06	.65217+06	.00000	.00000	.00000	.00000	.00000	.00000
	.65217+06	.36957+07	.65217+06	.00000	.00000	.00000	.00000	.00000	.00000
	.65217+06	.65217+06	.36957+07	.00000	.00000	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.16883+07	.00000	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.00000	.15217+07	.00000	.00000	.00000	.00000
	.00000	.00000	.00000	.00000	.00000	.14779+07	.00000	.00000	.00000
STEP= 4									


```

.00000 .00000 .00000 .00000 .00000 .30133+05
STEP= 17
STRINC -.5000+05 .0000 .0000 .0000 .0000 .0000
STRESS -.11098+05 -.42391+04 .0000 .0000 .0000 .0000
STRAIN -.65000+02 .00000 .00000 .00000 .00000 .00000
EPSEL -.65000+02 .00000 .00000 .00000 .00000 .00000
ICRACK 0
DIRCOS .10000+01 .00000 .00000 .00000 .00000 .00000
CMATRIX
.63622+06 .65217+06 .65217+06 .00000 .00000 .00000
.65217+06 .36957+07 .65217+06 .00000 .00000 .00000
.65217+06 .65217+06 .36957+07 .00000 .00000 .00000
.00000 .00000 .00000 .26953+06 .00000 .00000
.00000 .00000 .00000 .00000 .15217+07 .00000
.00000 .00000 .00000 .00000 .00000 .30133+05
STEP= 18
STRINC .5000+03 .0000 .0000 .0000 .0000 .0000
STRESS -.89847+08 -.39130+04 .00000 .00000 .00000 .00000
STRAIN -.60000+02 .00000 .00000 .00000 .00000 .00000
EPSEL -.65000+02 .00000 .00000 .00000 .00000 .00000
ICRACK 0
DIRCOS .10000+01 .00000 .00000 .00000 .00000 .00000
CMATRIX
.42217+07 .65217+06 .65217+06 .00000 .00000 .00000
.65217+06 .36957+07 .65217+06 .00000 .00000 .00000
.65217+06 .65217+06 .36957+07 .00000 .00000 .00000
.00000 .00000 .00000 .17313+07 .00000 .00000
.00000 .00000 .00000 .00000 .15217+07 .00000
.00000 .00000 .00000 .00000 .00000 .15217+07
STEP= 19
STRINC .5000+03 .0000 .0000 .0000 .0000 .0000
STRESS -.67546+04 -.35470+04 .00000 .00000 .00000 .00000
STRAIN -.55000+02 .00000 .00000 .00000 .00000 .00000
EPSEL -.65000+02 .00000 .00000 .00000 .00000 .00000
ICRACK 0
DIRCOS .10000+01 .00000 .00000 .00000 .00000 .00000
CMATRIX
.42217+07 .65217+06 .65217+06 .00000 .00000 .00000
.65217+06 .36957+07 .65217+06 .00000 .00000 .00000
.65217+06 .65217+06 .36957+07 .00000 .00000 .00000
.00000 .00000 .00000 .17313+07 .00000 .00000
.00000 .00000 .00000 .00000 .15217+07 .00000
.00000 .00000 .00000 .00000 .00000 .15217+07
STEP= 20
STRINC .5000+03 .0000 .0000 .0000 .0000 .0000
STRESS -.47649+04 -.32609+04 .00000 .00000 .00000 .00000
STRAIN -.50000+02 .00000 .00000 .00000 .00000 .00000
EPSEL -.65000+02 .00000 .00000 .00000 .00000 .00000
ICRACK 0
DIRCOS .10000+01 .00000 .00000 .00000 .00000 .00000
CMATRIX
.42217+07 .65217+06 .65217+06 .00000 .00000 .00000
.65217+06 .36957+07 .65217+06 .00000 .00000 .00000
.65217+06 .65217+06 .36957+07 .00000 .00000 .00000
.00000 .00000 .00000 .17313+07 .00000 .00000
.00000 .00000 .00000 .00000 .15217+07 .00000
.00000 .00000 .00000 .00000 .00000 .15217+07
STEP= 21
STRINC .5000+03 .0000 .0000 .0000 .0000 .0000
STRESS -.26541+04 -.29348+04 .00000 .00000 .00000 .00000

```

314

316


```

ICRACK 0
DIRCOS  .10000+01  .00000  .00000  .10000+01  .00000  .00000  .10000+01
CMATRIX
.4217+07  .65217+06  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .36957+07  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .65217+06  .36957+07  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .17313+07  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .15217+07
.15217+07
STEP 9
STRINC  .2111+04  .3260+03  .3260+03  .0000  .0000  .0000  .0000
STRESS  -.4363+04  -.1630+04  -.1630+04  .00000  .00000  .00000  .00000
STRAIN  -.25731-02  .10942+04  .10942+04  .00000  .00000  .00000  .00000
ESEL  -.35732-02  -.26019-05  -.26019-05  .00000  .00000  .00000  .00000
ICRACK 0
DIRCOS  .10000+01  .00000  .00000  .10000+01  .00000  .00000  .10000+01
CMATRIX
.4217+07  .65217+06  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .36957+07  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .65217+06  .36957+07  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .17313+07  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .15217+07
.15217+07
STEP 10
STRINC  .2111+04  .3260+03  .3260+03  .0000  .0000  .0000  .0000
STRESS  -.6074+04  -.1956+04  -.1956+04  .00000  .00000  .00000  .00000
STRAIN  -.30732-02  .10947+04  .10947+04  .00000  .00000  .00000  .00000
ESEL  -.35732-02  -.26019-05  -.26019-05  .00000  .00000  .00000  .00000
ICRACK 0
DIRCOS  .10000+01  .00000  .00000  .10000+01  .00000  .00000  .10000+01
CMATRIX
.4217+07  .65217+06  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .36957+07  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .65217+06  .36957+07  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .17313+07  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .15217+07
.15217+07
STEP 11
STRINC  .2111+04  .3260+03  .3260+03  .0000  .0000  .0000  .0000
STRESS  -.82249+04  -.22826+04  -.22826+04  .00000  .00000  .00000  .00000
STRAIN  -.35732-02  .10949+04  .10949+04  .00000  .00000  .00000  .00000
ESEL  -.35732-02  -.26019-05  -.26019-05  .00000  .00000  .00000  .00000
ICRACK 0
DIRCOS  .10000+01  .00000  .00000  .10000+01  .00000  .00000  .10000+01
CMATRIX
.63622+06  .65217+06  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .36957+07  .65217+06  .00000  .00000  .00000  .00000
.65217+06  .65217+06  .36957+07  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .26953+06  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .00000
.00000  .00000  .00000  .00000  .00000  .00000  .15217+07
.15217+07
.36133+05

```

7. LISTING OF MAIN PROGRAM FOR TESTING MATPAC PACKAGE

```

017 FOR MAIN1,MAIN1
C   MATERIAL PACKAGE TEST PROGRAM
   DIMENSION STRESS(6),STRAIN(6),EPSSEL(6),STRING(6),COEFF(15),
   *YCOF(25),DC(3,3),DCS(3,3),AST(3),C(6,6),TITLEF(12)
   READ(5,50) TITLE,ARM,ICOMPS,NELTYP,NSTEPS,NSPLIT
50  FORMAT(12A6/5I5)
   WRITE(6,60) TITLE
60  FORMAT(//25X,12A6//)
   WRITE(6,70) ARM,ICOMPS,NELTYP,NSTEPS,NSPLIT
70  FORMAT(   50H      ARM      ICOMPS      NELTYP      NSTEPS      NSPLIT/
15I10/)
   IF (ICOMPS.EQ.2) GO TO 200
   READ(5,80) COEFF
   WRITE(6,90) COEFF
80  FORMAT(8F10,4)
90  FORMAT(// 6H COEFF/(10E12,5) )
   READ(5,100) AST
   IF (ARM.GT.0.) READ (5,80) DCS
   WRITE(6,100) AST,DCS
100 FORMAT(4H AST, 3E12,4//4H DCS,9E12,5/)
   GO TO 300
200 READ(5,80) H1,KMOD ,SHRMOD
   READ(5,80) YCOF
   WRITE(6,110) H1,KMOD,SHRMOD ,YCOF
110 FORMAT(10H.....H1,KMOD,E12,5,10H.....SHRMOD,E12,5//6H YCOF/
   *(10E12,5))
300 DO 400 N=1,NSTEPS
   READ(5,60) STRING
   WRITE(6,130) *STRING
   CALL MATPAC (STRESS,STRAIN,STRING,EPSSEL,COEFF,H1,KMOD ,SHRMOD,
   *YCOF,DC,DCS,AST,ARM,ZK1,7K2,91PP,ICOMPS,NELTYP,C,ICRACK,NSPLIT)
130 FORMAT(6H STEP#,13//7H STRING,6E12,4)
   WRITE(6,140) STRESS,STRAIN,EPSSEL,ICRACK,DC,C
140 FORMAT(7H STRESS, 6E12,5/7H STRAIN,6E12,5/6H EPSSEL, 6E12,5/7H ICRA
   *CK,13/7H DIRCOS,9E12,5/8H CMATRIX/(6E12,5))
400 CONTINUE
   END

```

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APPENDIX II

PROCEDURE FOR SETTING UP INPUT FOR ANALYTICAL MODEL

1. INTRODUCTION

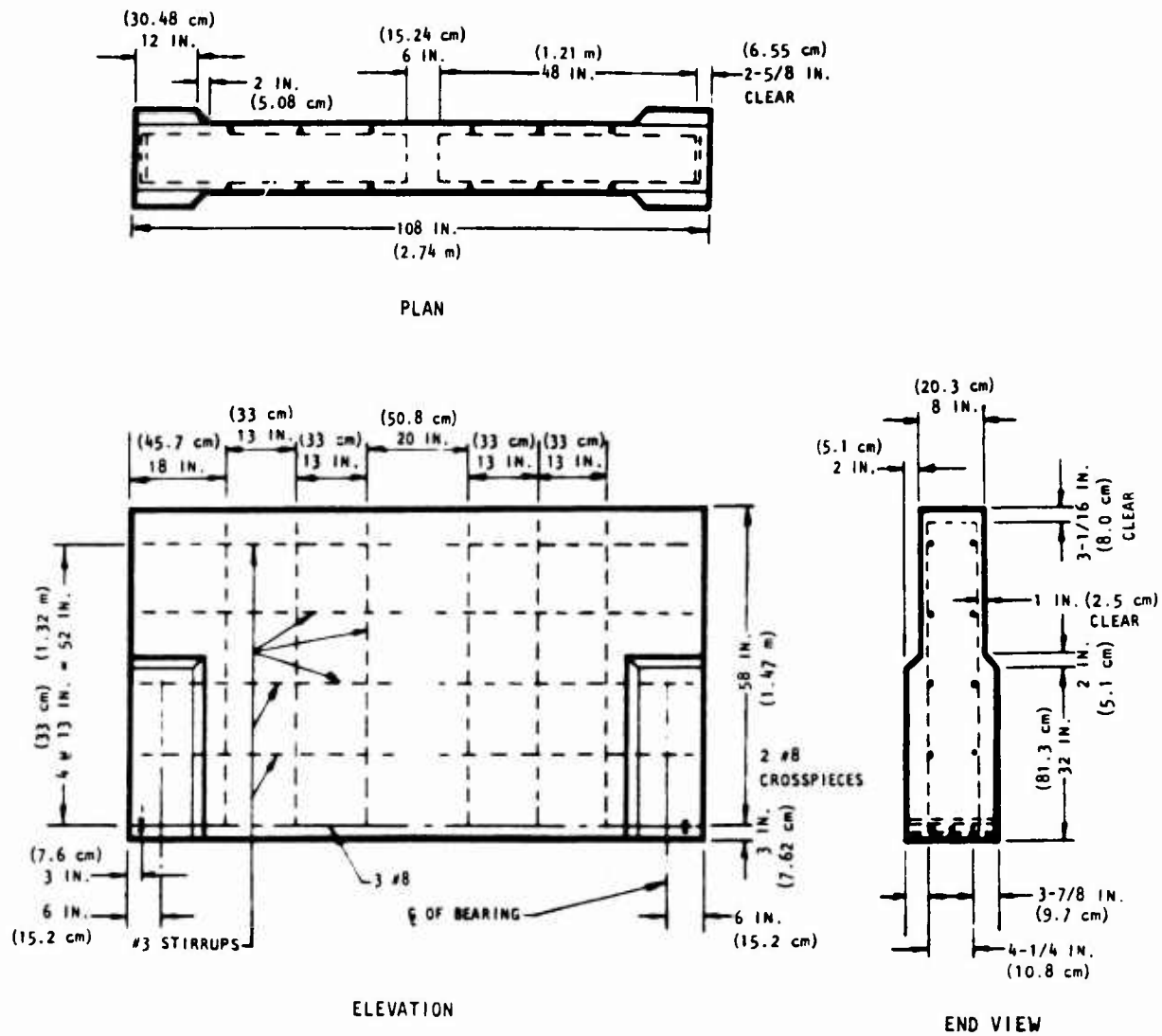
Several factors need to be included in the procedure of setting up finite element input for the present analytical model:

- a. Element size should be small enough to allow for a better description of stress and/or strain gradients.
- b. Elements should be divided to provide an area of steel not to exceed 5 percent of the area of concrete.
- c. Element width should be large enough to provide full bond at the start of loading and allow primary cracks to develop within the element (width should be greater than twice thickness of concrete cover).
- d. Integration time-step in a dynamic solution should be small enough to allow for better description of sharp rise time of the dynamic input.
- e. Element and nodal point numbers and integration time-step requirements are subject to computer budget limitations.

To illustrate the procedure followed to set up input for both static and dynamic cases, two examples were selected. Detailed description of the input will be given in the following subsections. The numerical results and discussions are included in section VI of this report.

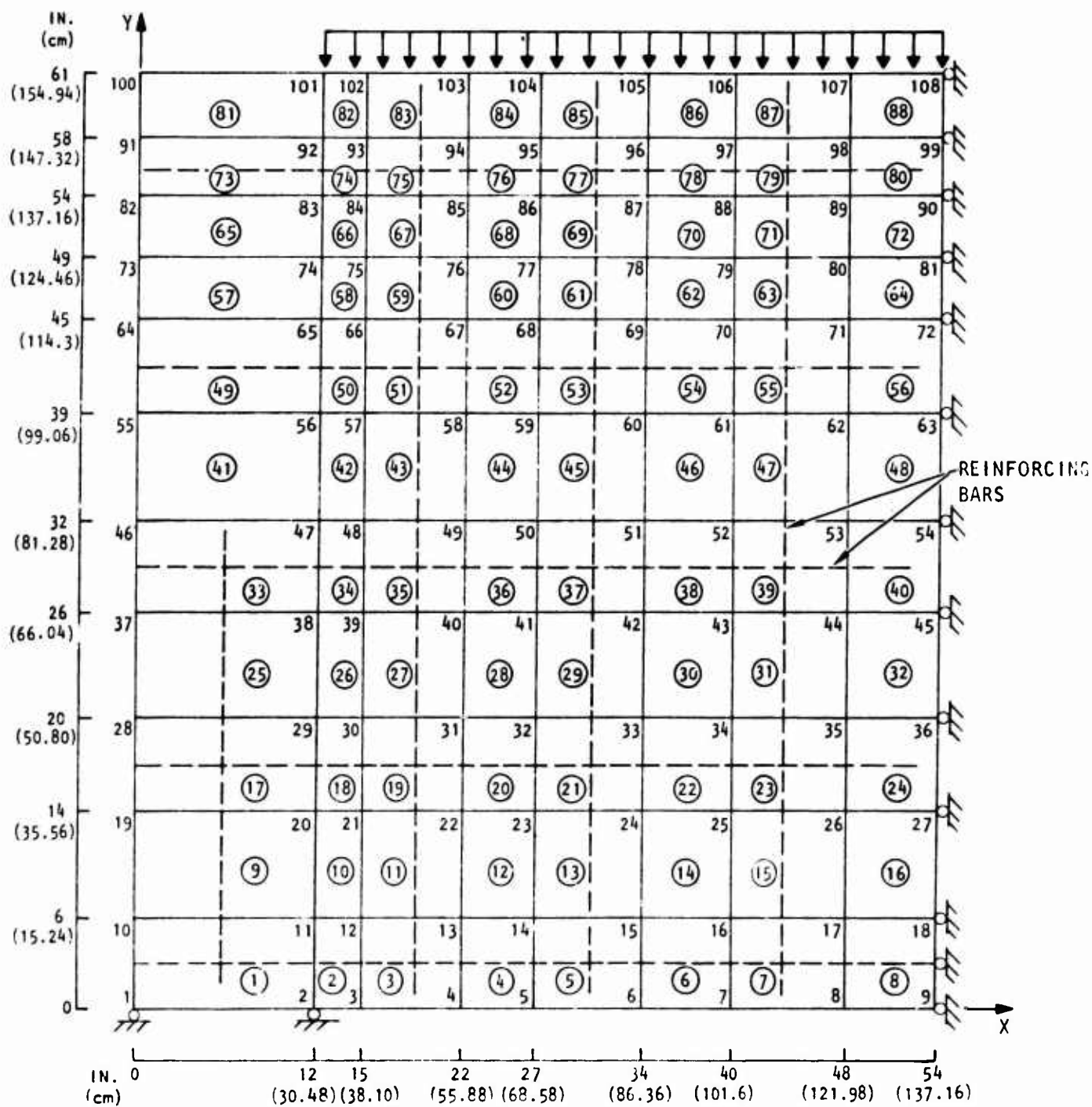
2. STATIC EXAMPLE

The geometry of beam 2S1.6-1 of static case S-1 is given in figure 109(a). The finite element mesh used in the analysis was shown in figure 56 and is reproduced as figure 109(b). Sizes and location of elements were chosen such that reinforcing bars were approximately in the center of the elements. The area of tension steel in Elements 1 through 8 is about 5 percent of the



(a) Geometry of beam

Figure 109. Static Beam 251.6-1, Case S-1 (Ref. 89)



(b) Finite element representation of beam

Figure 109. Static Beam 251.6-1, Case S-1 (Ref. 89) (concluded)

cross-sectional area of the elements (in the x-direction). Area of steel in the other two perpendicular directions ranges between 0.15 and 0.4 percent. These percentages do not exceed the 5 percent that is recommended for satisfactory distribution of steel in elements. The width of elements in the cracking zone ranges from 3 in. (7.6 cm) for Element 2 to 8 in. (20.3 cm) for Element 7. This width was dictated by the location of steel in the vertical direction and the need to monitor stress and/or strain gradient in the x-direction. The width is less than the 18 in. (45.7 cm) required for developing full bond. Therefore, the interpretation of the analytical results should account for this choice. The properties of steel and concrete were based on the data given in reference 89. Young's modulus was calculated using equation 13 as follows:

$$E = 33 W^{3/2} \sqrt{f'_c} \quad (108)$$

where

$$W = 145 \text{ lb per cubic ft (645 Kg/m}^3\text{)}$$

$$f'_c = 4160 \text{ psi (1992} \times 10^4 \text{ N/m}^2\text{)}$$

Therefore

$$E_c = 3.00 \times 10^6 \text{ psi (20685} \times 10^6 \text{ N/m}^2\text{)} \quad (109)$$

The steel directions coincide with the global directions. Therefore both NRB and DCS will be specified as zero values.

A sample of area of steel calculated for Elements 1 to 8 is given in table IX.

Table IX. Calculated Areas of Steel

Element No. A _s /A _c	1	2	3	4	5	6	7	8
x-direction	0.0330	0.05	0.050	0.050	0.050	0.050	0.0500	0.050
y-direction	0.0015	0.0	0.0040	0.000	0.004	0.000	0.0030	0.000
z-direction	0.0	0.0	0.0050	0.000	0.005	0.000	0.0045	0.000

The input parameters were:

NRB = 0 ICOMPS = 1 (for variable modulus model)
NELTYP = 3 (plane stress problem)

NSTEPS = 32 (number of static load steps)

NSPLIT = -1 (indicating strain input to material package)

The material properties in array COEFF are arranged as follows:

COEFF(1) = 3×10^6 psi ($20,685 \times 10^6$ N/m²) = E_c
COEFF(2) = 30×10^6 psi ($20,685 \times 10^7$ N/m²) = E_s
COEFF(3) = 4.16×10^3 psi ($28,683 \times 10^6$ N/m²) = f'_c
COEFF(4) = 4.8×10^4 psi ($33,096 \times 10^4$ N/m²) = f_y
COEFF(5) = 0.2 = ν
COEFF(6) = 365 psi (2517×10^3 N/m²) = f_t
COEFF(7) = -0.088 = n (slope of
cracking envelope)
COEFF(8) = 4.0 = n (dowel shear
coefficient)
COEFF(9) = 1.20×10^7 psi (8274×10^7 N/m²) = G_s
COEFF(10) = Rebonding coefficient not needed for this example
COEFF(11 through 15) = 0.0 (not necessary for this case)

3. DYNAMIC EXAMPLE

The geometry of the beam of case D-4 was given in figure 89. The finite element mesh used in the analysis was also shown in the same figure. Sizes and location of elements were chosen such that, whenever possible, reinforcing bars were approximately in the center of the elements. The width of element in the tension cracking zone along the x-axis ranges between 2.125 in. (5.4 cm) for Element 141 to 5.375 in. (13.7 cm) for Element 91. This width was dictated by the location of the vertical steel, the applied vertical loads, the location of beam supports, and the need to monitor stress and/or strain gradient in the x-direction. Since the width of the element in the cracked zone is less than the width recommended for developing full bond, the analytical model is expected to overestimate the stiffness at the early stages of the loading.

The division of the height of the beam, in the y-direction, into 10 elements was mainly dictated by budget limitations. This choice resulted in a bandwidth of 26, 176 nodal points, and 150 elements.

The area of steel in various elements is given in table X and indicates that a high steel ratio of 18 percent of cross-sectional area is used in the row of elements in the tension zone starting with Element 2 (Fig. 89).

This steel ratio is well above the 5 percent recommended for steel distribution, and the effect of its variation should be investigated in future work.

The properties of steel and concrete were based on data given in reference 91.

Since the steel directions coincide with the global directions, both NRB and DCS were specified as zero values.

The selection of integration time step Δt was guided by the rise time t_r of the dynamic input given in figure 90. A rise time of 18 msec was indicated in this figure. The time step Δt is related to the rise time t_r by the empirical relationship

Table X. Steel Ratio for Dynamic Example Case D-4

Element No.	Steel Ratio, A_s/A_c			Element No.	Steel Ratio, A_s/A_c		
	x	y	z		x	y	z
2	0.1802	0.0063	0.0206	82	0.1802	0.0039	0.0127
3 - 8	0	0.0063	0	83 - 88	0	0.0039	0
9	0.1333	0.0063	0.0254	89	0.1333	0.0039	0.0157
12	0.1802	0.0049	0.0160	92	0.1802	0.0031	0.0101
13 - 18	0	0.0049	0	93 - 98	0	0.0031	0
19	0.1333	0.0049	0.0198	99	0.1333	0.0031	0.0124
22	0.1802	0.0033	0.0105	102	0.1802	0.0053	0.0173
23 - 28	0	0.0033	0	103 - 108	0	0.0053	0
29	0.1333	0.0033	0.0130	109	0.1333	0.0053	0.0213
32	0.1802	0.0039	0.0127	112	0.1802	0.0039	0.0127
33 - 38	0	0.0039	0	113 - 118	0	0.0039	0
39	0.1333	0.0039	0.0157	119	0.1333	0.0039	0.0157
42	0.1802	0.0039	0.0127	122	0.1802	0.0039	0.0127
43 - 48	0	0.0039	0	123 - 128	0	0.0039	0
49	0.1333	0.0039	0.0157	129	0.1333	0.0039	0.0157
52	0.1802	0.0039	0.0127	132	0.1802	0.0039	0.0127
53 - 58	0	0.0039	0	133 - 138	0	0.0039	0
59	0.1333	0.0039	0.0157	139	0.1333	0.0039	0.0157
62	0.1802	0.0039	0.0127	142	0.1802	0.0039	0.0127
63 - 68	0	0.0039	0	143 - 148	0	0.0039	0
69	0.1333	0.0039	0.0157	149	0.1333	0.0039	0.0157
72	0.1802	0.0039	0.0127				
73 - 78	0	0.0039	0				
79	0.1333	0.0039	0.0157				

$$\Delta t \cong \frac{t_r}{15} \text{ to } \frac{t_r}{20} \quad (110)$$

or

$$\begin{aligned} \Delta t &= \frac{t_r}{18} \\ &= 0.001 \text{ sec} \end{aligned} \quad (111)$$

The input parameters were:

NRB = 0 ICOMPS = 1 (for variable modulus)

NELTYP = 3 (plane stress problem) NSTEPS = 50

NSPLIT = -1 (indicating strain input to material package)

The material properties in array COEFF are arranged as follows:

COEFF(1)	=	$3.74 \times 10^6 \text{ psi}$	$(25,787 \times 10^6 \text{ N/m}^2)$	=	E_c
COEFF(2)	=	$30 \times 10^6 \text{ psi}$	$(20,685 \times 10^7 \text{ N/m}^2)$	=	E_s
COEFF(3)	=	$3.26 \times 10^3 \text{ psi}$	$(22,478 \times 10^3 \text{ N/m}^2)$	=	f'_c
COEFF(4)	=	$4.58 \times 10^4 \text{ psi}$	$(31,759 \times 10^4 \text{ N/m}^2)$	=	f_y
COEFF(5)	=	0.2		=	ν
COEFF(6)	=	450 psi	$(3103 \times 10^3 \text{ N/m}^2)$	=	f_t
COEFF(7)	=	-0.138		=	η (slope of cracking envelope)
COEFF(8)	=	4.0		=	n (dowel shear coefficient)

$$C\emptyset EFF(9) = 1.2 \times 10^7 \text{ psi } (8274 \times 10^7 \text{ N/m}^2) = G_s$$

$$C\emptyset EFF 10) = 0.5 = \text{rebonding coefficient}$$

$$C\emptyset EFF 11 = 0.0 \text{ (not needed for this case)}$$

through 15)

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